Toward A Theory Of Emergent Matters

Hyun Seok Yang

(Institute for the Early Univese, Ewha Womans University)

Bayrischzell Workshop 2011
Noncommutativity and physics: Spacetime Quantum Geometry
Bayrischzell, May 20-23, 2011

Outline

Apology

This talk will be very speculative.
So please be patient for some tough ideas.

Lesson from Quantum Mechanics

Stability of atoms in classical mechanics

- Recall the Rutherford's solar system picture of the atom
- Classical electromagnetism predicts an intensive radiation from orbiting electrons
- Atoms are unstable and so our Universe too
- Quantum mechanics comes to rescue the stability of atoms
  $[x^i, p_k] = i\hbar\delta^i_k$: Noncommutative phase space
- Wave-particle duality in NC phase space: $\lambda = \frac{2\pi\hbar}{p}$

Quantum Field Theory

QFT = Special relativity $\oplus$ Quantum mechanics

- A particle is regarded as a point
- A particle has mass m and a stable ($\Delta t \to \infty$) particle is in an eigenstate of Hamiltonian
- Einstein relation: $E = mc^2$. A massive particle carries a huge energy !
- Nonsense for the point particle
- Layman's answer: Well, the concept of point particle is just an approximation. A particle has a tiny size and the huge energy may be stored there.
- If so, the stability problem of particle arises !
- Can the stability of particle be resolved by QFT ?

Noncommutative Space: θ-deformation

Stability of atoms: NC phase space $\bowtie$ Stability of particles: NC spacetime ?

- Gravity: Spacetime symmetry
  Electromagnetism: Internal symmetry
- Inner Automorphism in NC spacetime
  $[y^a, y^b]_\star = i\theta^{ab}$
- In NC spacetime, Internal symmetry $\Rightarrow$ Spacetime symmetry
- Gauge/Gravity duality in NC spacetime
- Gravity is emergent $\Rightarrow$ Spacetime should be emergent too !
- If spacetime is emergent, every structure supported on spacetime should be emergent too !

Background Independent Quantum Gravity

- 0-dimensional IKKT Matrix model: No *a priori* spacetime structure

$$S_{IKKT} = -\frac{1}{8\pi g_s}\mathrm{Tr}[\Phi^a, \Phi^b]^2 \qquad (1)$$

- Algebraic relations:

$$[\Phi^{[a}, [\Phi^b, \Phi^{c]}]] = 0, \qquad [\Phi_a, [\Phi^a, \Phi^b]] = 0. \qquad (2)$$

- Specify a vacuum: $\Phi^a_{\rm vac} = \frac{y^a}{\kappa} \Leftrightarrow \langle B_{ab}\rangle_{\rm vac} = (\theta^{-1})_{ab}$ which defines $\mathbf{R}^d_{NC}$: $[y^a, y^b]_\star = i\theta^{ab}$ where $a, b = 1, \cdots, d$.
- Expand $\Phi^a \equiv \frac{\theta^{ab}}{\kappa}\widehat{D}_b$ where $\widehat{D}_a = (B_{ab}y^b + \widehat{A}_a(y))$.
- IKKT Matrix model = NC U(1) gauge theory on $\mathbf{R}^d_{NC}$

$$S_{NC} = \frac{1}{4g^2_{YM}}\int d^d y(\widehat{F}_{ab} - B_{ab}) \star (\widehat{F}^{ab} - B^{ab}) \qquad (3)$$

where $\widehat{F}_{ab} = \partial_a\widehat{A}_b - \partial_b\widehat{A}_a - i[\widehat{A}_a, \widehat{A}_b]_\star$.

Emergent Geometry

Emergent geometry from NC gauge theory

- The relation between orthonormal frmaes $E_a \in \Gamma(TM)$, $E^a \in \Gamma(T^*M)$ in Einstein gravity and gauge theory bases $X_a \in \Gamma(TM)$, $V^a \in \Gamma(T^*M)$: $X_a = \lambda E_a$ and $E^a = \lambda V^a$ where $\lambda^2 = \det^{-1} V^a_b$.
- $ds^2 = E^a \otimes E^a = \lambda^2 \delta_{ab} V^a_c V^b_d dy^c dy^d$ where $V^a_c X^c_b = \delta^a_b$.

Einstein equations from NC gauge fields

- $\{D_a, \{D_b, D_c\}_\theta\}_\theta = \widehat{D}_a \widehat{F}_{bc} \;\Leftrightarrow\; X_{\widehat{D}_a \widehat{F}_{bc}} = [X_a, [X_b, X_c]]$
- Bianchi identity $\widehat{D}_{[a} \widehat{F}_{bc]} = 0 \Leftrightarrow R_{[abc]d} = 0$.
- Equations of motion $\widehat{D}_a \widehat{F}^{ab} = 0 \Leftrightarrow R_{ab} - \frac{1}{2} g_{ab} R = 8\pi G T_{ab}$.
  where $T_{ab} = T^{(M)}_{ab} + T^{(L)}_{ab}$.

- $\exists$ natural concept of Emergent Time: $(M, \omega = B)$
  Hamiltonian vector field $X_H : \iota_{X_H} \omega = dH \;\Rightarrow\; \frac{df}{dt} = X_H(f) = \{f, H\}_{\omega^{-1}}$

Gravity As A Large N Duality

$U(N \to \infty)$ Yang-Mills theory on $\mathbf{R}^{3,1}$ = NC $U(1)$ gauge theory on $\mathbf{R}^{3,1} \times \mathbf{R}^6_{NC}$

- $$S = -\frac{1}{g_{YM}^2} \int d^4 z \mathrm{Tr} \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \Phi^a)^2 - \frac{1}{4} [\Phi^a, \Phi^b]^2 \right) \tag{4}$$
  where we introduced 6 adjoint scalar fields Φ^a.
- Consider a vacuum $\Phi^a = y^a, \; a = 1, \cdots, 6$ and $A_\mu = 0$ where $[y^a, y^b]_\star = i\theta^{ab} : \mathbf{R}^6_{NC}$.
- Any background independent NC gauge theory can be expressed only in terms of covariant objects, i.e., $D_\mu(z, y) = \partial_\mu - iA_\mu(z, y)$ and $\Phi^a(z, y) = y^a + \theta^{ab} \widehat{A}_b(z, y)$.
- $$S = -\frac{1}{4g_{YM}^2} \int d^{10} X (F_{MN} - B_{MN})^2 \tag{5}$$
  with $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]_\star$.

Ward geometry

(Ward, CQG **7** (1990) L217)

- Vector fields $\Leftrightarrow$ NC fields $X^M = (D_\mu, \Phi^a)$:
  $\mathrm{ad}_{X^M}[f] \equiv [X^M, f]_\star = -i\theta^{ab}\frac{\partial X^M}{\partial y^b}\frac{\partial f}{\partial y^a} + \cdots \equiv X^a_M(z,y)\partial_a f(z,y) + \mathcal{O}(\theta^3)$.
- Let y^a be local coordinates on M_6, then locally,

$$A_\mu(z) = A^a_\mu(z,y)\frac{\partial}{\partial y^a}, \qquad X_a(z) = X^b_a(z,y)\frac{\partial}{\partial y^b}$$

- $f^{-1}(X_1, \cdots, X_4, X_1, \cdots, X_6)$ forms an orthonormal frame and hence defines a metric on $\mathbf{R}^{3,1} \times M_6$ where f is a scalar, a conformal factor, defined by $f^2 = \omega(X_1, \cdots, X_6)$

Emergent geometry from large N matrices

$$ds^2 = f^2\eta_{\mu\nu}dz^\mu dz^\nu + f^2\delta_{ab}V^a_c V^b_d(dy^c - \mathbf{A}^c)(dy^d - \mathbf{A}^d) \qquad (6)$$

where $\mathbf{A}^a = A^a_\mu dz^\mu$ and $V^a_c X^c_b = \delta^a_b$

Topological origin of matters

- A remarkable aspect of the large N gauge theory (53) is that it admits a rich variety of topological objects.
- Consider a stable class of time-independent solutions satisfying our asymptotic boundary condition and so matrices $\Phi^a(\mathbf{x})$ are nondegenerate along $\mathbf{S}^3 = \mathbf{R}^3 \cup \{\infty\}$ and so Φ^a defines a well-defined map

$$\Phi^a : \mathbf{S}^3 \to GL(N, \mathbf{C}) \tag{7}$$

  from $\mathbf{S}^3$ to the group of nondegenerate complex $N \times N$ matrices.
- If $\Phi^a \in \pi_3(GL(N, \mathbf{C}))$, the solution Φ^a will be stable under small perturbations, and the corresponding nontrivial element of $\pi_3(GL(N, \mathbf{C}))$ represents a topological invariant.
- In the stable regime where $N > \frac{3}{2}$, the homotopy groups of $GL(N, \mathbf{C})$ or $U(N)$ define a generalized cohomology theory, known as K-theory K(X). In our case, this group is given by

$$K(\mathbf{R}^{3,1}) = \pi_3(GL(N, \mathbf{C})) = \mathbf{Z}. \tag{8}$$

Atiyah-Bott-Shapiro: Clifford Modules

- ABS theorem: K-theory generators in (8) can be constructed in terms of Clifford module.
- The construction uses the gamma matrices $\Gamma^\mu : S_+ \to S_-$ to satisfy the Dirac algebra $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$ of the Lorentz algebra $SO(3,1)$ and $S_\pm$ be two irreducible spinor represenations of $Spin(3,1)$.
- Introduce a Dirac operator $\mathcal{D} : V \times S_+ \to V \times S_-$ such that $\mathcal{D} = \Gamma^\mu p_\mu + \cdots$ acting on a Hilbert space V as well as a spinor vector space $S_\pm$.
- The ABS construction implies that the Dirac operator $\mathcal{D}$ is a generator of $\pi_3(U(N))$ as a nontrivial topology in momentum space $(\mathbf{p}, \omega)$ and acts on low lying excitations near the vacuum which carry K-theory charges and so are stable.
- ABS construction shows that the Dirac operator $\mathcal{D}$ acts on collective (coarse-grained) modes of the solution (7) satisfying the Dirac equation

$$i\Gamma^\mu(\partial_\mu - ieA_\mu - iA_\mu^a T^a)\chi + \cdots = 0. \tag{9}$$

Emergent Matters from Stable Geometries

An explicit construction of Dirac operator depends on the topological class of the solution (7).

Noncommutative $U(1)$ instanton as Calabi-Yau 2-fold

- NC $U(1)$ instantons from $\Phi^a(a=1,2,3,4) \cong$ Calabi-Yau 2-folds
- $SU(2)$ holonomy group of CY 2-folds = $SU(2)$ gauge group for (fermionic) zero modes χ
- NC $U(1)$ instanton formed in extra dimensions may appear as a four-dimensional chiral fermion in $SU(2)$ representation. Leptons ?

Noncommutative Hermitian $U(1)$ instanton as a Calabi-Yau 3-fold

- NC Hermitian $U(1)$ instantons from $\Phi^a(a=1,\cdots,6) \cong$ Calabi-Yau 3-folds
- $SU(3)$ holonomy group of CY 3-folds = $SU(3)$ gauge group for (fermionic) zero modes χ
- NC Hermitian $U(1)$ instanton formed in extra dimensions may appear as a four-dimensional chiral fermion in $SU(3)$ representation. Quarks ?

Thank you for your attention and patience.

Feynman's Question F. J. Dyson, Am. J. Phys. 58, 209 (1990)

- Suppose that a particle exists with position q^i and velocity $\dot{q}_i$ satisfying commutation relations

$$[q^i, q^j] = 0, \qquad [q^i, \dot{q}_j] = i\hbar\delta^i_j. \tag{10}$$

- Feynman asks a question: What is the most general form of forces appearing in the Newton's equation consistent with the commutation relation (10) ?
- $m\frac{d\mathbf{v}}{dt} = e\big(\mathbf{E} + \mathbf{v} \times \mathbf{B}\big)$
- For $SU(2)$ or $SU(3)$ interactions, the quantum particle dynamics is defined by a symplectic structure on $T^*\mathbf{R}^3 \times F$ where the coordinates T^a of F satisfy the commutation relations
  $[T^a, T^b] = if^{abc}T^c, \quad [q^i, T^a] = 0$
- The generators of the $SU(n)$ symmetry on the Fock space: $T^a = a_i^\dagger \lambda^a_{ik} a_k$
- $i\Gamma^\mu(\partial_\mu - iA^a_\mu T^a)\chi + \cdots = 0$ where χ is a quark, e.g., for $n = 3$.