Bayrischzell workshop

Noncommutativity and Physics: Spacetime Quantum Geometry

Preliminary results for neutrino self energy in θ-exact covariant NCFT

Amon Ilakovac

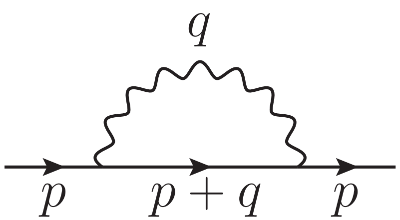
in colaboration with

R. Horvat, D. Kekez, P. Schupp, J. Trampetić
and J. You

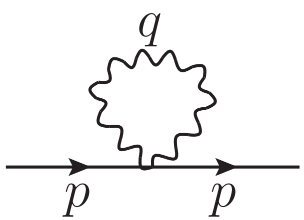
CONTENTS

Problem

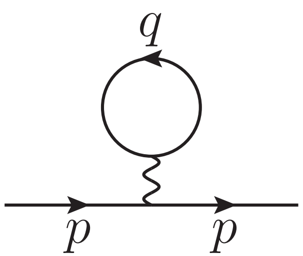
Evaluation of the neutrino self energy in the θ exact $U(1)$ noncomutative field theory with vertices obtained using Seiberg-Witten map.



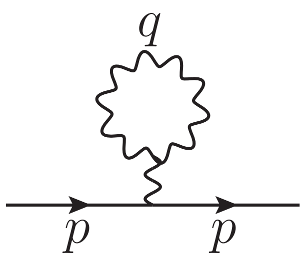
Σ_1



Σ_2

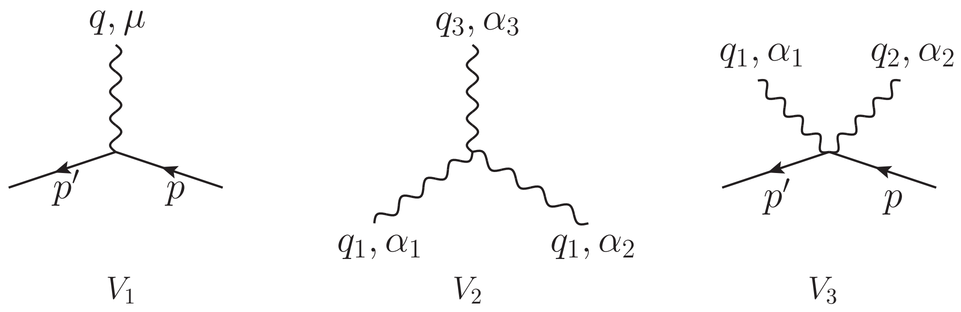


Σ_3



Σ_4

Vertices



$q = p - p'$ $\quad$ q_1, q_2, q_3 *incoming*

$$V_1^{\mu} = -iF(q,p)[\gamma^{\mu} q\theta p + \not{p}\tilde{q}^{\mu} - \not{q}\tilde{p}^{\mu}]P_{L,R}, \qquad F(q,p) = \frac{\sin\frac{1}{2}q\theta p}{\frac{1}{2}q\theta p}$$

$$\begin{aligned}
V_2^{\alpha_1\alpha_2\alpha_3} &= -2\sin\frac{1}{2}q_1\theta q_2[(q_1-q_2)^{\alpha_3}g^{\alpha_1\alpha_2} + (q_2-q_3)^{\alpha_1}g^{\alpha_2\alpha_3} + (q_3-q_1)^{\alpha_2}g^{\alpha_3\alpha_1}] \\
&- 2F(q_1,q_2)\Big[\theta^{\alpha_1\alpha_2}(q_2q_3q_1^{\alpha_3} - q_1q_3q_2^{\alpha_3}) + \theta^{\alpha_1\alpha_3}(q_2q_3q_1^{\alpha_2} - q_1q_2q_3^{\alpha_2}) \\
&\quad +\theta^{\alpha_2\alpha_3}(q_1q_3q_2^{\alpha_1} - q_1q_2q_3^{\alpha_1}) \\
&- g^{\alpha_1\alpha_2}(q_2^2\tilde{q}_1^{\alpha_3} + q_1^2\tilde{q}_2^{\alpha_3}) - g^{\alpha_1\alpha_3}(q_1^2\tilde{q}_3^{\alpha_2} + q_3^2\tilde{q}_1^{\alpha_2}) - g^{\alpha_2\alpha_3}(q_3^2\tilde{q}_2^{\alpha_1} + q_2^2\tilde{q}_3^{\alpha_1}) \\
&+ q_3^{\alpha_3}(\tilde{q}_2^{\alpha_1}q_3^{\alpha_2} + \tilde{q}_1^{\alpha_2}q_3^{\alpha_1}) + q_2^{\alpha_2}(\tilde{q}_1^{\alpha_3}q_2^{\alpha_1} + \tilde{q}_3^{\alpha_1}q_2^{\alpha_3}) + q_1^{\alpha_1}(\tilde{q}_2^{\alpha_3}q_1^{\alpha_2} + \tilde{q}_3^{\alpha_2}q_1^{\alpha_3})\Big]
\end{aligned}$$

$$
\begin{aligned}
V_3^{i_1 i_2}(p_1,p_2,q_1,q_2) &= 4i\frac{\sin\frac{p_1\wedge q_1}{2}\sin\frac{p_2\wedge q_2}{2}}{p_1\wedge q_1}\tilde{q}_1^{i_1}\gamma^{i_2} - 4i\frac{\sin\frac{p_1\wedge q_1}{2}\sin\frac{p_2\wedge q_2}{2}}{p_2\wedge q_2}\tilde{q}_2^{i_2}\gamma^{i_1}\\
&- 2i\frac{\sin\frac{q_1\wedge q_2}{2}\sin\frac{p_1\wedge p_2}{2}}{p_1\wedge p_2}(2\gamma^{i_2}\tilde{p}_2^{i_1} - \not{p}_2\theta^{i_1 i_2}) - 4i\frac{\sin\frac{p_1\wedge q_1}{2}\sin\frac{p_2\wedge q_2}{2}}{p_1\wedge q_1 p_2\wedge q_2}(\not{p}_2+\not{q}_2)\tilde{q}_1^{i_1}\tilde{q}_2^{i_2}\\
&+ 2i\not{q}_2\left[\frac{\sin\frac{q_1\wedge q_2}{2}\sin\frac{p_1\wedge p_2}{2}}{p_1\wedge p_2 k_1\wedge q_2}(p_2\wedge q_1\theta^{i_1 i_2} - 2\tilde{p}_2^{i_1}\tilde{q}_1^{i_2})\right.\\
&- \frac{\sin\frac{p_1\wedge q_2}{2}\sin\frac{p_2\wedge q_1}{2}}{p_1\wedge q_2 p_2\wedge q_1}2(\tilde{p}_2-\tilde{q}_1)^{i_1}\tilde{q}_1^{i_2} + \frac{\sin\frac{p_1\wedge q_2}{2}\sin\frac{p_2\wedge q_1}{2}}{p_1\wedge q_2}\theta^{i_1 i_2}\\
&+ \left.\left(\frac{\sin\frac{p_2\wedge q_1}{2}\sin\frac{p_1\wedge q_2}{2}}{p_2\wedge q_2 p_1\wedge q_2} + \frac{\sin\frac{p_1\wedge p_2}{2}\sin\frac{q_1\wedge q_2}{2}}{p_2\wedge q_2 q_1\wedge q_2}\right)(2\tilde{q}_1^{i_2}\tilde{p}_2^{i_1} + \theta^{i_1 i_2}q_1\wedge p_2 - \tilde{q}_1^{i_1}\tilde{q}_1^{i_2})\right]\\
&+ 2i\not{q}_1\left[\frac{\sin\frac{q_2\wedge q_1}{2}\sin\frac{p_1\wedge p_2}{2}}{p_1\wedge p_2 q_2\wedge q_1}(2\tilde{p}_2^{i_1}\tilde{q}_2^{i_2} - p_2\wedge q_2\theta^{i_1 i_2})\right.\\
&+ \frac{\sin\frac{p_1\wedge q_1}{2}\sin\frac{p_2\wedge q_2}{2}}{p_1\wedge q_1 p_2\wedge q_2}2(\tilde{p}_2+\tilde{q}_2)^{i_1}\tilde{q}_2^{i_2} - \frac{\sin\frac{p_1\wedge q_1}{2}\sin\frac{p_2\wedge q_2}{2}}{p_1\wedge q_1}\theta^{i_1 i_2}\\
&- \left.\left(\frac{\sin\frac{p_2\wedge q_2}{2}\sin\frac{p_1\wedge q_1}{2}}{p_2\wedge q_1 p_1\wedge q_1} + \frac{\sin\frac{p_2\wedge p_1}{2}\sin\frac{q_2\wedge q_1}{2}}{p_2\wedge q_1 q_2\wedge q_1}\right)(2\tilde{q}_2^{i_2}\tilde{p}_2^{i_1} + \theta^{i_1 i_2}q_2\wedge p_2 + \tilde{q}_2^{i_1}\tilde{q}_2^{i_2})\right]\\
&+ \{p_1\leftrightarrow p_2 \text{ and } i_1\leftrightarrow i_2\}
\end{aligned}
$$

Methods: parametrizations

1. Schwinger parametrization

$$\frac{1}{a^n} = \frac{1}{\Gamma(n)} \int_0^\infty e^{a\alpha} \alpha^{n-1} d\alpha$$

$$\frac{1}{a_1^{n_1} a_2^{n_2}} = \frac{1}{\Gamma(n_1)\Gamma(n_2)} \int_0^\infty \int_0^\infty e^{-a_1\alpha_1 - a_2\alpha_2} \alpha_1^{n_1-1} \alpha_2^{n_2-1} d\alpha_1 d\alpha_2$$

α_1 and α_2 are dimensionfull parameters.

2. Feynman parametrization

Used to combine the propagator denominators having the same maximal power of loop momentum (a_1 and a_2). Obtained from Schwinger parametrization putting $\alpha_1 = x\alpha$ and $\alpha_2 = (1-x)\alpha$ (x is dimensionless and α is dimensionfull parameter), and integrating over α

$$\frac{1}{a_1^{n_1} a_2^{n_2}} = \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} \int_0^1 \frac{x^{n_1-1}(1-x)^{n_2-1} dx}{[a_1 x + a_2(1-x)]^{n_1+n_2}}$$

3. "HQET" parametrization

Used to simplify a product of propagator denominators linear (a_1) and quadratic in loop momenta. Obtained from Schwinger parametrization putting $\alpha_1 = y\alpha$ and $\alpha_2 = \alpha$ (now both y and α are dimensionfull parameters) and integrating over α

$$\frac{1}{a_1^{n_1} a_2^{n_2}} = \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} \int_0^\infty \frac{y^{n_1-1} dy}{a_1 y + a_2}$$

Amplitudes and results

$$\Sigma_1 = \int \frac{\mu^{4-D} d^D q}{(2\pi)^D} \underbrace{\frac{(2 - e^{iq\theta p} - e^{-iq\theta p})}{(q\theta p)^2}}_{F(p,q)^2} \frac{1}{q^2} \frac{1}{(q+p)^2}$$

$$\times \Big[(q\theta p)^2 [(4-D)2(\not{p} + \not{q})] + (q\theta p)[\tilde{\not{q}}(2p^2 + 2pq) - \tilde{\not{p}}(2q^2 + 2pq)]$$

$$+[\not{p}(\tilde{q}^2(p^2 + 2qp) - q^2(\tilde{p}^2 + 2\tilde{p}\tilde{q})) + \not{q}(\tilde{p}^2(q^2 + 2pq) - p^2(\tilde{q}^2 + 2\tilde{p}\tilde{q}))] P_{L,R} \Big]$$

$$= \{(4-D)[2I_{10} - I_{1+} - I_{1-}] + [2I_{20} - I_{2+} - I_{2-}] + [2I_{30} - I_{3+} - I_{3-}]\}$$

$$\equiv (4-D) I_{10+-} + I_{20+-} + I_{30+-}$$

$$\varepsilon I_{10+-} = \frac{i}{(4\pi)^2} 2\not{p} \varepsilon J_{1,2} = \frac{i}{(4\pi)^2} 2\not{p}$$

$$I_{20+-} = \frac{i}{(4\pi)^2} \frac{\tilde{\tilde{\not{p}}} p^2}{\tilde{p}^2} 4 J_{1,2}$$

$$I_{30+-} = \frac{i}{(4\pi)^2} \frac{\not{p}((\tilde{p}^2)^2 - p^2 \tilde{\tilde{p}}^2)}{(\tilde{p}^2)^2} J_{3a} + \frac{i}{(4\pi)^2} \frac{\not{p}(\tilde{p}^2(-2+\varepsilon) - p^2 Tr\theta^2) + \tilde{\tilde{\not{p}}}(2p^2)}{\tilde{p}^2} J_{3b}$$

$$
\begin{aligned}
J_{1,2} &= \frac{1}{\varepsilon} + \frac{1}{2}\ln \mu^2\tilde{p}^2 + \left(1 - \frac{\gamma}{2} + \frac{1}{2}\ln(-4\pi) - \psi^0(2)\right) \\
J_{3a} &= -\frac{1}{\varepsilon}8(-4\pi)^{\frac{\varepsilon}{2}} - \ln \mu^2\tilde{p}^2 + \left(-4 - 6\ln 2 - 4\psi^0(2) - \frac{1}{3}p^2\tilde{p}^2\right) \\
&+ 4\sum_0^\infty \frac{\sqrt{\pi}}{\Gamma(n+\frac{5}{2})}\frac{(p^2\tilde{p}^2)^{n+1}}{(n+1)!2^{4n+4}} \\
&\quad \left[n\left(\frac{1}{4}\ln\frac{p^2\tilde{p}^2}{4} - \psi^0(n+3)\right) + (n+1)(\psi^0(n+2) - \psi^0(2n+4))\right] \\
&- 2\sum_0^\infty \frac{\sqrt{\pi}}{\Gamma(n+\frac{5}{2})}\frac{\Gamma(n+\frac{3}{2})}{\Gamma(n+\frac{7}{2})}\frac{(p^2\tilde{p}^2)^{n+2}}{(n+1)!2^{4n+8}} \\
&\quad \left[\ln p^2\tilde{p}^2 - \psi^0\left(n+\frac{5}{2}\right) - \psi^0(n+1) + 2(\psi^0(n+3) - \psi^0(2n+6))\right]
\end{aligned}
$$

$$
\begin{aligned}
J_{3b} &= -\frac{1}{\varepsilon}2(-4\pi)^{\frac{\varepsilon}{2}} - \ln\mu^2\tilde{p}^2 - \psi^0\left(n+\frac{3}{2}\right) + 2\psi^0(2) + 4 \\
&+ \sum_0^\infty \frac{\sqrt{\pi}}{\Gamma(n+\frac{5}{2})}\frac{(p^2\tilde{p}^2)^{n+1}}{(n+1)!2^{4n+4}}\left(-\frac{1}{2}\ln\frac{p^2\tilde{p}^2}{4} + \psi^0(2n+4)\right) \\
&- \frac{1}{2}\sum_0^\infty \frac{\sqrt{\pi}}{\Gamma(n+\frac{5}{2})}\frac{\Gamma(n+\frac{1}{2})}{\Gamma(n+\frac{3}{2})}\frac{(p^2\tilde{p}^2)^{n+1}}{(n+1)!2^{4n+4}} \\
&\left(\ln p^2\tilde{p}^2 - 2\ln 2 - \psi^0(n+1) - \psi^0(2n+4) - \psi^0\left(n+\frac{3}{2}\right) + \psi^0\left(n+\frac{1}{2}\right)\right)
\end{aligned}
$$

$$
\Sigma_2 = 0
$$

Status

1. The self energy diagram Σ_1 is evaluated. It contains usual UV $(1/\varepsilon)$ divergences and logarithmic IR divergences with UV/IR mixing $(\ln \mu^2 \tilde{p}^2)$.

2. The diagram Σ_2 is equal zero.

Thank you