Exercise 3: Weyl rescaling in two dimensions

Show that in two dimensions, the effect of a Weyl rescaling $g_{mn} \to g_{mn} e^{2\Lambda}$ on the Einstein-Hilberg action is

$$\int d^2 \sigma \sqrt{-\det(g)} R \to \int d^2 \sigma \sqrt{-\det(g)} \left(R - 2D^2 \Lambda \right) \,, \tag{1}$$

where R is the Ricci scalar and D is the covariant derivative with respect to the Levi-Civita connection.

Exercise 4: Weyl rescaling and the energy-momentum tensor

Show that invariance of any action under Weyl rescaling implies the vanishing of the trace of the enery-momentum tensor !