Exercise Sheet 5 (String Theory, LVA Nr. 136.005) due 2nd of May

Exercise 9: The Virasoro algebra

- Show that the Virasoro generators,

$$L_n = -\frac{1}{2}\sum_k \alpha_k \cdot \alpha_{n-k} \tag{1}$$

satisfy the classical Poisson brackets

$$\{L_m, L_n\}_{PB} = i(m-n)L_{m+n}\,. \tag{2}$$

- Consider 2π periodic functions $f(\theta)$. Reparametrizations

$$\theta \to \theta + a(\theta) \tag{3}$$

are generated by

$$D = ia(\theta)\frac{\partial}{\partial\theta}\,. \tag{4}$$

Show that the elements of the basis of such generators,

$$ie^{-in\theta}\frac{\partial}{\partial\theta}\,, \tag{5}$$

form the same Virasoro algebra, eq.(2). Here, there is commutator of two such transformations acting on an arbirary function instead of the Poisson bracket.