Extending the Neutrino Sector of a Noncommutative Standard Model

Jiangyang You

A collaboration with Raul Horvat, Amon Ilakovac, Dalibor Kekez, Peter Schupp and Josip Trampetic

Bayrischzell Workshop 2011
Noncommutativity and Physics: Spacetime Quantum Geometry

Outline

Neutrino

- Two features in desire:

  1. Seesaw mechanism provides a robust mass and mixing generation mechanism (sadly not an NC mechanism).

  2. Neutral particles can Couple to a $U_\star(1)$ gauge field via a commutator
  $$D_\mu \nu = \partial_\mu \nu - i[A_\mu \stackrel{\star}{,} \nu].$$

NCSM

- One model, the “minimal” NCSM model (Calmet et.al., Eur.Phys.J.C 23 (2002) 363-376; Melic et.al., Eur.Phys.J.C 42 (2005) 483-497&499-504...) to extend.
- Seiberg-Witten map eases the gauge invariance checking, formal invariance under infinitesimal noncommutative gauge transformation requested.

Model

- Constructions starts by coupling right hand neutrino to the $U_\star(1)_Y$ hypercharge field

$$D_\mu N_R = \partial_\mu N_R - i[B^0_\mu \stackrel{\star}{,} N_R].$$

- We pick up the same Yukawa "sandwich" as in the minimal NCSM

$$\mathcal{Y} \sim \bar{\Psi} \star H \star \Psi'.$$

  and seek for formal noncommutative gauge invariance.

- For simplicity we try to fix the fermion Seiberg-Witten map to be the same across the action.

Yukawa terms

- The gauge invariance can be ensured by appropriate hybrid gauge transformation, e.g.

$$\delta_\Lambda \Psi = i\left(R_l(\Lambda) \star \Psi + \Psi \star R_r(\Lambda)\right)$$

and hybrid Seiberg-Witten map for each field

$$\begin{aligned}
\delta_\Lambda(\bar\Psi \star H \star \Psi') = & - i(\bar\Psi \star R_l(\Lambda) + R_r(\Lambda) \star \bar\Psi) \star H \star \Psi' + \bar\Psi \star \delta_\Lambda H \star \Psi' \\
& + i\bar\Psi \star H \star (R'_l(\Lambda) \star \Psi' + \Psi' \star R'_r(\Lambda)) \\
:= & - iR_r(\Lambda) \star \bar\Psi \star H \star \Psi' + i\bar\Psi \star H \star \Psi' \star R_r(\Lambda) \\
& - i\bar\Psi \star R_l(\Lambda) \star H \star \Psi' + i\bar\Psi \star H \star R'_l(\Lambda) \star \Psi' \\
& + i\bar\Psi \star (R_l(\Lambda) \star H - H \star R'_l(\Lambda)) \star \Psi'.
\end{aligned}$$

- For our case

$$\delta_{\Lambda_{Y\star}} \mathcal{V}_R = ikg_Y[\Lambda_{Y\star} \stackrel{\star}{,} \mathcal{V}_R],$$

$$\delta_{\Lambda_{Y\star}} L_R = ig_Y\left[(-1+k)\Lambda_{Y\star} \star L_R - kL_R \star \Lambda_{Y\star}\right]$$

$$\delta_{\Lambda_{Y\star}} \Psi_L = ig_Y\left[(-\frac{1}{2}+k)\Lambda_{Y\star} \star \Psi_L - k\Psi_L \star \Lambda_{Y\star}\right],$$

$$\delta_{\Lambda_{Y\star}} H^d = ig_Y\left[(-\frac{1}{2}+k)\Lambda_{Y\star} \star H^d + (1-k)H^d \star \Lambda_{Y\star}\right]$$

$$\delta_{\Lambda_{Y\star}} (H^d)^c = ig_Y\left[(-\frac{1}{2}+k)\Lambda_{Y\star} \star (H^d)^c - k(H^d)^c \star \Lambda_{Y\star}\right].$$

Majorana mass

- Yukawa terms fix the gauge transformation for fermions, these will then restrict the choice of Majorana mass terms.
- Seesaw subtypes request different Majorana mass terms
  1. Singlet majorana term $i\Psi_R^T \star H^s \star \sigma_2 \Psi_R$ for seesaw type I works with the transformations specified above if
  $$\delta_{\Lambda_{Y\star}} H^s = ike_Y [\Lambda_{Y\star} \stackrel{\star}{,} H^s].$$
  2. Problem with Doublet majorana term via triplet Higgs
  $$\delta_{\Lambda_{Y\star}} \left((\psi_L^*)^T \star \left(\frac{i}{\sqrt{2}} \tau \cdot \Delta \cdot \tau_2 \right) \star \sigma_2 \psi_L^* \right) = ie_Y \frac{1}{2} \left[(\psi_L^*)^T \star \Lambda_{Y\star} \star \left(\frac{i}{\sqrt{2}} \tau \cdot \Delta \cdot \tau_2 \right) \star \sigma_2 \psi_L^* \right.$$
  $$\left. + (\psi_L^*)^T \star \delta_{\Lambda_{Y\star}} \left(\frac{i}{\sqrt{2}} \tau \cdot \Delta \cdot \tau_2 \right) \star \sigma_2 \psi_L^* + (\psi_L^*)^T \star \left(\frac{i}{\sqrt{2}} \tau \cdot \Delta \cdot \tau_2 \right) \star \sigma_2 \psi_L^* \star \Lambda_{Y\star} \right].$$
  3. Seesaw mechanism type III with triplet neutrinos works.

Properties

- Default feature of this model are the seesaw mechanism type I (and III) and noncommutative photon/Z-right hand neutrino coupling

$$\hat{S}_{\nu_R} = \int d^4x \; - ke \tan\vartheta_W \left(\bar{\nu}_R\gamma^\mu[z_\mu \stackrel{\star}{,} \nu_R] + i\theta^{ij}\left(\left(\partial_\mu\bar{\nu}_R\right)\gamma^\mu\left(z_i \star_2 \partial_j\nu_R\right)\right.\right.$$
$$\left.\left. - \left(z_i \star_2 \partial_j\bar{\nu}_R\right)\gamma^\mu\left(\partial_\mu\nu_R\right)\right)\right) + ke\left(\bar{\nu}_R\gamma^\mu[a_\mu \stackrel{\star}{,} \nu_R]\right.$$
$$\left. + i\theta^{ij}\left(\left(\partial_\mu\bar{\nu}_R\right)\gamma^\mu\left(a_i \star_2 \partial_j\nu_R\right) - \left(a_i \star_2 \partial_j\bar{\nu}_R\right)\gamma^\mu\left(\partial_\mu\nu_R\right)\right)\right)$$

- Induced features include noncommutative photon-neutrino (L/R), photon/Z-singlet Higgs coupling etc..

$$
\begin{aligned}
\hat{S}_L = \int d^4x\, \frac{g_L}{\sqrt{2}} & \left(\bar{\nu}_L \not{w}^+ \star l_L + \frac{i}{2}\theta^{ij} \Big((\partial_\mu \bar{\nu}_L)\gamma^\mu (w_i^+ \bullet (\partial_j l_L)) + ((\partial_i \bar{\nu}_L) \bullet w_j^+)\gamma^\mu (\partial_\mu l_L) \Big) \right) \\
+ \frac{g_L}{\sqrt{2}} & \left(\bar{l}_L \not{w}^- \star \nu_L + \frac{i}{2}\theta^{ij} \Big((\partial_\mu \bar{l}_L)\gamma^\mu (w_i^- \bullet (\partial_j \nu_L)) + ((\partial_i \bar{l}_L) \bullet w_j^-)\gamma^\mu (\partial_\mu \nu_L) \Big) \right) \\
+ \frac{e}{\sin 2\vartheta_W} & \left(\bar{\nu}_L \not{z} \star \nu_L + \frac{i}{2}\theta^{ij} \left((\partial_\mu \bar{\nu}_L)\gamma^\mu (z_i \bullet (\partial_j \nu_L)) + ((\partial_i \bar{\nu}_L) \bullet z_j)\gamma^\mu (\partial_\mu \nu_L) \right) \right) \\
- \frac{e}{\tan 2\vartheta_W} & \left(\bar{l}_L \not{z} \star l_L + \frac{i}{2}\theta^{ij} \left((\partial_\mu \bar{l}_L)\gamma^\mu (z_i \bullet (\partial_j l_L)) + ((\partial_i \bar{l}_L) \bullet z_j)\gamma^\mu (\partial_\mu l_L) \right) \right) \\
- e & \left(\bar{l}_L \not{a} \star l_L + \frac{i}{2}\theta^{ij} \left((\partial_\mu \bar{l}_L)\gamma^\mu (a_i \bullet (\partial_j l_L)) + ((\partial_i \bar{l}_L) \bullet a_j)\gamma^\mu (\partial_\mu l_L) \right) \right) \\
- \int d^4x\, ke \tan\vartheta_W & \left(\bar{\psi}_L \gamma^\mu [z_\mu \stackrel{\star}{;} \psi_L] + i\theta^{ij} \left(\left(\partial_\mu \bar{\psi}_L\right) \gamma^\mu \left(z_i \star_2 \partial_j \psi_L \right) - \left(z_i \star_2 \partial_j \bar{\psi}_L \right) \gamma^\mu \left(\partial_\mu \psi_L \right) \right) \right) \\
+ \int d^4x\, ke & \left(\bar{\psi}_L \gamma^\mu [a_\mu \stackrel{\star}{;} \psi_L] + i\theta^{ij} \left(\left(\partial_\mu \bar{\psi}_L\right) \gamma^\mu \left(a_i \star_2 \partial_j \psi_L \right) - \left(a_i \star_2 \partial_j \bar{\psi}_L \right) \gamma^\mu \left(\partial_\mu \psi_L \right) \right) \right)
\end{aligned}
$$

......

Outlook

- We build an NCSM model action with seesaw mechanism and noncommutative neutrino-photon coupling included.
- Vertices and Phenomenology associated are subjects for further studies.
- More seesaw could be possible if we loose certain assumption.
- Some work has been carried out on the θ-exact one loop neutrino self-energy, which is covered in the talk by Amon Ilakovac.

Bonus feature

- It is possible to expand the right hand neutrino sector with respect to the formal power of fields or the power of coupling constant, but not θ.
- A simpler action

$$S = \int d^4x \, -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i\bar{\Psi}\gamma^{\mu} \left(\partial_{\mu}\Psi - i \left[A_{\mu} \stackrel{\star}{,} \Psi\right]\right)$$

has been subjected to loop calculations for some time.

$$
\begin{aligned}
S_{bb\bar{\nu}_R\nu_R} = \int d^4x\, & \bar{\nu}_R \star \gamma^\mu [b^0_\mu \stackrel{\star}{,} \nu_R] - i(\theta^{ij} b^0_i \star_2 \partial_j \bar{\nu}_R) \star \partial\!\!\!/ \nu_R - i\bar{\nu}_R \star \partial\!\!\!/ (\theta^{ij} b^0_i \star_2 \partial_j \nu_R) \\
& - (\theta^{ij} b^0_i \star_2 \partial_j \bar{\nu}_R) \star \gamma^\mu [b^0_\mu \stackrel{\star}{,} \nu_R] - \bar{\nu}_R \star \gamma^\mu [b^0_\mu \stackrel{\star}{,} \theta^{ij} b^0_i \star_2 \partial_j \nu_R] \\
& - \bar{\nu}_R \star \gamma^\mu [\frac{1}{2}\theta^{ij} b^0_i \star_2 (\partial_j b^0_\mu + f_{j\mu}) \stackrel{\star}{,} \nu_R] + i(\theta^{ij} b^0_i \star_2 \partial_j \bar{\nu}_R) \star \partial\!\!\!/ (\theta^{kl} b^0_k \star_2 \partial_l \nu_R) \\
& + i \left(\frac{1}{2}\theta^{ij}\theta^{kl} \left[(b^0_k \star_2 (\partial_l b^0_i + f_{li})) \star_2 \partial_j \bar{\nu}_R + 2 b^0_i \star_2 (\partial_j (b^0_k \star_2 \partial_l \bar{\nu}_R)) \right] \right. \\
& \left. - \frac{1}{2}\theta^{ij}\theta^{kl} a_i \star_2 (\partial_k a_j \star_2 \partial_l \bar{\nu}_R) - \frac{1}{2}\theta^{ij}\theta^{kl} \left[-b^0_i \partial_k \bar{\psi} (\partial_j b^0_l + f_{jl}) + \partial_k \partial_i \bar{\nu}_R b^0_j b^0_l \right]_{\star_3} \right) \star \partial\!\!\!/ \nu_R \\
& + i\bar{\nu}_R \star \partial\!\!\!/ \left(\frac{1}{2}\theta^{ij}\theta^{kl} \left[(b^0_k \star_2 (\partial_l b^0_i + f_{li})) \star_2 \partial_j \nu_R + 2 b^0_i \star_2 (\partial_j (b^0_k \star_2 \partial_l \nu_R)) \right] \right. \\
& \left. - \frac{1}{2}\theta^{ij}\theta^{kl} b^0_i \star_2 (\partial_k b^0_j \star_2 \partial_l \nu_R) - \frac{1}{2}\theta^{ij}\theta^{kl} \left[-b^0_i \partial_k \nu_R (\partial_j b^0_l + f_{jl}) + \partial_k \partial_i \nu_R b^0_j b^0_l \right]_{\star_3} \right)
\end{aligned}
$$

Thanks!

$$f \bullet g = \cdot \left(\frac{e^{\frac{i}{2}\theta^{ij}\partial_i \otimes \partial_j} - 1}{\frac{i}{2}\theta^{ij}\partial_i \otimes \partial_j} \right) f \otimes g.$$

$$f \star_2 g = \cdot \frac{e^{\frac{i}{2}\theta^{ij}\partial_i \otimes \partial_j} - e^{-\frac{i}{2}\theta^{ij}\partial_i \otimes \partial_j}}{2i(\frac{1}{2}\theta^{ij}\partial_i \otimes \partial_j)} f \otimes g$$

$$= f(x_1) \frac{\sin \frac{\partial_1 \wedge \partial_2}{2}}{\frac{\partial_1 \wedge \partial_2}{2}} g(x_2)\Big|_{x_1 = x_2 = x}.$$

$$[f(x)g(x)h(x)]_{\star_3} = \int dp_1 e^{ip_1 x} \tilde{f}(p_1) \int dp_2 e^{ip_2 x} \tilde{g}(p_2) \int dp_3 e^{ip_3 x} \tilde{h}(p_3)$$

$$\cdot \left[\frac{\sin(\frac{p_2 \wedge p_3}{2}) \sin(\frac{p_1 \wedge (p_2 + p_3)}{2})}{\frac{(p_1 + p_2) \wedge p_3}{2} \, \frac{p_1 \wedge (p_2 + p_3)}{2}} + \frac{\sin(\frac{p_1 \wedge p_3}{2}) \sin(\frac{p_2 \wedge (p_1 + p_3)}{2})}{\frac{(p_1 + p_2) \wedge p_3}{2} \, \frac{p_2 \wedge (p_1 + p_3)}{2}} \right].$$

$$\Sigma_1 = \frac{1}{(4\pi)^{\frac{D}{2}}} 2\not{\tilde{p}} p^2 \left(\frac{\mathrm{tr}\,\theta\theta}{\tilde{p}^2} + \frac{2\tilde{\tilde{p}}^2}{\tilde{p}^4} \right) \left[(p^2)^{\frac{D}{2}-2} \Gamma\left(2 - \frac{D}{2}\right) B\left(\frac{D}{2} - 1, \frac{D}{2}\right) - 2\int_0^1 dx (1-x) K_0 \left[\left(x(1-x) p^2 \tilde{p}^2 \right)^{\frac{1}{2}} \right] \right]$$

$$+ \frac{1}{(4\pi)^{\frac{D}{2}}} \left\{ 2\left(\not{\tilde{p}} \left(1 - \frac{D}{2}\right) + \frac{p^2 \not{\tilde{\tilde{p}}}}{\tilde{p}^2} - \frac{\mathrm{tr}\,\theta\theta}{2} \frac{p^2 \not{\tilde{p}}}{\tilde{p}^2} \right) - \frac{\not{\tilde{p}}}{\tilde{p}^4} \left(\tilde{\tilde{p}}^2 p^2 - \tilde{p}^4 \right) \right\}$$

$$\cdot \frac{1}{2} \int_0^1 dx (1-x) (\tilde{p}^2)^{2-\frac{D}{2}} \frac{\pi}{\sin \frac{D\pi}{2}} \left[\left(x(1-x) p^2 \tilde{p}^2 \right)^{\frac{D}{2}-1} \Gamma\left(\frac{1}{2}\right) \cdot {}_1\tilde{F}_2 \left(\frac{1}{2}; \frac{3}{2}, \frac{D}{2}; \frac{x(1-x)p^2\tilde{p}^2}{4} \right) \right.$$

$$\left. - 2^{D-2} \Gamma\left(\frac{3-D}{2}\right) \cdot {}_1\tilde{F}_2 \left(\frac{3-D}{2}; \frac{4-D}{2}, \frac{5-D}{2}; \frac{x(1-x)p^2\tilde{p}^2}{4} \right) \right]$$

$$
\begin{aligned}
\Sigma_2 \propto \int \frac{d^D p}{(2\pi)^D} \frac{g_{i_1 i_2}}{p^2} \Bigg\{ & -4i \frac{\sin^2 \frac{p\wedge k}{2}}{p\wedge k} \tilde{k}^{i_1} \gamma^{i_2} - 4i \frac{\sin^2 \frac{p\wedge k}{2}}{p\wedge k} \tilde{k}^{i_2} \gamma^{i_1} - 4i \frac{\sin^2 \frac{p\wedge k}{2}}{(p\wedge k)^2} (-\not{p} \\
& + \not{k}) \tilde{k}^{i_1} \tilde{k}^{i_2} + 2i\not{k} \Big[(-p\wedge k \theta^{i_1 i_2} + 2\tilde{p}^{i_1} \tilde{k}^{i_2}) - \frac{\sin^2 \frac{p\wedge k}{2}}{(p\wedge k)^2} 2(-\tilde{p} - \tilde{k})^{i_1} \tilde{k}^{i_2} - \frac{\sin^2 \frac{p\wedge k}{2}}{p\wedge k} \theta^{i_1 i_2} \\
& + \frac{\sin^2 \frac{p\wedge k}{2}}{(p\wedge k)^2} (-2\tilde{k}^{i_2} \tilde{p}^{i_1} - \theta^{i_1 i_2} k\wedge p - \tilde{k}^{i_1} \tilde{k}^{i_2}) \Big] + 2i\not{k} \Big[(-2\tilde{p}^{i_1} \tilde{k}^{i_2} + p\wedge k \theta^{i_1 i_2}) \\
& + \frac{\sin^2 \frac{p\wedge k}{2}}{(p\wedge k)^2} 2(-\tilde{p} + \tilde{k})^{i_1} \tilde{k}^{i_2} + \frac{\sin^2 \frac{p\wedge k}{2}}{p\wedge k} \theta^{i_1 i_2} - \frac{\sin^2 \frac{p\wedge k}{2}}{(p\wedge k)^2} (-2\tilde{k}^{i_2} \tilde{p}^{i_1} - \theta^{i_1 i_2} k\wedge p + \tilde{k}^{i_1} \tilde{k}^{i_2}) \Big] \\
& + 4i \frac{\sin^2 \frac{p\wedge k}{2}}{(p\wedge k)^2} \tilde{k}^{i_2} \gamma^{i_1} + 4i \frac{\sin^2 \frac{p\wedge k}{2}}{p\wedge k} \tilde{k}^{i_1} \gamma^{i_2} - 4i \frac{\sin^2 \frac{p\wedge k}{2}}{(p\wedge k)^2} (\not{p} \\
& + \not{k}) \tilde{k}^{i_2} \tilde{k}^{i_1} + 2i\not{k} \Big[(p\wedge k \theta^{i_2 i_1} - 2\tilde{p}_2^{i_2} \tilde{k}^{i_1}) - \frac{\sin^2 \frac{p\wedge k}{2}}{(p\wedge k)^2} 2(\tilde{p} - \tilde{k})^{i_2} \tilde{k}^{i_1} + \frac{\sin^2 \frac{p\wedge k}{2}}{p\wedge k} \theta^{i_2 i_1} \\
& + \frac{\sin^2 \frac{p\wedge k}{2}}{(p\wedge k)^2} (2\tilde{k}^{i_1} \tilde{p}^{i_2} + \theta^{i_2 i_1} k\wedge p - \tilde{k}^{i_2} \tilde{k}^{i_1}) \Big] + 2i\not{k} \Big[(2\tilde{p}^{i_2} \tilde{k}^{i_1} - p\wedge k \theta^{i_2 i_1}) \\
& + \frac{\sin^2 \frac{p\wedge k}{2}}{(p\wedge k)^2} 2(\tilde{p} + \tilde{k})^{i_2} \tilde{k}^{i_1} - \frac{\sin^2 \frac{p\wedge k}{2}}{p\wedge k} \theta^{i_2 i_1} - \frac{\sin^2 \frac{p\wedge k}{2}}{(p\wedge k)^2} (2\tilde{k}^{i_1} \tilde{p}^{i_2} + \theta^{i_2 i_1} k\wedge p + \tilde{k}^{i_2} \tilde{k}^{i_1}) \Big] \Bigg\} \\
= 0
\end{aligned}
$$

- In commutative QED

$$A^{\mu}(q) = \int d^4x \left\langle \Omega \left| \bar{\psi}(x) e^{-iqx} \gamma^{\mu} \psi(x) \right| \Omega \right\rangle = \int d^4x\, e^{-iqx} \left\langle \Omega \left| j^{\mu}(x) \right| \Omega \right\rangle = 0$$

- In NCQED

$$A^{\mu}_{\star}(q) = \int d^4x \left\langle \Omega \left| \bar{\psi}(x) \star e^{-iqx} \gamma^{\mu} \star \psi(x) \right| \Omega \right\rangle \sim \mathrm{tr} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^{\mu}}{\not{k}} e^{-i\frac{q\theta k}{2}} \sim \frac{\tilde{q}^{\mu}}{\tilde{q}^4}$$

- $\star$ commutator does not improve either

$$\mathrm{tr} \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^{\mu}}{\not{k}} \sin \frac{q\theta k}{2} \sim \frac{\tilde{q}^{\mu}}{\tilde{q}^4}$$