Exercise 1: Relativistic Point particles

Consider the action of a relativistic point particle moving in space-time with background metric $g_{\mu\nu}$

$$S_{NG} = -m \int d\mathbf{s} = -m \int d\mathbf{t} \sqrt{g_{\mu\nu}} \frac{\partial X^{\mu}}{\partial t} \frac{\partial X^{\nu}}{\partial t} \tag{1}$$

- a) Show that the action does not depend on the parametrization of the worldline !
- **b**) What is the non-relativistic limit of this action ?

Exercise 2: The 'Polyakov' action

Consider the action,

$$S_P = \frac{m}{2} \int dt \sqrt{h_{tt}} \left(h_{tt}^{-1} g_{\mu\nu} \frac{\partial X^{\mu}}{\partial t} \frac{\partial X^{\nu}}{\partial t} - 1 \right) , \qquad (2)$$

where $h_{tt}(t)$ is a metric on the worldline. As this action contains no derivatives of h_{tt} , h_{tt} is not a dynamical field and we can integrate it out, i.e. solve its equations of motion and reinsert the solution back into the action.

a) Show that this reproduces the action S_{NG} in eq. (1) !