pseudo-force	transformation	simpler force
centrifugal, Coriolis	rotations	0

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pseudo-force	transformation	simpler force	geometry
centrifugal, Coriolis	rotations	0	Euclid
magnetic	Lorentz	electric	Minkowski
gravitational	general coordinate	0	Riemann

pseudo-force	transformation	simpler force	geometry	time
centrifugal, Coriolis	rotations	0	Euclid	absolute
magnetic	Lorentz	electric	Minkowski	universal
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pseudo-force	transformation	simpler force	geometry	time
centrifugal, Coriolis	rotations	0	Euclid	absolute
magnetic	Lorentz	electric	Minkowski	universal
gravitational	general coordinate	0	Riemann	proper τ
elect.-magn., weak, strong	gauge	gravitational	NCG	$\Delta\tau \sim 10^{-41}$ s

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new physics	discrete spectra	

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ansatz	$\nu = g\,({n_2}^{q} - {n_1}^{q})$ Balmer-Rydberg	
discrete param.	$q \in \mathbb{Z}$	
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ansatz	$\nu = g\,({n_2}^q - {n_1}^q)$ Balmer-Rydberg	Yang-Mills-Higgs models
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Maxwell
Oskar Klein
Gordon
Dirac
Weyl
Elie Cartan
Majorana
Yukawa
Brout
Englert

G

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$g, \lambda, \mu \in \mathbb{R}_+$

$g_Y \in \mathbb{C}$

$$
\begin{aligned}
\mathcal{L}[A,\psi,\varphi] = \quad & \tfrac{1}{2}\,\mathrm{tr}\,(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu) \\
& + g\ \mathrm{tr}\,(\partial_\mu A_\nu [A^\mu, A^\nu]) \\
& + g^2\,\mathrm{tr}\,([A_\mu, A_\nu][A^\mu, A^\nu]) \\
& + \bar{\psi}\,\partial\!\!\!/\psi \\
& + ig\,\bar{\psi}(\tilde{\rho}_L \oplus \tilde{\rho}_R)(A_\mu)\,\gamma^\mu \psi \\
& + \tfrac{1}{2}\,\partial_\mu \varphi^* \partial^\mu \varphi \\
& + \tfrac{1}{2}\,g\,\{(\tilde{\rho}_S(A_\mu)\varphi)^* \partial^\mu \varphi + \partial_\mu \varphi^* \tilde{\rho}_S(A_\mu)\varphi\} \\
& + \tfrac{1}{2}\,g^2\,(\tilde{\rho}_S(A_\mu)\varphi)^* \tilde{\rho}_S(A^\mu)\varphi \\
& + \lambda\,\varphi^*\varphi\varphi^*\varphi \\
& - \tfrac{1}{2}\,\mu^2\,\varphi^*\varphi \\
& + g_Y\,\bar{\psi}\varphi\psi + \bar{g}_Y\,\bar{\psi}\varphi^*\psi
\end{aligned}
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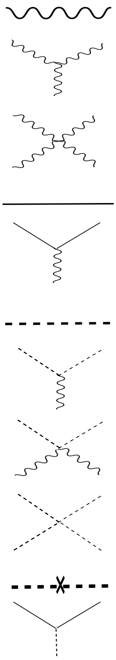

Properties:

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$$G = U(1), \quad \mathcal{H}_L = \mathcal{H}_R, \quad \mathcal{H}_S = \{0\},$$

Yang-Mills Lagrangian = Maxwell's Lagrangian with $g = e/\sqrt{\epsilon_0}$.

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- The Yang-Mills-Higgs Lagrangian defines a perturbatively renormalizable quantum field theory if the Yang-Mills anomaly

$$a_{YM} = \mathrm{tr}\,\left[\tilde{\rho}_L(A)^3\right] - \mathrm{tr}\,\left[\tilde{\rho}_R(A)^3\right]$$

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- The Yang-Mills-Higgs action is invariant under general coordinate transformations.

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new physics	discrete spectra	forces from gauge bosons
ansatz	$\nu = g\,({n_2}^q - {n_1}^q)$ Balmer-Rydberg	Yang-Mills-Higgs models
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Continuous parameters:

$$g_1 = 0.3574 \pm 0.0001, \quad g_2 = 0.6518 \pm 0.0003, \quad g_3 = 1.218 \pm 0.01,$$

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$$g_Y\text{'s} \quad \rightsquigarrow \quad \text{fermion masses and mixings.}$$

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new physics	discrete spectra	forces from gauge bosons
ansatz	$\nu = g\,(n_2{}^q - n_1{}^q)$ Balmer-Rydberg	Yang-Mills-Higgs models
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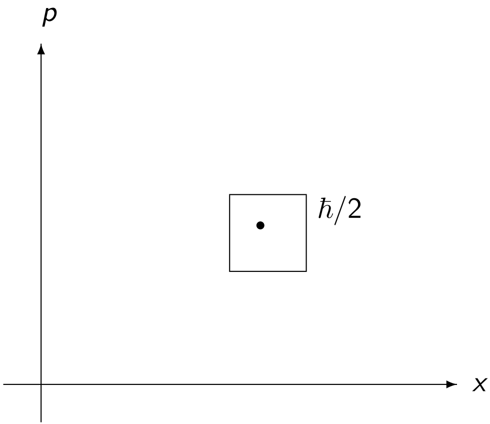
$$[x, p] = i\hbar 1 \quad \Rightarrow \quad \Delta x \Delta p \geq \tfrac{1}{2}\hbar.$$



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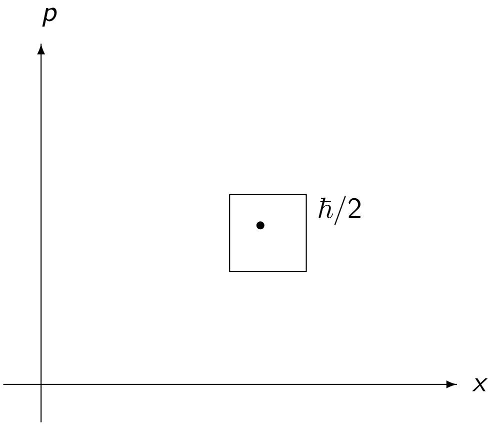

$\mathcal{A}$ = algebra of observables, $\mathcal{H}$ = unitary representation, $\partial\!\!\!/$ = Dirac operator

Reconstruction **theorem** (Connes, hep-th/9603053): There is a one-to-one correspondence between *commutative* (even) real spectral triples

$$(\mathcal{A}, \mathcal{H}, \partial\!\!\!/, J, (\chi))$$

and (even-dimensional) compact Riemannian spin manifolds M.

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$$\mathcal{A} = \mathcal{C}^\infty(M),$$

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$$\partial\!\!\!/ = i\gamma^\mu(\partial/\partial x^\mu + \tfrac{1}{4}\omega_{\mu ab}\gamma^a\gamma^b),$$

$$J = \gamma^0\gamma^2 \circ \text{complex conjugation},$$

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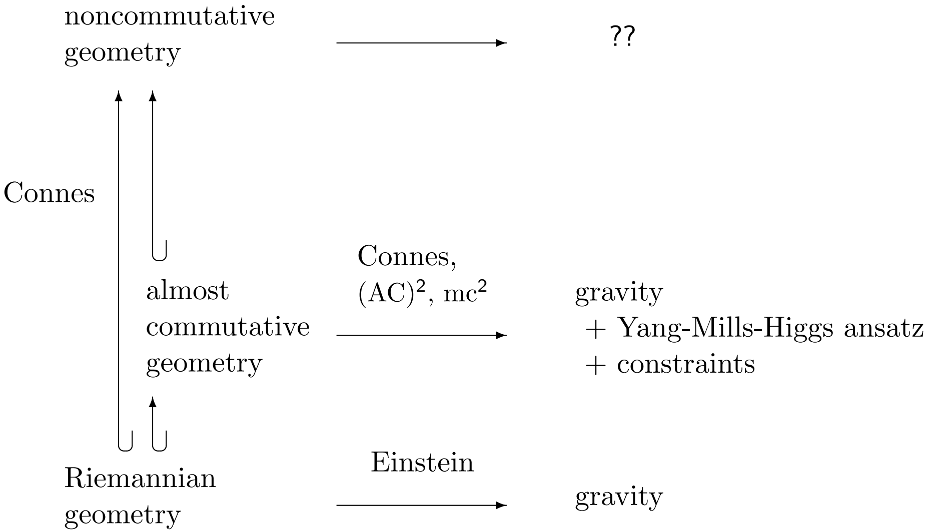
Example (D. Hamilton, 1996)


noncommutative geometry
??
Connes
Connes, (AC)², mc²
almost commutative geometry
gravity
+ Yang-Mills-Higgs ansatz
+ constraints
Einstein
Riemannian geometry
gravity

Constraints on discrete parameters:

-

$$G = \mathrm{Aut}(\mathcal{A})$$

Examples: $\mathrm{Aut}(\mathcal{C}^\infty(M))^e = \mathrm{Diff}(M)^e$, $\mathrm{Aut}(\mathbb{H}) = SU(2)/\mathbb{Z}_2$, $\mathrm{Aut}(M_3(\mathbb{C})) = SU(3)/\mathbb{Z}_3$

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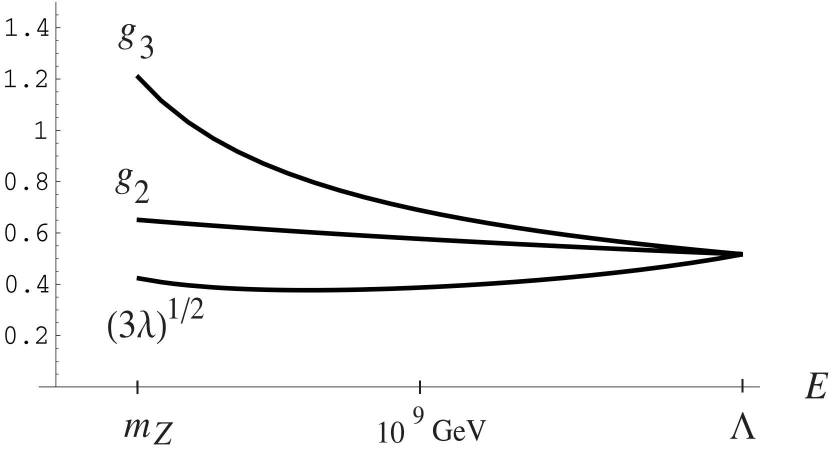
if we believe in big desert and standard renormalisation group flow.



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Noncommutative geometry beyond the standard model:

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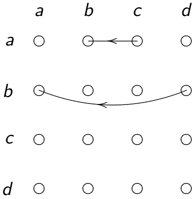
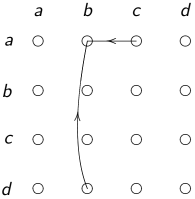
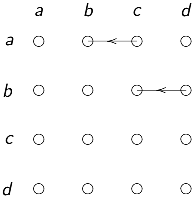


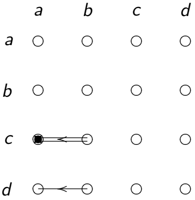
diag. 1



diag. 2



diag. 3



diag. 4

Jureit & Stephan 2007: the irreducible Krajewski diagrams with 4 or less simple algebras in *KO* dimension 6. Diag. 4 yields the standard model with one generation of fermions and a massless neutrino.

Higgs-mass predictions in the literature:

arXiv:0708.3344 [hep-ph]

- over 100 predictions from 114 GeV to 10^{18} GeV leaving
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a few classes of models:

- $su(2) \oplus u(1) \subset su(2|1)$, 4 predictions
- super symmetry, 45 predictions
- super string (inspired), 2 predictions
- E-theory, $m_H = 161.8033989$ GeV
- extra dimensions, 11 predictions
- cancelation of a particular 1-loop divergence, 8 predictions
  e.g. quadratic: 309 GeV by Decker & Pestieau 1979, Veltman 1981
- lattice gauge theory, 2 predictions
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