An explicit construction of G_4 Fluxes in F-Theory

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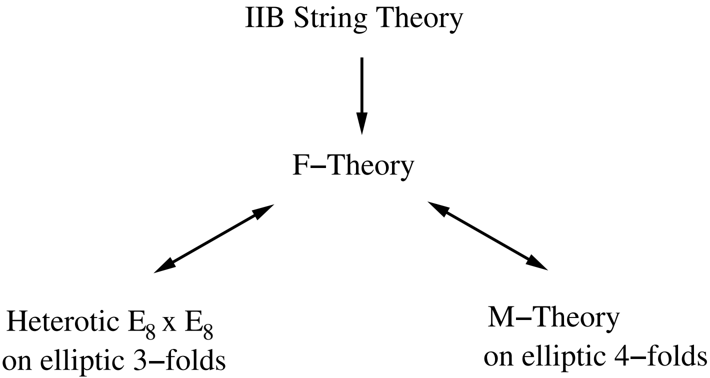
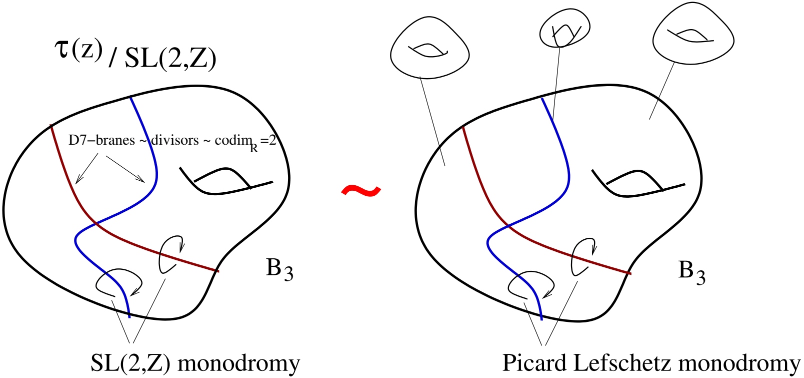
- ⋆ Introduction
- ⋆ $K3$
- ⋆ CY_4
- ⋆ Deconstruction ?

⋆ F-Theory is definded by various dualities/limits.



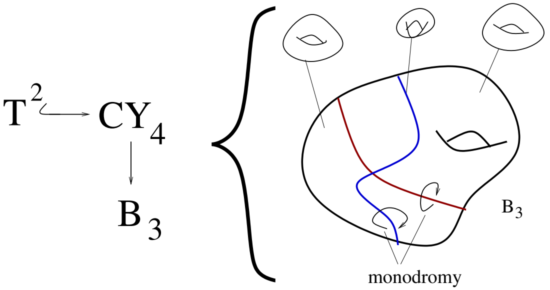
The basic idea of F-Theory

⋆ F-Theory geometrizes the $SL(2,\mathbb{Z})$ self-duality of type IIB string theory by interpreting it as the modular group of an auxiliary torus.



F-Theory and type IIB

⋆ The whole fibration has to be a Calabi-Yau 4-fold.



- ⋆ Brane stacks ∼ singularities encode non-abelian gauge groups, matter, etc...
- ⋆ More groups and reps. than in perturbative IIB string theory are possible → F-Theory GUTs .
- ⋆ There exists a (weak coupling) limit in which F-Theory reduces to type IIB.

F-Theory and M-Theory

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- ⋆ M-theory on CY_4 is dual to type IIB on B_3 in the limit of vanishing fibre volume: ‘F-theory limit’.
  This is a decompactification limit on the type IIB side !
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Fluxes

- ⋆ G_4 on CY_4 in M-Theory $\leftrightarrow$ F_2 and G_3.
- ⋆ These fluxes should not break Lorentz symmetry
  $\rightarrow$ G_4 must have 'one leg along the fibre'.
- ⋆ In homology, this means G_4 should not intersect the fibre or divisors in the base.
- ⋆ Supersymmetry demands that G_4 is of type $(2, 2)$.

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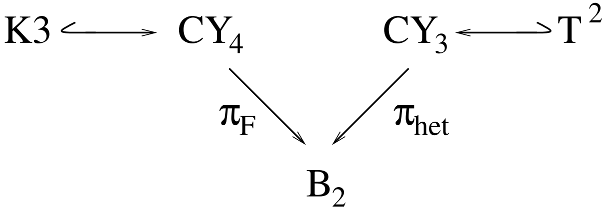

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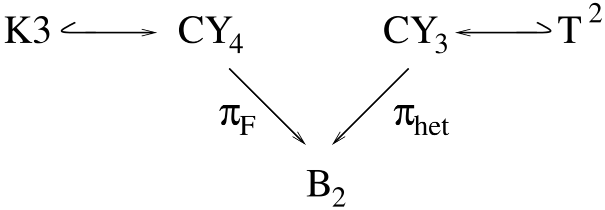
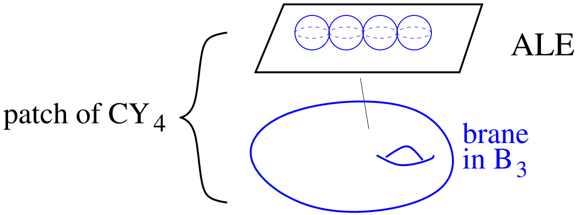


- ⋆ On CY_3 on can use the spectral cover/divisor to construct gauge bundles.
- ⋆ $\text{het}_{E_8 \times E_8} \leftrightarrow$ F-Theory duality makes similar techniques availiable in F-Theory GUTs.
- ⋆ This is used a lot in 'local' model building...

Geometry of gauge groups

- Let's focus on a stack of branes with some non-abelian gauge group on some divisor S of the base.
- Resolving the singularity, we obtain a fibration of some ALE space over S.
- Similiar in spirit to $\text{het}_{E_8 \times E_8} \leftrightarrow$ F-Theory duality: Fibration of $\mathbb{P}^1$s $\sim$ spectral cover.



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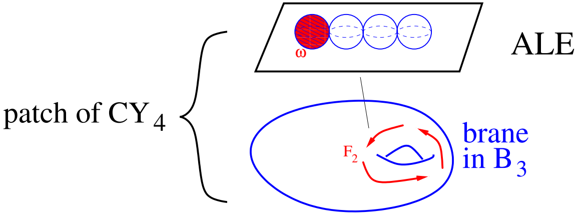

Fluxes: Can write $G_4 \sim \omega \wedge F_2$, or use heterotic analogue,...
[Donagi, Wijnholt; Marsano et al.;...]
More sophisticated *global* approach: [Grimm et al.]

Our work

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Why is this interesting ?

- ⋆ fluxes are needed for chirality.
- ⋆ needed for moduli stabilization.
- ⋆ want to compute D3-brane (M2-brane) tadpole.

our favourite toy: K3

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Elliptic fibrations can be described by a Weierstrass model:

$$y^2 = x^3 + f_4 x z^4 + g_6 z^6$$

in a $\mathbb{P}^2_{1,2,3}(z,x,y)$ bundle over the base B (for K3: $B = \mathbb{P}^1$).

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(This is well-known in general. We want to start from the Weierstrass model, however !)

K3: the trick

The Weierstrass model can be reparametrized and rewritten as

$$Y_- Y_+ + z^6 a_6 = XQ\,.$$

where $$Y_\pm = y \pm \tfrac{1}{2} z^3 a_3 \quad , \quad Q = X(X + \tfrac{1}{4} z^2 b_2) + \tfrac{1}{2} z^4 b_4\,.$$

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Adjusting f_4, g_6 such that $a_6 \equiv 0$, our $K3$ gains an extra cycle

$$\sigma = \{Y_\pm = 0\} \cap \{X = 0\}\,.$$

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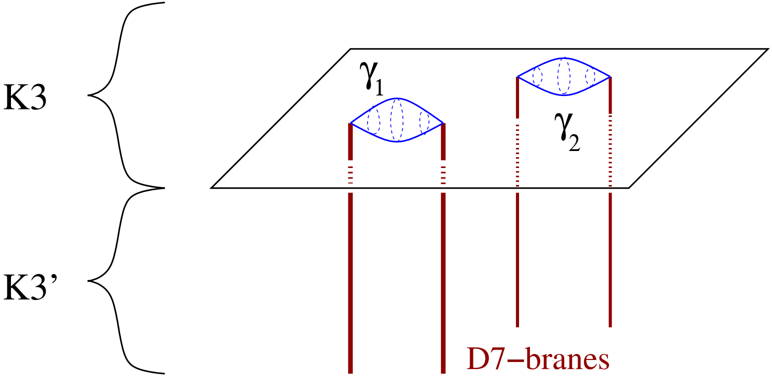
We find a new integral $(1,1)$ cycle

$$\gamma_\pm = \sigma_\pm - \beta - 2\phi\,.$$

which does not intersect base or fibre !

K3: interpretation in weak coupling limit



In the weak coupling limit, a configuration for which $a_6 \equiv 0$ can be achived by putting four $D7$-branes such that

$$\int_{\gamma_1} \Omega^{2,0}(K3) - \int_{\gamma_2} \Omega^{2,0}(K3) = 0\,.$$

Hence the $(1,1)$ cycle γ is given by $\gamma_1 - \gamma_2$.

Fourfolds

For fourfolds, we can do a similar trick. We simply demand that a_6 factorizes, i.e. $a_6 = \rho\tau$:

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Note: if ρ or τ is a constant, the flux is zero.

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Note: The cycles exists throughout moduli space, they just cease to be holomorphic. This way of discussing moduli stabilization is not availiable in the standard case of threefolds !

Tadpole

- We can compute the D3-brane tadpole

$$-\frac{1}{2}\int_{CY_4} G_4 \wedge G_4 = -\int_{B_3} c_1(B_3) \wedge [\rho][\tau]$$

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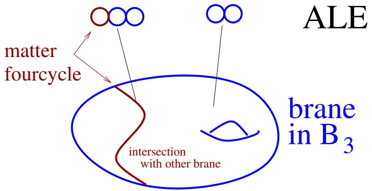
$$-\frac{1}{2}\int_{CY_4} G_4 \wedge G_4 = -\int_{B_3} c_1(B_3) \wedge [\rho][\tau]$$

The constraint $a_6 = \rho\tau$ is well known from [Collinucci, Denef, Esole] for fluxed D7 branes.

Using this identification, we find a match with the type IIB result for the tadpole in the weak coupling limit !

Chirality

In F-Theory, matter arises from intersections of branes.
Over these loci, the singularity enhances.



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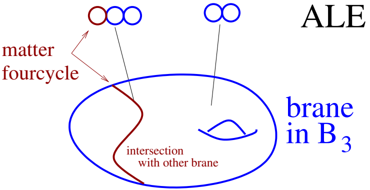

- ⋆ We can construct similar fluxes in situations with intersecting branes/ non-abelian gauge groups.
- ⋆ We can then compute chirality by integrating this flux over matter four-cycles [Hayashi et al.;Donagi, Wijnholt;...]

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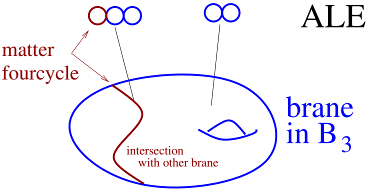

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We can have chirality without destroying the gauge symmetry !

D7 deconstruction

Algebraic cycles similarly appear in the description of D7-branes via ‘deconstruction’, i.e. as coherent sheaves defined by an exact sequence [Denef, Collinucci, Esole]:

$$0 \to \bar{D}9 \xrightarrow{T} D9 \to D7 \to 0\,.$$

- $D9$, $\bar{D}9$ are just vector bundles on B_3.
- Where the tachyon matrix T is invertible, the map T is onto. The D7 branes are at $\det(T) = 0$, i.e. they are determinantal varieties.
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- The construction works such that the extra algebraic cycles are carrying fluxes.

In this description the locus of the *D7*-brane and the fluxes on its worldvolume form one object.

F-theory deconstruction ?

We can write an exact sequence of coherent sheaves

$$0 \to \mathcal{E} \xrightarrow{M} \mathcal{F} \to \mathcal{G} \to 0\,,$$

which encodes our Calabi-Yau fourfold together with G_4 as $\mathcal{G}$.

- ⋆ The matrix M is given by

$$M = \begin{pmatrix} 0 & X & \rho & Y_+ \\ -X & 0 & -Y_- & \tau \\ -\rho & Y_- & 0 & Q \\ -Y_+ & -\tau & -Q & 0 \end{pmatrix}$$

- ⋆ Its Pfaffian gives CY_4 in terms of the Weierstrass model. On CY_4, the rank of M goes down by two, defining a vector bundle V of rank two.

$$G_4 \sim c_2(V)$$

Conclusions

- ⋆ We have found explicit realizations of G_4 corresponding to F_2 on branes in terms of algebraic cycles in F-theory.
- ⋆ We can compute the D3-tadpoles and chirality indices. They match with the corresponding type IIB results in the weak coupling limit.
- ⋆ This was used by [Weigand et al] to construct chirality inducing flux in $SU(5)$ models.

It seems there is much more to uncover:

- ⋆ What is the precise relation to the constructions of [Grimm et al.] or [Marsano er al.]?
- ⋆ Can G_4 together with the elliptic CY_4 be described as a coherent sheaf ?
- ⋆ What is the physical meaning of $M, \mathcal{E}, \mathcal{F}$?