Exercise Sheet 10 (String theory, LVA Nr. 136.005) due June 20

Exercise 16: Consider the three-dimensional round sphere on which the string is defined (other directions are considered trivial) as follows:

a) the metric is given by:

$$G_{\mu\nu}\, dX^\mu dX^\nu = R^2 \Big(d\psi^2 + \sin^\psi \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \Big)$$

b) the antisymmetric B-field is given by

$$B_{\mu\nu}\, dX^\mu \wedge dX^\nu = R^2 \Big(\psi - sin\psi \cos\psi \Big) \sin\theta \, d\theta \wedge d\phi$$

- Show that
$$\frac{1}{4\pi^2 l_s^2} \int\limits_{S^3} H \, \in \, \mathbb{Z}, \quad H = dB \tag{1}$$
and $R^2/l^2 = k \in \mathbb{Z}$.
- Show that $\beta^G = \beta^B = 0$ and the central charge is
$$c = \beta^\Phi = 3 - \frac{6}{k} + \mathcal{O}(1/k^2)$$

Exercise 17: Show that varying the effective closed string action (taken from the lecture) and using

$$\delta R = (R_{\mu\nu} - \nabla_\mu \nabla_\nu)\delta G^{\mu\nu} + G_{\mu\nu}\Box\delta G^{\mu\nu},$$

one obtains the beta-functions as the equations of motion.