Fuzzy extra dimensions and particle physics models

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Motivation

Motivation

Unification of fundamental interactions

Exciting proposal: Extra dimensions may exist in nature

- Unification might be achieved in higher dimensions.

Upon Compactification and subsequent Dimensional Reduction to four dimensions

- Try to make contact with low energy phenomenology.

Motivation

Starting from an $\mathcal{N}=1$ susy theory in 10D and performing a toroidal reduction $\rightsquigarrow$ $\mathcal{N}=4$ susy in 4D (no chiral fermions).

In order to achieve $\mathcal{N}=1$ susy in four dimensions...

- ...use appropriate manifolds to describe the extra dimensions (e.g. Calabi-Yau, $SU(3)$-structure manifolds).
- ...use orbifolds.
  - From a $4D$ perspective: use orbifold techniques to project $\mathcal{N}=4$ SYM to a theory with less susy (Kachru-Silverstein '98).

Here: starting from $\mathcal{N}=4$ SYM study the possibility to reveal vacua which...

- ...dynamically develop fuzzy extra dimensions.
- ...could be realistic.

Motivation

Important results from the dimensional reduction of higher-dimensional gauge theories on fuzzy spaces (Aschieri-Madore-Manousselis-Zoupanos '04,'05):

- Appearance of non-abelian gauge theories in four dimensions starting from an abelian gauge theory in higher dimensions.
- Renormalizability of the theory both in four as well as in higher dimensions.

Difficulty: Chiral Fermions.

In order to further justify the renormalizability and try to obtain chiral theories...

...Reverse approach (Aschieri-Grammatikopoulos-Steinacker-Zoupanos '06):

- Start with 4D renormalizable gauge theory with appropriate content of scalars.
- Find minima of the potential and determine vacua where fuzzy extra dimensions are dynamically generated.

Essentially a bottom-up approach...

Related idea: "deconstruction" of dimensions
(Arkani-Hamed-Cohen-Georgi '01)

- Start with a four-dimensional, renormalizable theory.
- Build the extra dimensions dynamically.

Inclusion of fermions (Steinacker-Zoupanos '07, A.C.-Steinacker-Zoupanos '09).

- For a single fuzzy sphere $S_N^2 \subset \mathbb{R}^3 \rightarrow$ tangent space group is $SO(3)$ and differential calculus is $3D$, odd-dimensional $\rightsquigarrow$ not a good candidate for chirality.
- But...the above mechanism can be also used to dynamically generate other fuzzy spaces.
- Simplest good candidate: a product of two fuzzy spheres $S_N^2 \times S_N^2 \subset \mathbb{R}^6$, with tangent space group $SO(6)$ and $6D$ differential calculus.

The analysis of the fermionic spectrum showed:

- Models with mirror fermions, i.e. fermions with the same quantum numbers as their ordinary counterparts but with opposite chirality,
  i.e. two chiral sectors with opposite chirality.
- The two sectors can be exactly separated.
- The mirrors may have larger mass than the observable ones at low energy $\rightsquigarrow$ the electroweak breaking has to be studied further in this context.

But...the chiral character of the weak force is a central ingredient in any successful particle theory.
Further step in order to achieve chirality: Perform orbifold projection...

Orbifolds of $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ SYM theory

- Gauge group: $SU(3N)$.
- Spectrum:
  - Gauge fields $A_\mu, \mu = 1, \ldots, 4$
  - 6 real scalars ϕ_a (or 3 complex $\phi_i, i = 1, 2, 3$)
  - 4 Majorana fermions $\psi_p, p = 1, \ldots, 4$
- Also, global $SU(4)_R$ R-symmetry:
  - gauge fields $\rightarrow$ singlets
  - real scalars $\rightarrow$ in **6**
  - fermions $\rightarrow$ in **4**
- Interactions encoded in the superpotential:

$$W_{\mathcal{N}=4} = Tr(\epsilon_{ijk}\Phi^i\Phi^j\Phi^k),$$

where Φ^i the three adjoint chiral supermultiplets.

Orbifolds

Consider the discrete group $\mathbb{Z}_3$ as subgroup of $SU(4)_R$

- maximal embedding in $SU(4)_R \rightsquigarrow$ no susy
- embedding in $SU(3)$ subgroup $\rightsquigarrow \mathcal{N} = 1$ susy
- embedding in $SU(2)$ subgroup $\rightsquigarrow \mathcal{N} = 2$ susy

Here we discuss the case of $\mathcal{N} = 1$ models.

$\mathbb{Z}_3$ acts non-trivially on the various fields, depending on their transformation properties under the R-symmetry and the gauge group.

Orbifold projection: keep the fields which are invariant under the (combined) $\mathbb{Z}_3$ action.

The projected theory has the following spectrum:

- Gauge group: $SU(N) \times SU(N) \times SU(N)$.
- Complex scalars and fermions ($\mathcal{N} = 1$ susy):

$$(N, \overline{N}, 1) + (\overline{N}, 1, N) + (1, N, \overline{N}).$$

  Chiral representations...
- Three families.

The projected superpotential has the same form as before, encoding the interactions among the surviving fields in the projected theory.

Search for vacua of the projected theory which can be interpreted as dynamically generated fuzzy extra dimensions.

Dynamical generation of (twisted) fuzzy sphere

(A.C.-Steinacker-Zoupanos '10)

The mechanism...

The (scalar) potential of the projected theory is

$$V_{\mathcal{N}=1}(\phi) = \frac{1}{4} Tr([\phi_i, \phi_j]^\dagger [\phi_i, \phi_j]).$$

Clearly there is no vacuum with non-vanishing commutators.

Required modification: add $\mathcal{N} = 1$ Soft Susy Breaking (SSB) terms

$$V_{SSB} = \frac{1}{2}\delta_{ij}(\phi_i)^\dagger \phi_j + \frac{1}{2}\epsilon_{ijk}\phi_i\phi_j\phi_k + h.c.$$

Soft terms: explicit susy breaking terms, they do not introduce quadratic divergencies
(scalar mass terms, trilinear scalar couplings, gaugino masses).
SSB potential is necessary for the theory to have a chance to become realistic.

Vacuum

The full potential is:

$$V = V_{\mathcal{N}=1} + V_{SSB} + V_D,$$

where V_D includes the D-terms, and it is non-negative.

The vacuum of the model is given by the minimum of the potential.

The potential can be brought in the form:

$$V = \frac{1}{4} F_{ij}^{\dagger} F_{ij} + V_D,$$

where we have defined

$$F_{ij} = [\phi_i, \phi_j] - i\epsilon_{ijk}(\phi_k)^{\dagger}.$$

Vacuum

The minimum is obtained when the following relations are satisfied,

$$
\begin{aligned}
[\phi_i, \phi_j] &= i\epsilon_{ijk}(\phi_k)^\dagger, \\
\phi_i(\phi_i)^\dagger &= R^2.
\end{aligned}
$$

These relations define the twisted fuzzy sphere and they are compatible with the orbifold projection.
It is related to the ordinary fuzzy sphere via

$$\phi_i = \Omega\, \tilde{\phi}_i,$$

for some $\Omega \neq 1$ which satisfies $\Omega^3 = 1, \quad [\Omega, \phi_i] = 0, \quad \Omega^\dagger = \Omega^{-1}$ and $(\tilde{\phi}_i)^\dagger = \tilde{\phi}_i$.

Then $[\tilde{\phi}_i, \tilde{\phi}_j] = i\epsilon_{ijk}\tilde{\phi}_k, \quad \tilde{\phi}_i\tilde{\phi}_i = R^2 \quad \rightsquigarrow$ the ordinary fuzzy sphere.

A solution leading to such a vacuum may be expressed as

$$\phi_i = \Omega \left(\mathbb{1}_3 \otimes (\lambda_i^{(N-n)} \oplus 0_n) \right),$$

$\lambda_i^{(N-n)}$: the generators of the corresponding rep. of $SU(2)$.

- The gauge symmetry $SU(N)^3$ is broken down to $SU(n)^3$.
- Moreover, there exists a finite Kaluza-Klein tower of massive states.
- Therefore the vacuum can be interpreted as spontaneously generated fuzzy extra dimensions.
- At this intermediate scale the theory behaves as a higher dimensional theory.
- The fluctuations of the vacuum correspond to the internal components of the higher-dimensional gauge field, which describe in a well-defined way the gauge theory on the (twisted) fuzzy sphere,

$$\phi_i = \lambda_i + A_i \rightsquigarrow \text{covariant coordinates}$$

Particle physics models

$SU(3)^3$ model

- Let us consider the case: $N = n + 3$
- The relevant decomposition of each $SU(N)$ factor is:

$$SU(N) \supset SU(n) \times SU(3) \times U(1)$$

- Then the decomposition of the representation $(N, \overline{N}, 1) + (\overline{N}, 1, N) + (1, N, \overline{N})$ of $SU(N)^3$ is:

$$SU(n) \times SU(n) \times SU(n) \times SU(3) \times SU(3) \times SU(3)$$
$$(n, \overline{n}, 1; 1, 1, 1) + (1, n, \overline{n}; 1, 1, 1) + (\overline{n}, 1, n; 1, 1, 1) +$$
$$+(1, 1, 1; 3, \overline{3}, 1) + (1, 1, 1; 1, 3, \overline{3}) + (1, 1, 1; \overline{3}, 1, 3) +$$
$$+(n, 1, 1; 1, \overline{3}, 1) + (1, n, 1; 1, 1, \overline{3}) + (1, 1, n; \overline{3}, 1, 1) +$$
$$+(\overline{n}, 1, 1; 1, 1, 3) + (1, \overline{n}, 1; 3, 1, 1) + (1, 1, \overline{n}; 1, 3, 1).$$

- Applying the above mechanism we can write down the vacuum solution:

$$\phi_i = \Omega\,[\mathbb{1}_3 \otimes (\lambda_i^{(N-3)} \oplus 0_3)],$$

interpreted in terms of twisted fuzzy sphere $\tilde{S}^2_{N-3}$.

- This vacuum solution amounts to vevs for the fields:

$$\langle(n, \overline{n}, 1; 1, 1, 1)\rangle, \langle(1, n, \overline{n}; 1, 1, 1)\rangle, \langle(\overline{n}, 1, n; 1, 1, 1)\rangle.$$

- The gauge group is broken down to the "trinification" group

$$SU(3)_c \times SU(3)_L \times SU(3)_R$$

(Glashow '84)

- The matter fields transform as $(\overline{3}, 1, 3) + (3, \overline{3}, 1) + (1, 3, \overline{3})$ $\rightsquigarrow$ chiral fermions.

The quarks of the first family transform under the gauge group as

$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \sim (3, \overline{3}, 1),$$

$$q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \sim (\overline{3}, 1, 3),$$

and the leptons transform as

$$\lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix} \sim (1, 3, \overline{3}).$$

- Concerning the rest of the fermions we can form invariants (Yukawas)...
  - $(1, n, \overline{n}; 1, 1, 1)\langle(n, \overline{n}, 1; 1, 1, 1)\rangle(\overline{n}, 1, n; 1, 1, 1)$ + cyclic
  - $\underbrace{(\overline{n}, 1, 1; 1, 1, 3)}_{\text{fermion}}\underbrace{\langle(n, \overline{n}, 1; 1, 1, 1)\rangle}_{\text{boson}}\underbrace{(1, n, 1; 1, 1, \overline{3})}_{\text{fermion}}$ etc.

$\rightsquigarrow$ finite Kaluza-Klein tower of massive fermionic modes.

- Also: the 1-loop β-function of the model is zero.
- Phenomenological studies on $SU(3)^3$ have shown that it can be rendered finite even to all orders and it can lead to predictions (Ma-Mondragon-Zoupanos '04).

Similarly, other particle physics models can be constructed.

- $SU(4)_c \times SU(2)_L \times SU(2)_R$ (Pati-Salam gauge group)
- matter fields in $(4,1,2) + (\bar{4},2,1) + (1,2,2)$
- quarks and leptons:

$$f \sim (4,2,1) \quad = \quad \begin{pmatrix} d & u \\ d & u \\ d & u \\ e & \nu \end{pmatrix},$$

$$f^c \sim (\bar{4},1,2) \quad = \quad \begin{pmatrix} d^c & d^c & d^c & e^c \\ u^c & u^c & u^c & \nu^c \end{pmatrix},$$

- $SU(4)^3$ with matter fields in $(4,1,\bar{4}) + (\bar{4},4,1) + (1,\bar{4},4)$
- quarks and leptons:

$$f = \begin{pmatrix} d & u & y & x \\ d & u & y & x \\ d & u & y & x \\ e & \nu & a & v \end{pmatrix} \sim (4,\bar{4},1), \quad f^c = \begin{pmatrix} d^c & d^c & d^c & e^c \\ u^c & u^c & u^c & \nu^c \\ y^c & y^c & y^c & a^c \\ x^c & x^c & x^c & v^c \end{pmatrix} \sim (\bar{4},1,4).$$

Further breaking

Having obtained e.g. $SU(3)^3$, it can be subsequently treated as an ordinary *GUT* and its spontaneous breakdown can be studied (Babu-He-Pakvasa '86, Lazarides-Panagiotakopoulos '93, Ma et.al. '04).

Here:

- Study whether the breaking of $SU(3)^3$ to the MSSM and furthermore to the $SU(3) \times U(1)_{em}$ can be performed in the above spirit.
- Require that the breaking takes place...
  - ...without adding any additional supermultiplets.
  - ...at one step, i.e. without breaking first to an intermediate gauge group.

- The field content consists of 3 families in

$$(\overline{3},1,3)+(3,\overline{3},1)+(1,3,\overline{3})$$

- The superpotential is

$$W^{(proj)}_{\mathcal{N}=1}=YTr(\lambda q^c q)+Y'\epsilon_{ijk}\epsilon_{abc}(\lambda_{ia}\lambda_{jb}\lambda_{kc}+q^c_{ia}q^c_{jb}q^c_{kc}+q_{ia}q_{jb}q_{kc}).$$

- The scalar potential includes the corresponding SSB terms
- We can give vevs to neutral directions of

$$\lambda=\begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix},$$

i.e. ν,ν^c,S (GUT breaking) and N,N^c (EW breaking).

- In order to apply our mechanism we need trilinear terms involving the fields which can acquire vev.
- These terms exist and they are: $\nu\nu^c N^c$ and $NN^c S$.

Relation to twisted fuzzy sphere:

- Transform the lepton matrix: $\lambda'^i = \Omega_3 \lambda^i$, $\Omega_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$
- Then:

$$\lambda' = \begin{pmatrix} E & N^c & e \\ \nu^c & e^c & S \\ N & E^c & \nu \end{pmatrix}.$$

- Vacuum Solution (superscripts are family indices):

$$\lambda'^1 = \begin{pmatrix} 0 & k_1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda'^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & k_2 \\ 0 & 0 & 0 \end{pmatrix}, \lambda'^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_3 & 0 & 0 \end{pmatrix}.$$

$\rightsquigarrow$ vevs to N, N^c and S.

The above matrices satisfy: $[\lambda'^i, \lambda'^j] = ih_{ijk}(\lambda'^k)^\dagger$,
where we have defined: $h_{ijk} \equiv \frac{k_i k_j}{k_k}\epsilon_{ijk}$.

$\rightsquigarrow$ Twisted fuzzy sphere with more than one scales included...
At least 2 needed anyway for GUT and EW symmetry breaking.

Different transformation of the lepton matrix and the same vacuum $\rightsquigarrow$ vevs to ν, ν^c and N^c.

Therefore all the neutral directions of λ acquire vev $\rightsquigarrow$
Spontaneous symmetry breaking through twisted fuzzy spheres.

Conclusions

- Extra dimensions serve as an arena for unification of fundamental interactions.
- The attempt to make contact between physics studied in accelerators and fundamental theories in higher dimensions is still under way.
- We show that within a four-dimensional and renormalizable field theory, fuzzy extra dimensions can be generated dynamically...
- ...leading to low-energy models which have phenomenological relevance.
- Using orbifold techniques we build chiral unification models with fuzzy extra dimensions.
- The spontaneous symmetry breaking down to the MSSM and $SU(3)_c \times U(1)_Q$ is understood in this framework.

Conclusions

Prospects of further work:

- Investigate the higher-dimensional origin of the soft supersymmetry breaking sector - Identify the resulting soft terms with those of the MSSM.
- More detailed study of the $SU(3)^3$ model - Effect of the Kaluza-Klein tower on the low-energy phenomenology? - Finiteness? - Predictions?
- Further study of Pati-Salam vacua?
- Inclusion of gravity - Supergravity compactifications with fuzzy extra dimensions?
- Relation of non-commutativity to non-geometric backgrounds?