

Exercise Sheet 9 (String theory, LVA Nr. 136.005) due June 06

Exercise 13: Using the standard oscillator (anti-)commutation relations and the expressions

$$Q_{BRST} = \sum_{-\infty}^{\infty} : L_m^X c_{-m} : - \frac{1}{2} \sum_{-\infty}^{\infty} (n-m) : c_{-m} c_{-n} b_{n+m} : - c_0, \quad U = \sum_{-\infty}^{\infty} c_{-m} b_m$$

,

- Calculate $[Q_{BRST}, U]$ and $\{Q_{BRST}, b_m\}$.
- Using the above results, show that from $Q_{BRST}^2 = 0$ it follows that the total central charge is vanishing (d=26).

Exercise 14: Show that the Polyakov action (including ghosts) is invariant with respect to the BRST transformations (i.e. its variation is proportional to a total derivative).

Exercise 15: (Not compulsory, but gives a bonus) Consider closed string with

$$L_0 + \tilde{L}_0 = \frac{\alpha'}{2} p^2 + N + \tilde{N}, \quad \mathbf{N}_{bc} = \sum_{-\infty}^{\infty} n : b_n c_{-n} :, \quad (1)$$

where

$$\mathbf{N} = \sum_{-\infty}^{\infty} \alpha_{\mu n} \alpha_{-n}^{\mu}, \quad \tilde{\mathbf{N}} = \sum_{-\infty}^{\infty} \tilde{\alpha}_{\mu n} \tilde{\alpha}_{-n}^{\mu}.$$

Compute the quantities

$$\text{Tr} \left(q^{\mathbf{N}} \bar{q}^{\tilde{\mathbf{N}}} \right) \text{ and } \text{Tr} \left(q^{\mathbf{N}_{bc}} \bar{q}^{\tilde{\mathbf{N}}_{bc}} \right), \quad q = e^{2\pi i \tau}. \quad (2)$$

Give an interpretation of the results.