Noncommutativity and Physics: Spacetime Quantum Geometry

Photon-neutrino interaction in θ-exact covariant noncommutative field theory

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Collaboration with

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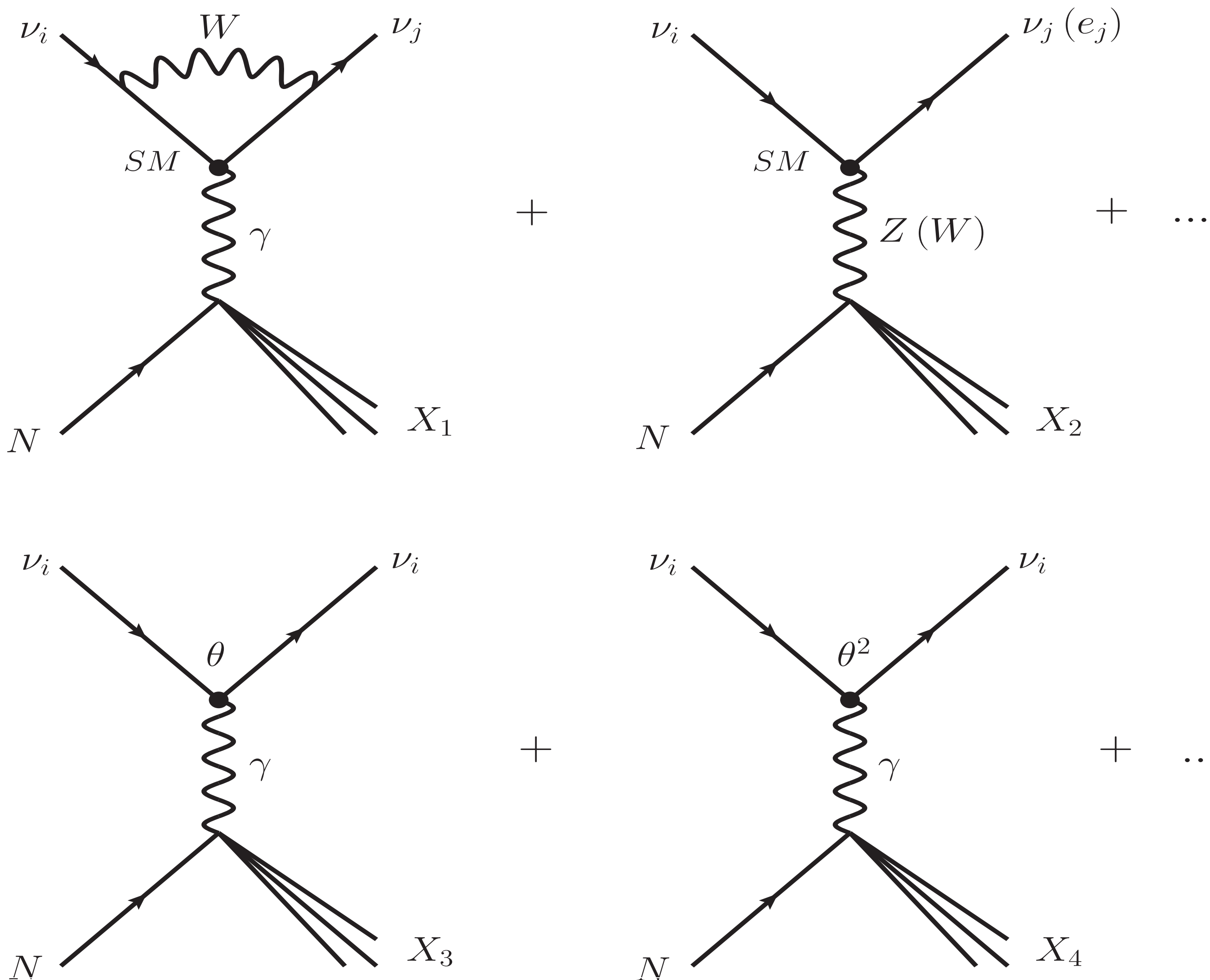
Contents

Spacetime NC and UHECR-ν experiments



Neutrino-photon NC interactions

[P. Schupp, J.Trampetic, J. Wess and G. Raffelt, *The photon neutrino interaction in non-commutative gauge field theory and astrophysical bounds,* Eur. Phys. J. C **36** (2004) 405]

Neutrino-photon interaction introduced via:

$\star$–commutator with covariant derivative

$$D_\mu \Psi = \partial_\mu \Psi - i\kappa e \left[A_\mu \star \Psi - \Psi \star A_\mu \right]$$

The action for a neutral fermion that couples to an Abelian gauge boson in the adjoint of NC U(1),

$$\begin{aligned} S &= \int d^4x \left(\bar{\Psi} \star i\gamma^\mu D_\mu \Psi - m \bar{\Psi} \star \Psi \right) \\ \Psi &= \psi + e\theta^{\nu\rho} a_\rho \partial_\nu \psi + \mathcal{O}(\theta^2) \\ A_\mu &= a_\mu + e\theta^{\rho\nu} a_\nu \left[\partial_\rho a_\mu - \frac{1}{2} \partial_\mu a_\rho \right] + \mathcal{O}(\theta^2) \end{aligned}$$

ν **N** $\longrightarrow \nu$ + *anything*

The gauge invariant action of order θ^1 and $\kappa = 1$

$$S = \int d^4x\, \bar{\psi} \left[\frac{e}{2} f_{\mu\nu}(i\theta^{\mu\nu\rho}\partial_\rho - \theta^{\mu\nu} m)\right] \psi .$$

Feynman rule $\Gamma^{\mu}_{\binom{\mathrm{L}}{\mathrm{R}}}(\nu\bar{\nu}\gamma) = ie\frac{1}{2}(1 \mp \gamma_5)\theta^{\mu\nu\tau} k_\nu q_\tau , \quad m = 0$

[R. Horvat, D. Kekez and J. Trampetić, *Spacetime noncommutativity and ultrahigh energy cosmic ray experiments* Phys. Rev. D **83**, 065013 (2011)] Fig 3. gives:

$$\frac{d^2\sigma_{\mathrm{NC}}}{dx\,dy} = \mathcal{I}\, \frac{2\pi\alpha^2}{E_\nu M_N (xy)^2} \left[(1-y)F_2^\gamma + y^2 x F_1^\gamma + y(1-y/2)xF_3^\gamma\right] .$$

$$\mathcal{I} = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \,(\frac{kck'}{2\Lambda_{\mathrm{NC}}^2})^2 , \quad \text{with} \quad c^{\mu\nu} = \theta^{\mu\nu}\Lambda_{\mathrm{NC}}^2$$

$$\approx \left((c_{01} - c_{13})^2 + (c_{02} - c_{23})^2\right) \frac{E_\nu^3 M_N}{4\Lambda_{\mathrm{NC}}^4}\, x\, y\, (1-y) .$$

Process reveal stronger energy dependence than expected

$$E_\nu^{1/2} s^{1/4}/\Lambda_{\rm NC} \lesssim 1\,, \quad s = 2E_\nu M_N$$

Results are given for $(c_{01}-c_{13})^2 + (c_{02}-c_{23})^2 = 1$.
Employing $\sigma_{exp} = 4 \times 10^{-3}$ mb [for neutrino flux (FKRT-Fodor et al J.Cosm.Astropart.Phys. 11 (2003) 015] from RICE Collaboration search results at $E^\nu = 10^{11}$ GeV,

$$\sigma(\theta)/\sigma_{exp} \implies \Lambda_{\rm NC} \gtrsim 455 \ \ {\rm TeV}$$

$$\implies \left[\frac{\sigma(\theta^2)}{\sigma(\theta)}\right]_{\Lambda_{\rm NC}=455 \ \ {\rm TeV}} \simeq 10^4 \quad \text{Unacceptable!}$$

ν **N** $\rightarrow \nu$ + *anything*

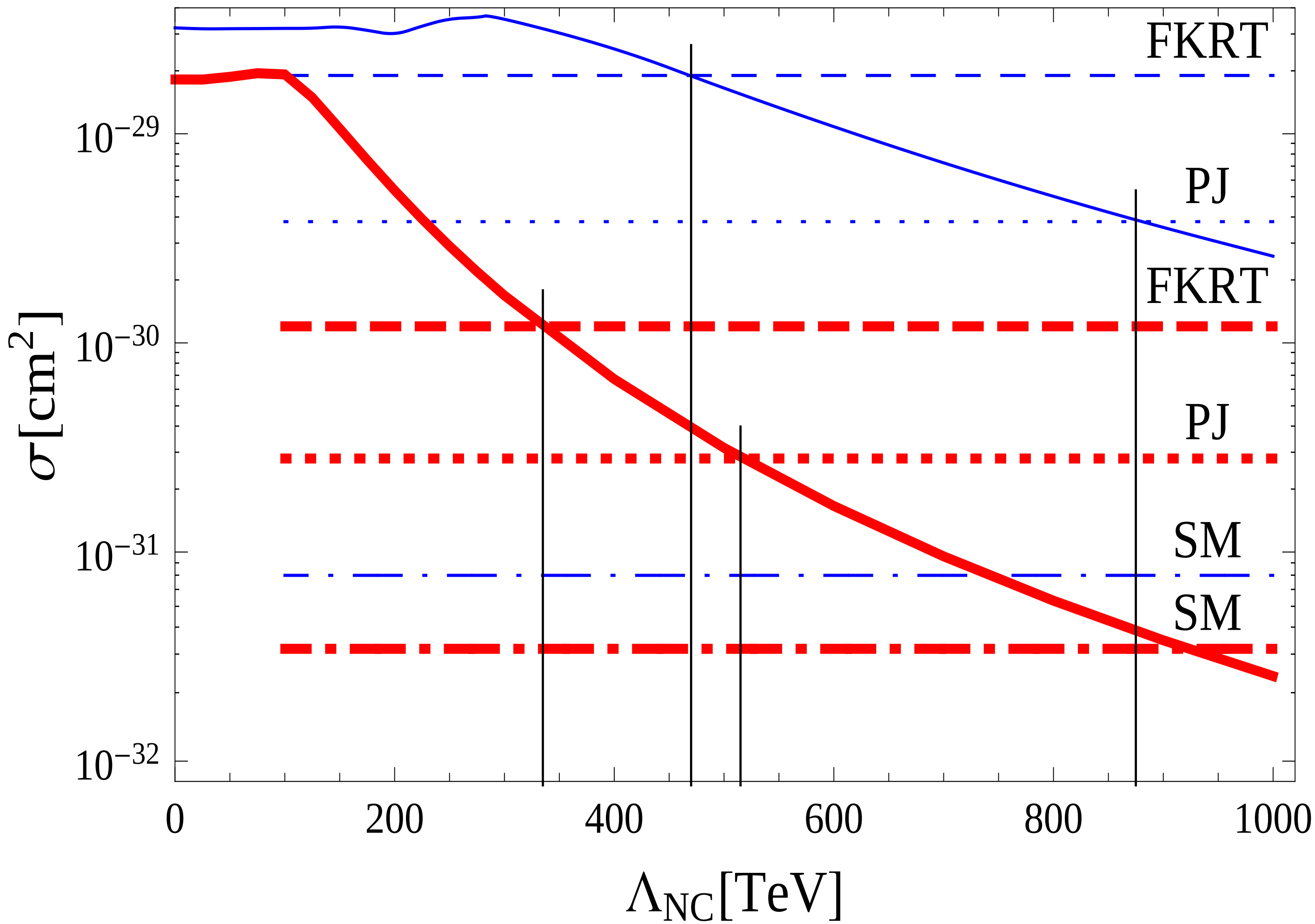
[R. Horvat, D. Kekez and J. Trampetić, *Spacetime noncommutativity and ultrahigh energy cosmic ray experiments* Phys. Rev. D **83**, 065013 (2011)]

The simplest possible modeling: $\Psi \rightarrow \psi\,, \quad A_\mu \rightarrow a_\mu \quad \rightarrow$
$\rightarrow S_{\rm NC}(\theta) = -ie \int d^4x\, \bar\psi\, \gamma^\mu (a_\mu \star \psi - \psi \star a_\mu) \quad \rightarrow$
expansion/resummation of $\star$-product gives Feynman rule,
$\Gamma^\mu_{\binom{\rm L}{\rm R}}(\bar\nu\nu\gamma) = ie(1 \pm \gamma_5)\gamma^\mu \sin(\frac{q\theta k}{2})$ and the following integral

$$
\begin{aligned}
\mathcal{I} &= \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, 4\sin^2(\frac{kck'}{2\Lambda^2_{\rm NC}}) \\
&= 2\,(1-\cos(A)J_0(B))\;, \\
A &= \frac{E_\nu E'_\nu}{\Lambda^2_{\rm NC}} c_{03}(\cos\vartheta - 1)\;, \\
B &= \frac{E_\nu E'_\nu}{\Lambda^2_{\rm NC}} \sin\vartheta\ \text{sign}(c_{01}-c_{03})\sqrt{(c_{01}-c_{13})^2+(c_{02}-c_{23})^2}.
\end{aligned}
$$

ν **N** $\longrightarrow \nu$ + *anything*

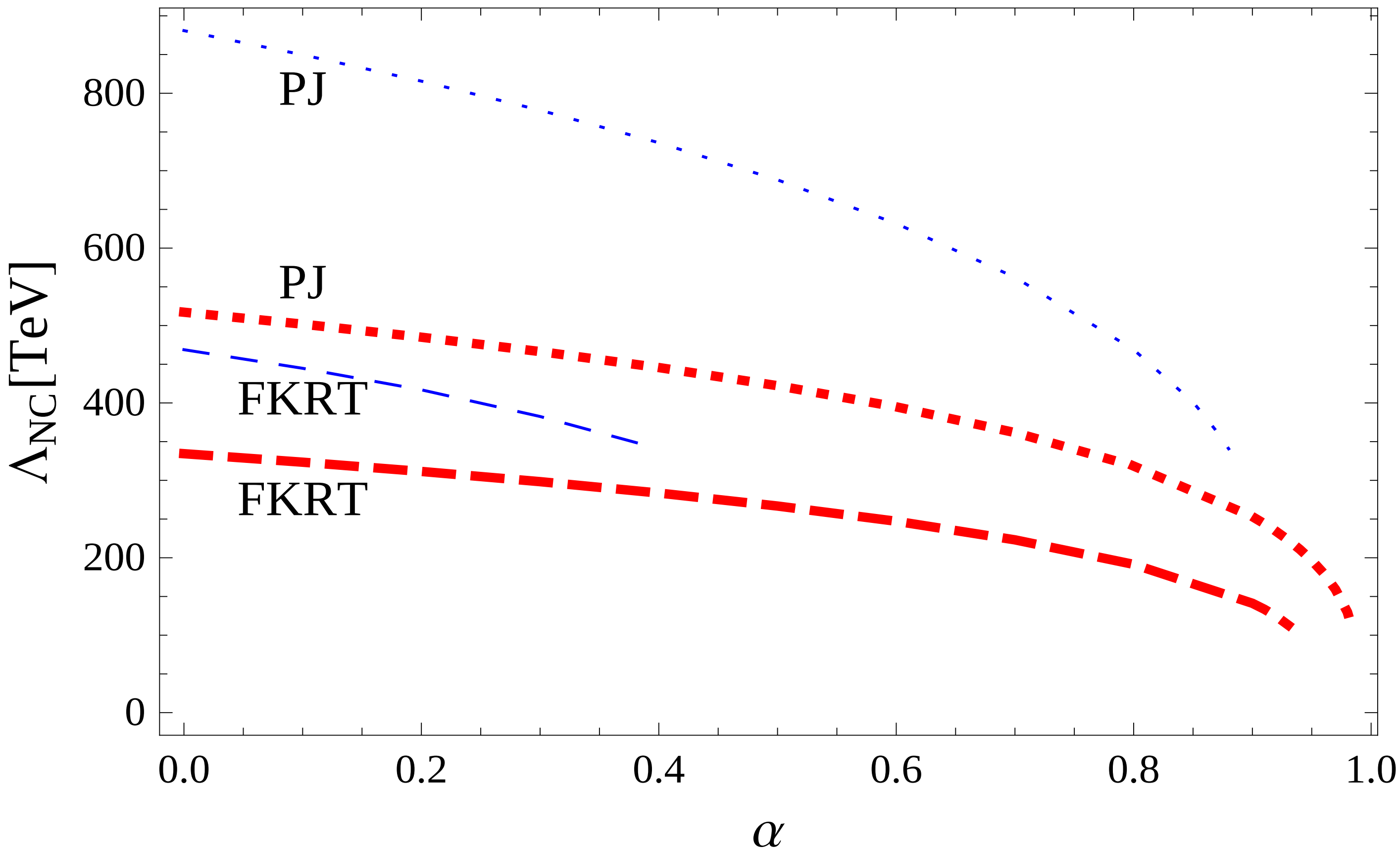
[R. Horvat, D. Kekez and J. Trampetić, *Spacetime NC and UHECR experiments* PRD 83, 065013 (2011)]



$\sigma[cm^2]$ versus $\Lambda_{\rm NC}$ for $E_\nu = 10^{10}$ GeV (thick lines) and $E_\nu = 10^{11}$ GeV (thin lines).

ν **N** $\longrightarrow \nu$ + *anything*

[R. Horvat, D. Kekez and J. Trampetić, *Spacetime NC and UHECR experiments* PRD 83, 065013 (2011)]



The intersections of our curves with the RICE results (cf. Fig.1) as a function of the fraction of Fe nuclei in the UHE cosmic rays. The terminal point on each curve represents the highest fraction of Fe nuclei above which no useful information on $\Lambda_{\rm NC}$ can be inferred with our method.

θ-exact model properties / What do we want? / Wishing list?

[P. Schupp and J. You, *UV/IR mixing in NC QED defined by Seiberg-Witten map*, *JHEP* **08** (2008) 107

* Direct neutrino-photon coupling in θ-exact NCFT
* Model based on the Seiberg-Witten mapping
* No charge quantization problem
* Any gauge group and arbitrary matter repres.
* Covariant NCSM Yukawa couplings OK
* Unitarity is OK for: $\theta^{ij} \neq$, $\theta^{0i} = 0$;
* Covariant generalization of $\theta^{0i} = 0$ to:

$$\theta_{\mu\nu}\theta^{\mu\nu} \;=\; -\theta^2 = \frac{2}{\Lambda_{\rm NC}^4}\left(\vec{B}_\theta^2 - \vec{E}_\theta^2\right) > 0$$

* UV/IR mixing and/or Renormalisability $\leftrightarrow$ Quantum Gravity
* Holography distinct UV/IR connection $\rightarrow$ $\Lambda_{\rm IR}/\Lambda_{\rm NC}/\Lambda_{\rm UV}/M_{Pl}$

Photon-neutrino interaction in θ-exact covariant NCFT

[P. Schupp and J. You, *UV/IR mixing in NC QED defined by Seiberg-Witten map*, *JHEP* **08** (2008) 107

$$S = \int \left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\Psi}\left(\not{D} - m_\nu\right)\Psi\right) d^4x$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu \stackrel{\star}{,} A_\nu];\ D_\mu\Psi = \partial_\mu\Psi - i[A_\mu \stackrel{\star}{,} \Psi]$$

At least three known methods for θ-exact computations:

- The closed formula derived using deformation quantization based on Kontsevich formality maps ,

- the relationship between open Wilson lines in the commutative and noncommutative picture and

- direct recursive computations using consistency conditions

direct deduction from the recursion and consistency relations:

$$\delta_\Lambda A_\mu \equiv i[\Lambda \stackrel{\star}{,} A_\mu] = A_\mu[a_\mu + \delta_\lambda a_\mu] - A_\mu[a_\mu] + \mathcal{O}(\lambda^2)\,,$$

$$\delta_\Lambda \Psi \equiv i[\Lambda \stackrel{\star}{,} \Psi] = \Psi[a_\mu + \delta_\lambda a_\mu, \psi + \delta_\lambda \psi] - \Psi[a_\mu, \psi] + \mathcal{O}(\lambda^2)\,,$$

$$\Lambda[[\lambda_1, \lambda_2], a_\mu] =$$

$$[\Lambda[\lambda_1, a_\mu] \stackrel{\star}{,} \Lambda[\lambda_2, a_\mu]] + i\delta_{\lambda_1}\Lambda[\lambda_2, a_\mu] - i\delta_{\lambda_2}\Lambda[\lambda_1, a_\mu]\,,$$

$\gamma_{pl} \to \bar{\nu}\nu$ in θ-exact covariant NCGFT

[P. Schupp and J. You, *UV/IR mixing in NC QED defined by Seiberg-Witten map*, *JHEP* **08** (2008) 107

With the ansatz

$\Lambda = \hat{\Lambda}[a_\mu]\lambda = (1 + \hat{\Lambda}^1[a_\mu] + \hat{\Lambda}^2[a_\mu] + \mathcal{O}(a^3))\lambda\,,$

$\Psi = \hat{\Psi}[a_\mu]\psi = (1 + \hat{\Psi}^1[a_\mu] + \hat{\Psi}^2[a_\mu] + \mathcal{O}(a^3))\psi\,,$ starting with the fermion field Ψ, at lowest order we have $i[\lambda \stackrel{\star}{,} \psi] = \hat{\Psi}[\partial\lambda]\psi$ from $[f \stackrel{\star}{,} g] = i\theta^{ij}\left(\frac{\partial f(x)}{\partial x^i}\right)\frac{\sin(\frac{\partial_x\theta\partial_y}{2})}{\frac{\partial_x\theta\partial_y}{2}}\left(\frac{\partial g(y)}{\partial y^i}\right)\Big|_{x=y}$ we observe that

$\hat{\Psi}[a_\mu] = -\theta^{ij}a_i \star_2 \partial_j$ where $f \star_2 g = f(x)\frac{\sin\frac{\partial_x\wedge\partial_y}{2}}{\frac{\partial_x\wedge\partial_y}{2}}g(y)\Big|_{x=y}$.

Gauge transformation Λ similar $\hat{\Lambda}^1 = -\frac{1}{2}\theta^{ij}a_i \star_2 \partial_j$.

Lowest order consistency relation:

$$-\partial_\mu(\tfrac{1}{2}\theta^{ij}a_i \star_2 \partial_j\lambda) - i[\lambda \stackrel{\star}{,} a_\mu] = A^2_\mu[a_\mu + \partial_\mu\lambda] - A^2_\mu[a_\mu]$$

$\gamma_{pl} \to \bar{\nu}\nu$ in θ-exact covariant NCGFT

[P. Schupp and J. You, *UV/IR mixing in NC QED defined by Seiberg-Witten map*, *JHEP* **08** (2008) 107

We obtain

$$A_\mu = a_\mu - \frac{1}{2}\theta^{ij} a_i \star_2 (\partial_j a_\mu + f_{j\mu}) + \mathcal{O}(a^3)\,,$$
$$\Psi = \psi - \theta^{ij} a_i \star_2 \partial_j \psi + \mathcal{O}(a^2)\psi\,,$$
$$\Lambda = \lambda - \frac{1}{2}\theta^{ij} a_i \star_2 \partial_j \lambda + \mathcal{O}(a^2)\lambda\,,$$
$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$$
$$\mathcal{L} = \bar{\psi}\gamma^\mu [a_\mu \stackrel{\star}{,} \psi]$$
$$-(\theta^{ij} a_i \star_2 \partial_j \bar{\psi})(i\partial\!\!\!/ - m_\nu)\psi - \bar{\psi}(i\partial\!\!\!/ - m_\nu)(\theta^{ij} a_i \star_2 \partial_j \psi) + \bar{\psi}\mathcal{O}(a^2)\psi\,.$$

To extract Feynman rules in an appropriate form, we use the arithmetic property

$$i\theta^{ij}\partial_i f \star_2 \partial_j g = [f \stackrel{\star}{,} g]$$

to obtain the effective neutrino-photon Lagrangian density

$\gamma_{pl} \to \bar{\nu}\nu$ in θ-exact covariant NCGFT

[R. Horvat, D. Kekez, P. Schupp, J. Trampetić and J. You, *Photon-neutrino interaction in θ-exact covariant noncommutative field theory*, arXiv:1103.3383v1]

$$\mathcal{L} = -(\theta^{ij} a_i \star_2 \partial_j \bar{\psi})(i\partial\!\!\!/ - m_\nu)\psi - \bar{\psi}(i\partial\!\!\!/ - m_\nu)(\theta^{ij} a_i \star_2 \partial_j \psi) + i\bar{\psi}\gamma^\mu(\theta^{ij}\partial_i a_\mu \star_2 \partial_j \psi) + \mathcal{O}(a^2\bar{\psi}\psi)\,.$$

Generalized star product $\star_2$ turns into a function,

$$F(q,k) = \frac{\sin\frac{q\theta k}{2}}{\frac{q\theta k}{2}}$$

$$F(q,k) = F(k,q) = F(k,k') = F(k',k)\,, \quad q = k - k'$$

$$\Gamma^\mu = iF(q,k)\left[(k\!\!\!/ - m_\nu)\tilde{q}^\mu + (q\theta k)\gamma^\mu - q\!\!\!/\tilde{k}^\mu\right], \;\; \tilde{k}^\mu = \theta^{\mu j}k_j\,.$$

$\gamma_{pl} \to \bar{\nu}\nu$ in θ-exact covariant NCGFT

Next we start with the action for a neutral massless free fermion field

$$S = \int \bar{\psi}\gamma^\mu\partial_\mu\psi \, d^4x = \int \bar{\psi} \star \gamma^\mu\partial_\mu\psi \, d^4x \,,$$

and we lift the factors in the action via generalized SW maps $\hat{\Psi}[a_\mu]$ and $\hat{\Phi}[a_\mu]$ to NC status as follows; (if the SW maps $\hat{\Psi}$, $\hat{\Phi}$ and $\hat{\Lambda}$ satisfy)

$$S = \int \hat{\Psi}(\bar{\psi})\gamma^\mu\hat{\Phi}(\partial_\mu\psi) \, d^4x = \int \hat{\Psi}(\bar{\psi}) \star \gamma^\mu\hat{\Phi}(\partial_\mu\psi) \, d^4x \,.$$

$$\delta_\lambda(\hat{\Psi}(\bar{\psi})) = i[\hat{\Lambda}(\lambda) \stackrel{\star}{,} \hat{\Psi}(\bar{\psi})], \;\; \delta_\lambda(\hat{\Phi}(\partial_\mu\psi)) = i[\hat{\Lambda}(\lambda) \stackrel{\star}{,} \hat{\Phi}(\partial_\mu\psi)]$$

$\hat{\Psi}(\psi) = \psi - \theta^{ij} a_i \star_2 \partial_j\psi$, and neutral fields $\delta\psi = \delta(\partial_\mu\psi) = 0$, we notice that we can in principle use the same map also for $\hat{\Phi}$:

$$\hat{\Phi}_{\text{alt}}(\partial_\mu\psi) = \hat{\Psi}(\partial_\mu\psi) = \partial_\mu\psi - \theta^{ij} a_i \star_2 (\partial_j\partial_\mu\psi) + \mathcal{O}(a^2)\psi \,.$$

This action construction is quite unusual from the point of gauge theory, as it yields a covariant derivative term without introducing a covariant derivative:

$$S = \int \Big(i\bar{\psi}\gamma^\mu \partial_\mu \psi - i \left(\theta^{ij}\partial_j \bar{\psi} \star_2 a_i\right) \gamma^\mu \partial_\mu \psi +$$

$$i\bar{\psi}\gamma^\mu \left(\theta^{ij} a_i \star_2 \partial_\mu \partial_j \psi\right) \Big) d^4x \; + \mathcal{O}(a^2) \longrightarrow \;\; \Gamma^\mu = iF(q,k)\tilde{q}^\mu \not{k}$$

Second choice for $\hat{\Phi}$:

$$\hat{\Phi}(\partial_\mu \psi) = D^\star_\mu \hat{\Psi}(\psi) = \partial_\mu \hat{\Psi}(\psi) - i[A_\mu \stackrel{\star}{,} \hat{\Psi}(\psi)] =$$

$$\partial_\mu \psi - \theta^{ij} a_i \star_2 \partial_j \partial_\mu \psi + \theta^{ij} f_{i\mu} \star_2 \partial_j \psi + O(a^2)\psi \,,$$

based on the well-known NC QED-type covariant derivative gives

$$\Gamma^\mu = iF(q,k) \left[\not{k} \tilde{q}^\mu + (q\theta k)\gamma^\mu - \not{q} \tilde{k}^\mu \right].$$

$\gamma_{pl} \to \bar{\nu}\nu$ in θ-exact covariant NCGFT

[R. Horvat, D. Kekez, P. Schupp, J. Trampetić and J. You, *Photon-neutrino interaction in θ-exact covariant noncommutative field theory*, arXiv:1103.3383v1]

Photon dispersion relation in a stellar plasma

$$q^2 \equiv \mathrm{E}_\gamma^2 - \mathbf{q}_\gamma^2 \overset{def.}{=} \omega_{pl}^2 \overset{calc.}{=} \mathcal{R}e\, \Pi_T(q_0, |\vec{q}| = 0) = \frac{e^2T^2}{9},$$

$$|M_{\rm NC}(\gamma_{pl.} \to \bar{\nu}\nu)|^2 = 4e^2(F(q,k))^2(q\theta k)^2\left(q^2 + 2m_\nu^2\right),$$

$$\Gamma_{\rm NC}(\gamma_{pl.} \to \bar{\nu}_{\binom{L}{R}}\nu_{\binom{L}{R}}) = \frac{\alpha\,\omega_{pl}}{4\pi}\int\limits_0^\pi \sin\vartheta d\vartheta \int\limits_0^{2\pi} d\phi \sin^2\frac{q\theta k}{2}$$

$$= \frac{1}{4}\alpha\,\omega_{pl}\int\limits_{-1}^{1} dx\left[1 - (\cos Ax)J_0(B\sqrt{1-x^2})\right],$$

$$A \equiv \frac{c_{03}\,\omega_{pl}^2}{2\Lambda_{\rm NC}^2}, \quad B \equiv \frac{\omega_{pl}^2}{2\Lambda_{\rm NC}^2}\sqrt{c_{01}^2 + c_{02}^2}.$$

Rate for $\gamma_{pl} \rightarrow \bar{\nu}\nu$

[R. Horvat, D. Kekez, P. Schupp, J. Trampetić and J. You, *Photon-neutrino interaction in θ-exact covariant noncommutative field theory*, arXiv:1103.3383v1]

Partial width:

$$\Gamma_{\rm NC}(\gamma_{\rm pl} \rightarrow \bar{\nu}_{\binom{\rm L}{\rm R}}\nu_{\binom{\rm L}{\rm R}}) = \frac{\alpha}{2}\omega_{pl}\left(1 - \frac{\sin\xi}{\xi}\right), \quad \xi = \frac{\omega_{pl}^2}{2\Lambda_{\rm NC}^2}$$

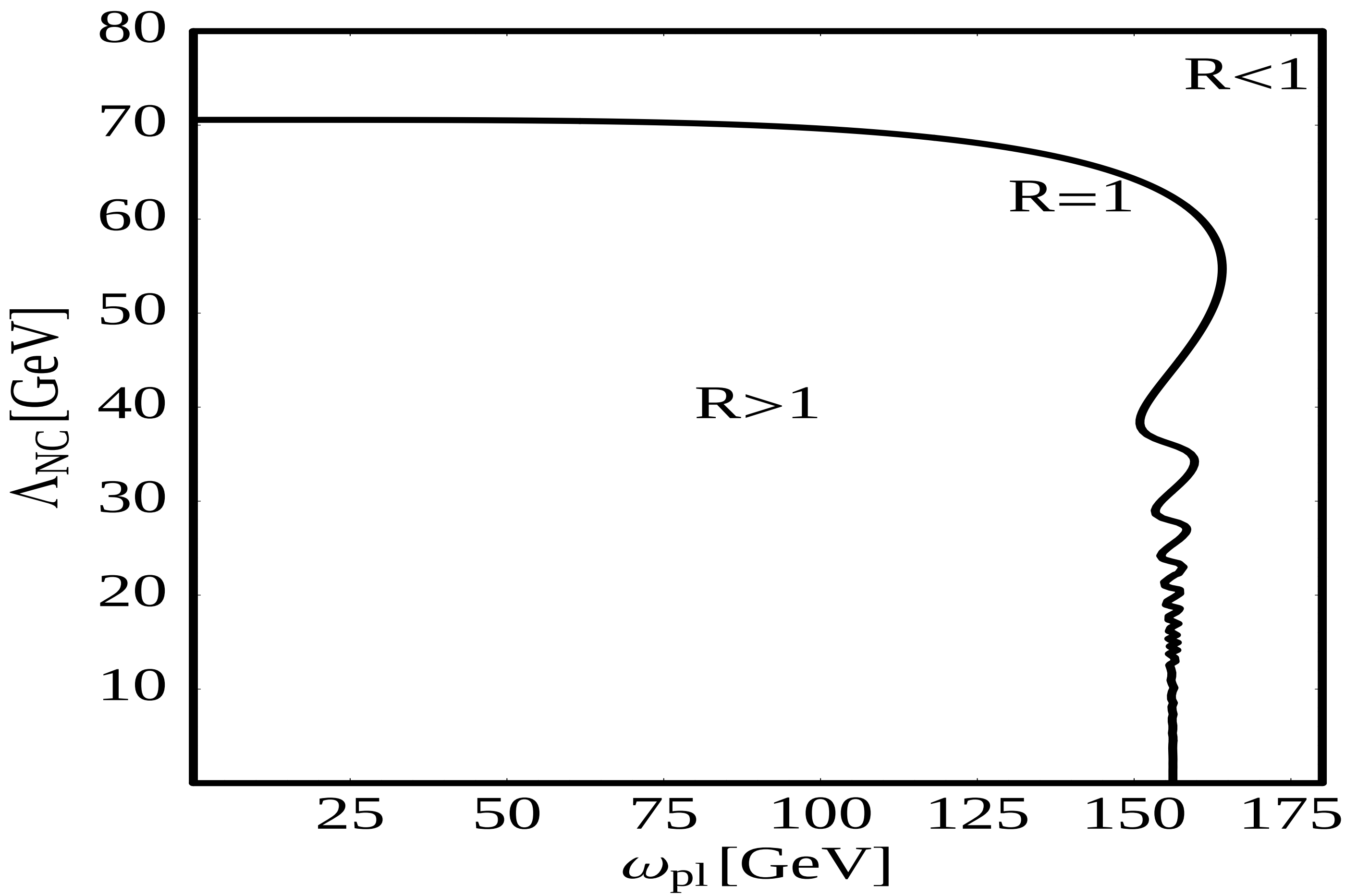
$$R \equiv \frac{\sum_{flavors}\Gamma_{\rm NC}(\gamma_{pl.} \rightarrow \bar{\nu}_L\nu_L + \bar{\nu}_R\nu_R)}{\sum_{flavors}\Gamma_{\rm SM}(\gamma_{pl.} \rightarrow \bar{\nu}_L\nu_L)}$$

$$= \frac{3\cdot 48\pi^2\alpha^2}{(c_{\nu_e}^2 + c_{\nu_\mu}^2 + c_{\nu_\tau}^2)\, G_F^2\, \omega_{pl}^4}\left(1 - \frac{\sin\xi}{\xi}\right),$$

For ν_e, we have $c_{\rm v} = \frac{1}{2} + 2\sin^2\Theta_W$, while for ν_μ and ν_τ we have $c_{\rm v} = -\frac{1}{2} + 2\sin^2\Theta_W$. For $1 - \frac{\sin\xi}{\xi} = \frac{1}{6}\xi^2 - \frac{1}{120}\xi^4 + \ldots$, the ω_{pl} dependence vanishes.

$R/\Lambda_{\rm NC}/\omega_{pl}$

[R. Horvat, D. Kekez, P. Schupp, J. Trampetić and J. You, *Photon-neutrino interaction in θ-exact covariant noncommutative field theory*, arXiv:1103.3383v1]



The plot of scale $\Lambda_{\rm NC}$ versus the plasmon frequency ω_{pl} with $R = 1$

Neutrino charge radius

[P. Minkowski, P. Schupp, and J. Trampetic, *Neutrino dipole moments and charge radii in NC spacetime*, EPJC 37 (2004) 123]; [R. Horvat, D. Kekez, P. Schupp, J. Trampetić and J. You, *Photon-neutrino interaction in θ-exact covariant NCFT*, arXiv:1103.3383v1]

$$\Gamma(\gamma_{pl.} \to \bar{\nu}_{\rm L}\nu_{\rm L}) = \frac{\alpha}{144}\frac{q^6}{E_\gamma}\left|\langle r^2_\nu\rangle\right|^2 \to \left|\langle r^2_\nu\rangle\right| = \lim_{\omega_{pl}\to 0}\frac{6\sqrt{2}}{\omega^2_{pl}}\sqrt{1-\frac{\sin\xi}{\xi}}\,.$$

The limit $\omega_{pl} \to 0$ picks up only the first term that corresponds to the θ^1 result. This implies that there are no θ-exact corrections to the θ^1 charge radius which was obtained earlier

$$|\langle r^2_\nu\rangle| = \frac{\sqrt{3}}{\Lambda^2_{\rm NC}}\,.$$

Very stringent bound on $\langle r^2_{\nu_R}\rangle$ based on SN1987A . With $\langle r^2_{\nu_R}\rangle \lesssim 2\times 10^{-33}{\rm cm}^2$, one obtains $\Lambda_{\rm NC} \gtrsim 0.6$ TeV.

BBN from $f^{\pm}\,\nu_R \to f^{\pm}\,\nu_R$

[R. Horvat and J. Trampetic, *Constraining spacetime noncommutativity with primordial nucleosynthesis*, Phys. Rev. D **79** (2009) 087701]

Energy density of 3 light ν_R at nucleosynthesis time ($T \sim 1$MeV) is equivalent to the effective additional number of doublet neutrino specis ΔN_ν ($\lesssim 1$):

$$3\left(\frac{T_{\nu_R}}{T_{\nu_L}}\right)^4 \lesssim \Delta N_{\nu,max}\ , \qquad \frac{T_{\nu_R}}{T_{\nu_L}} = \left[\frac{g_{*S}(T_{\nu_L})}{g_{*S}(T_{dec})}\right]^{1/3}\ ,$$

here g_{*S} are degrees of freedom specifying the entropy of the still interacting species

$$\sigma_{scatt}(f^{\pm}\,\nu_R \to f^{\pm}\,\nu_R) \simeq 36\,\alpha^2 E^2/\Lambda^4_{\rm NC}, \qquad E \simeq 9T\ .$$

ν_R decouple at T_C when thermally averaged scatt. rate Γ_{scatt} and H -expansion rate of the Univ. in radiation-dominated epoch are about equal $\Gamma_{scatt}(T_{dec}) \simeq H(T_{dec})$.

BBN from $f^{\pm}\,\nu_R \to f^{\pm}\,\nu_R$

[R. Horvat and J. Trampetic, *Constraining spacetime noncommutativity with primordial nucleosynthesis*, Phys. Rev. D **79** (2009) 087701]

$$\Gamma_{scatt}(T_{dec}) \;=\; < n_{scatt}\,\sigma_{scatt}\,v >, \quad n_{scatt} \simeq 0.18T^3$$
$$H(T_{dec}) \;\simeq\; 1.66 g_*^{1/2} T^2/M_{Pl}, \quad g_* \simeq g_{*S}\;.$$

This and σ_{scatt} gives

$$T_{dec} \simeq 0.5\,\alpha^{-2/3} M_{Pl}^{-1/3} \Lambda_{\rm NC}^{4/3}\;.$$

Imposing conservative bound $\Delta N_{\nu,max} = 1, (e,\mu,s)$ enforces constraint $T_{dec} > T_C$ (- critical temperature for deconfinment restoration phase transition);

$$T_{dec} \stackrel{<}{\sim}\; 200{\rm MeV} \Longrightarrow \Lambda_{\rm NC} \stackrel{>}{\sim} 3{\rm TeV}.$$

For $\Delta N_{\nu,max} < 0.2,$ (all charged lepton and quarks) we have

$$T_{dec} \stackrel{<}{\sim} 300{\rm GeV} \Longrightarrow \Lambda_{\rm NC} \stackrel{>}{\sim} 10^3{\rm TeV}.$$

BBN from $\Gamma(\gamma_{pl} \to \bar{\nu}\nu)$

[R. Horvat, D. Kekez, P. Schupp, J. Trampetić and J. You, *Photon-neutrino interaction in θ-exact covariant NCFT*, arXiv:1103.3383v1]

The RH neutrino is commonly considered to decouple at the temperature T_{dec} satisfying the condition with the Hubble expansion rate

$$\Gamma(\gamma_{pl.} \to \bar{\nu}_R \nu_R) \simeq H(T_{dec}) \simeq 1.66\, g_* \frac{T_{dec}^2}{M_{Pl}}, \quad \omega_{pl} = \frac{eT_{dec}}{3}\, g_*^{ch},$$

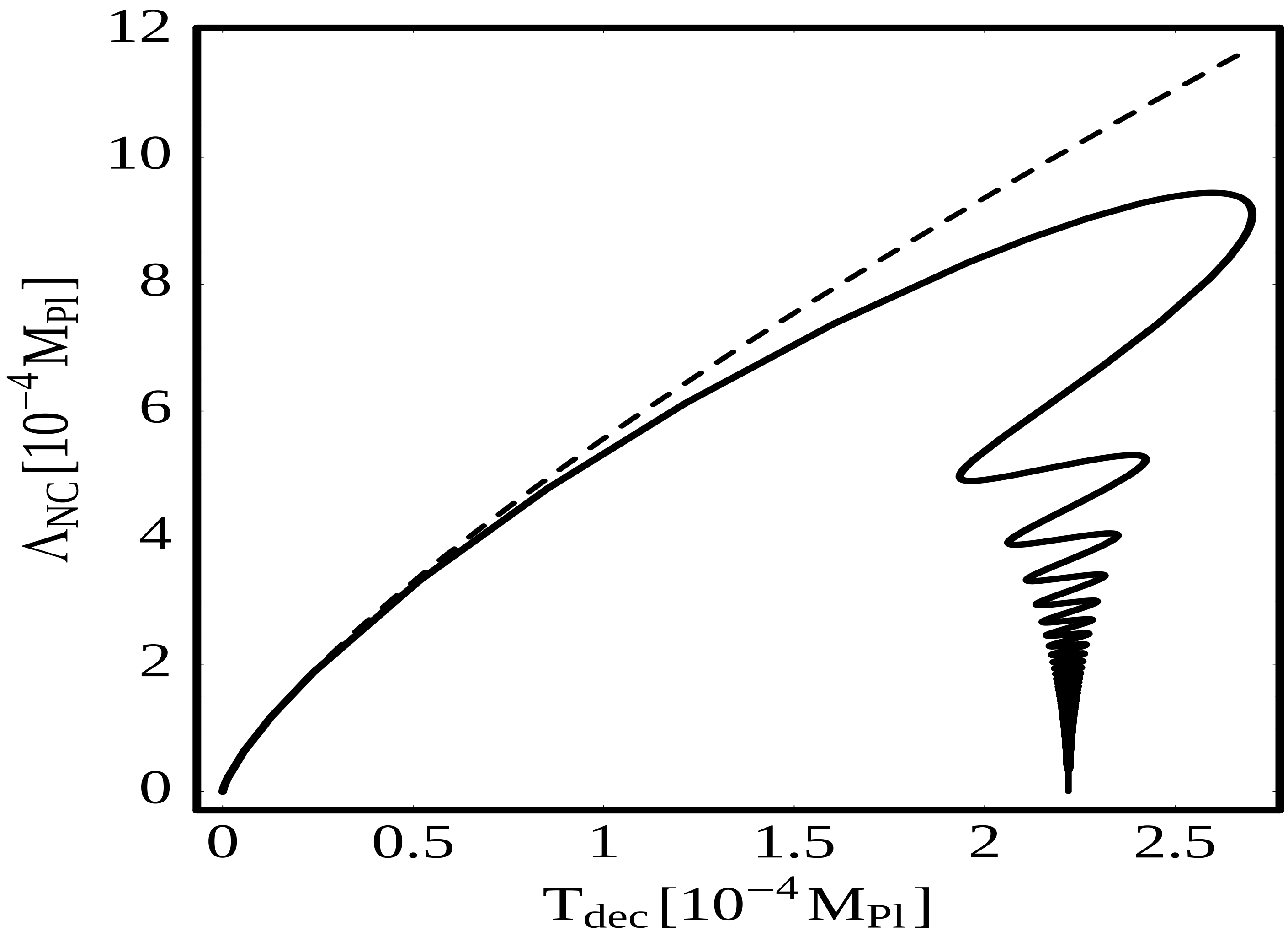
Computing the decoupling temperature T_{dec} based on the assumption that the decay rate is solely due to the NC effects and comparing with lower bounds on T_{dec} that can be inferred from observational data, we can determine lower bounds on the NC scale $\Lambda_{\rm NC}$ from

$$T_{dec} \simeq \frac{M_{pl} e^3 g_*^{ch}}{39.84 \pi g_*} \left(1 - \frac{\sin\xi}{\xi}\right), \quad \xi = \frac{e^2 (g_*^{ch})^2 T_{dec}^2}{18 \Lambda_{\rm NC}^2}$$

$$T_{dec} \simeq 2.22 \times 10^{-4} M_{Pl} \left(1 - \frac{\sin\xi}{\xi}\right) \stackrel{\xi \to 0}{\Longrightarrow} \Lambda_{\rm NC} > 3.68(887)\,{\rm TeV}.$$

BBN from $\Gamma(\gamma_{pl} \to \bar{\nu}\nu)$

[R. Horvat, D. Kekez, P. Schupp, J. Trampetić and J. You, *Photon-neutrino interaction in θ-exact covariant noncommutative field theory*, arXiv:1103.3383v1]



The plot of the scale $\Lambda_{\rm NC}$ versus T_{dec} for perturbative/exact solution (dashed/full curve).

NCFT and Holography

[R. Horvat and J. Trampetić, *Constraining NCFT with holography*, *JHEP* **01** (2011) 112]

UV/IR mixing in NCFT - understood via nonplanar loops. In a theory without UV completion ($\Lambda_{\rm UV} \to \infty$) phases, becomes inefficient to control the vanishing momenta, i.e., the original UV divergences reappear as IR div. Theory thus becomes an effective QFT with the UV and the IR cutoffs obeying

$$\Lambda_{\rm UV}\Lambda_{\rm IR} \sim \Lambda_{\rm NC}^2 \; .$$

From absolute Bekenstein-Hawking bound $S_{\rm BH} \sim L^2 M_{Pl}^2$, and properties of effective QFT in a box of size L, (IR, UV cutoffs), with respect to black hole physics stringent constraint is obeyed

$$\Lambda_{\rm UV}^3 \Lambda_{\rm IR}^{-3} \lesssim M_{Pl}^{3/2} \Lambda_{\rm IR}^{-3/2} \sim S_{\rm BH}^{3/4} \; .$$

NCFT and Holography

[R. Horvat and J. Trampetić, *Constraining NCFT with holography*, *JHEP* **01** (2011) 112]

$$\Lambda_{\rm IR} \gtrsim \Lambda_{\rm NC}\left(\frac{\Lambda_{\rm NC}}{M_{Pl}}\right)^{1/3}, \quad \Lambda_{\rm UV} \lesssim \Lambda_{\rm NC}\left(\frac{M_{Pl}}{\Lambda_{\rm NC}}\right)^{1/3}$$

Considering the muon $\Delta(g_\mu - 2) \sim \frac{\alpha}{\pi}\left[\left(\frac{m_\mu}{\Lambda_{\rm UV}}\right)^2 + \left(\frac{\Lambda_{\rm IR}}{m_\mu}\right)^2\right]$

$\Lambda_{\rm NC} \gtrsim m_\mu \longrightarrow \Delta(g_\mu - 2)_{\rm IR} \sim \frac{\alpha}{\pi}\left(\frac{\Lambda_{\rm IR}}{m_\mu}\right)^2$, , this together with

$\frac{g_\mu - 2}{2}({\rm Exp} - {\rm SM}) = (22 - 26)\times 10^{-10}$, gives

$$m_\mu \lesssim \Lambda_{\rm NC} \lesssim 0.1 \text{ TeV}$$

$$\Lambda_{\rm IR} \lesssim 10^{-1}\ \text{ MeV} \text{ and } 10^5\ \text{ MeV} \lesssim \Lambda_{\rm UV} \lesssim 10^5\ \text{ TeV}$$

$\Lambda_{\rm NC} \lesssim m_\mu \longrightarrow \Delta(g_\mu - 2)_{\rm IR} \sim \frac{\alpha}{\pi}\left(\frac{m_\mu}{\Lambda_{\rm UV}}\right)^2$, gives

$$10^{-4}\text{ MeV} \lesssim \Lambda_{\rm NC} \lesssim {\rm m}_\mu$$

$$\Lambda_{\rm UV} \gtrsim 10^2\ \text{ GeV} \text{ and } 10^{-1}\ \text{ MeV} \lesssim \Lambda_{\rm IR} \lesssim 10^{-13}\ \text{ MeV}$$

SUMMARY

1. The θ-exact covariant NCFT, motivated by UHECR-ν.
2. Action, SW map based, is covariant and gauge invariant.
3. It posses UV/IR mixing in photon self-energy.
4. Neutrino 2-point function indicate interesting new behavior regarding UV/IR and Renormalizability :) !
5. Physical quantities as a functions of energy behaves correctly for full energy scale.

SUMMARY

6. In plasmon physics: Decay rate becomes finite - good defined function of $\Lambda_{\rm NC}$ and ω_{pl} (see Fig.).

7. Neutrino charge radius: there are no θ-exact corrections to θ^1-results.

8. In BBN from plasmon decay, the $\Lambda_{\rm NC}$ becomes finite - good defined function of $T_{\rm dec}$ (see Fig.).

9. Connection of effective NCFT with Holography via $\Lambda_{\rm IR}/\Lambda_{\rm NC}/\Lambda_{\rm UV}/M_{Pl} \to$ window to Quantum Gravity?

Limits on $\Lambda_{\rm NC}$ from theory and experiments

DECAYS: 1$\rightarrow$ 2

* $Z \rightarrow \gamma\gamma \quad \Rightarrow \Lambda_{\rm NC} > 1000$ GeV, [Buric,Latas,Radovanovic, JT]

* $J/\psi \rightarrow \gamma\gamma \quad \Rightarrow \Lambda_{\rm NC} \;\; > \;\; 250$ GeV, [C. Tamarit, J.T.]

* $\Upsilon \rightarrow \gamma\gamma \quad \Rightarrow \Lambda_{\rm NC} \;\; > \;\; 250$ GeV, [C. Tamarit, J.T.]

* $K \rightarrow \pi\gamma \qquad \Rightarrow \Lambda_{\rm NC} \;\; > \;\; 43$ GeV, [Melic, Passek, J.T.]

SCATTERINGS: 2$\rightarrow$ 2

* $e^+e^- \rightarrow \gamma\gamma \Rightarrow \Lambda_{\rm NC} > 141$ GeV, [OPAL Coll.-2003]

* $\gamma\gamma \rightarrow \bar{f}f \qquad \Rightarrow \Lambda_{\rm NC} > 200$ GeV, [T. Ohl et al.]

* $\bar{f}f \rightarrow Z\gamma \qquad \Rightarrow \Lambda_{\rm NC} > 1000$ GeV, [T. Ohl et al.]

* $e^+e^- \rightarrow W^+W^- \Rightarrow \Lambda_{\rm NC} \in [.1, 1]$ TeV, [Conley, Hewett]

* $WW \rightarrow WW \Rightarrow \Lambda_{\rm NC} \in [.5, 5]$ TeV, [Conley, Hewett]

g-2:

* $\mu^+ - \mu^- \Rightarrow \Lambda_{\rm NC} \;\; > \;\; 1000$ GeV, [A. Joseph]

Limits on $\Lambda_{\rm NC}$ from theory and experiments

ASTROPHYSICS

DECAYS: 1$\rightarrow$ 2

* $\gamma_{\rm pl} \rightarrow \nu\bar{\nu} \quad \Rightarrow \Lambda_{\rm NC} > 81$ GeV, [Schupp, JT, Wess, Raffelt]

NEUTRINO DIPOLE MOMENTS:

* $({\rm d_{mag}})^{\rm Dirac} \quad \Rightarrow \Lambda_{\rm NC} > 1.8$ TeV, [Minkowski et al.]
* $({\rm d^{el}_{mag}})^{\rm Majorana} \Rightarrow \Lambda_{\rm NC} > 150$ TeV, [Minkowski et al.]
* $\frac{dipole_{NC}}{anapole_{SM}} \Rightarrow \Lambda_{\rm NC} > 10$ TeV, [Ettefaghi, Haghighat]

SCATTERINGS - Supernova SN1987A:

* $f\bar{f} \rightarrow \nu_R\bar{\nu}_R \quad \Rightarrow \Lambda_{\rm NC} > 3.7$ TeV, [Ettefaghi, Haghighat]
* $f\nu_R \rightarrow f\nu_R \quad \Rightarrow \Lambda_{\rm NC} < 1.1$ TeV, [Ettefaghi, Haghighat]

COSMOLOGY

* BBN$_{(e,\mu,s)} \Rightarrow \Lambda_{\rm NC} > 3$ TeV, [R. Horvat, J.T.]
* BBN$_{(all\ fermions)} \Rightarrow \Lambda_{\rm NC} > 10^3$ TeV, [R. Horvat, J.T.]