Restrictions on infinite sequences of type IIB vacua

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Motivation: the landscape

Outline of the talk

- Type IIB flux compactifications
  - No-go theorem by Maldacena and Nunez
  - GKP's evasion strategy
- Type IIB moduli stabilization
  - Calabi-Yau geometry
  - Flux vacua
  - No-go theorem by Ashok and Douglas
- Sequences in type IIB approaching D-limits
  - Mirror Quintic
  - LCS point
  - numerical results

  } one-parameter CY

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Sector	IIA	IIB
NS⊗NS R⊗R	$g_{\mu\nu}$ $B_{(2)}$ ϕ $C_{(1)}, C_{(3)}, \ldots, C_{(7)}$	$g_{\mu\nu}$ $B_{(2)}$ ϕ $C_{(0)}, C_{(2)}, \ldots, C_{(8)}$
NS⊗R R⊗NS	Ψ_M λ Ψ'_M λ'	Ψ_M λ Ψ'_M λ'

$$F_{(p+1)} = dC_{(p)}$$

$$*F_{(10-p-1)} = F_{(p+1)}$$

$$\begin{aligned} S^{IIB}_{Bose} &= S_{NS} + S_R + S_{CS} \\ &= \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left[R - \frac{1}{2}\frac{\partial_M\tau\partial^M\bar{\tau}}{(\mathrm{Im}\,\tau)^2} - \frac{1}{2}\frac{|G_{(3)}|^2}{\mathrm{Im}\,\tau} - \frac{1}{2}|\tilde{F}^2_{(5)}|\right] + S_{CS} \end{aligned}$$

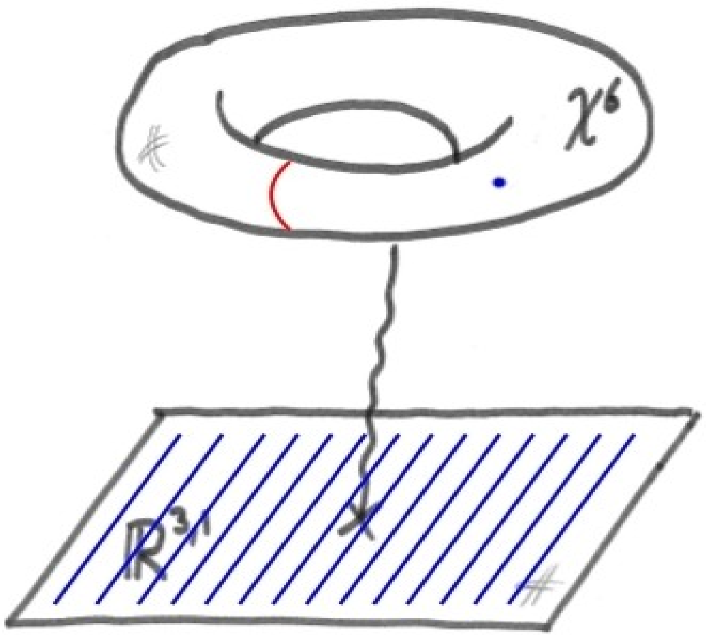
$$\tau = C_{(0)} + ie^{-\phi}$$

$$G_{(3)} = F_{(3)} - \tau H_{(3)}$$

$$\tilde{F}_{(5)} = F_{(5)} - \frac{1}{2}C_{(2)} \wedge H_{(3)} + \frac{1}{2}B_{(2)} \wedge F_{(3)}$$

Semi classical approx. of space-time $\mathcal{M}^{10} = \mathbb{R}^{3,1} \times X^6$

warped metric ansatz:



$$g_{MN} = \begin{pmatrix} e^{2A(y)}\eta_{\mu\nu} & 0 \\ 0 & e^{-2A(y)}\tilde{g}_{mn}(y) \end{pmatrix}$$

$$\begin{cases} \tau = \tau(y) \\ G_{(3)} \in H^3(X, \mathbb{Z}) \\ \tilde{F}_{(5)} = (1 + *)\left[d\alpha(y) \wedge d\mathrm{Vol}_4\right] \end{cases}$$

Einstein's equations:

$$\tilde{\nabla}^2 e^{4A} = e^{2A}\frac{1}{2}\frac{|G_{(3)}|^2}{\mathrm{Im}\,\tau} + e^{-6A}\left(|\partial\alpha|^2 + |\partial e^{4A}|^2\right)$$

$$\overset{\int_X}{\Longrightarrow} \quad A = \mathrm{const}\,, \quad \alpha = \mathrm{const} \quad \text{and} \quad G_{(3)} = 0$$

No-go theorem [Maldacena-Nunez '00]

Including only fluxes $\Longrightarrow$ 4D geometry cannot be M (or dS)

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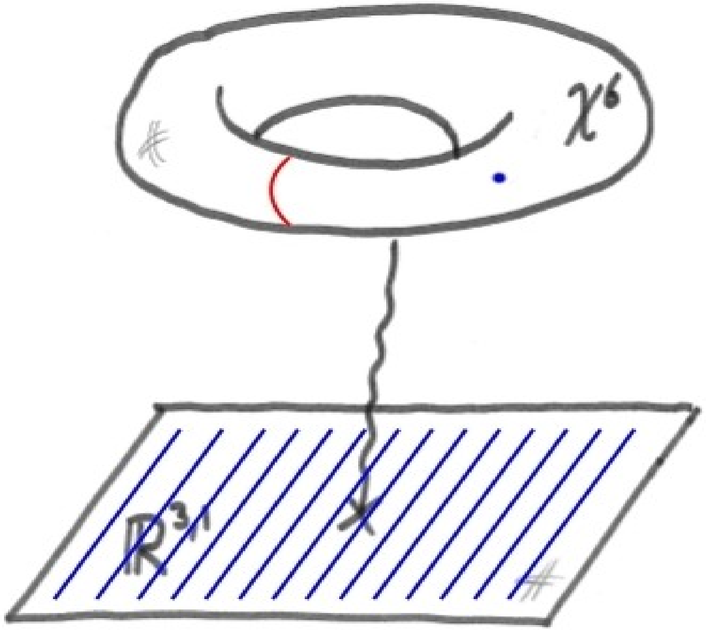

$$\begin{cases} \tau = \tau(y) \\ G_{(3)} \in H^3(X,\mathbb{Z}) \\ \tilde{F}_{(5)} = (1+*)\left[d\alpha(y) \wedge d\mathrm{Vol}_4\right] \end{cases}$$

1) Einstein's equations: $S = S^{IIB}_{Bose} + S_{loc}$ $\longrightarrow$ $T_{MN} = T^{IIB}_{MN} + T^{loc}_{MN}$

$$\tilde{\nabla}^2 e^{4A} = e^{2A}\frac{1}{2}\frac{|G_{(3)}|^2}{\mathrm{Im}\,\tau} + e^{-6A}\left(|\partial\alpha|^2 + |\partial e^{4A}|^2\right) + k^2 e^{2A}\left(T^m_m - T^\mu_\mu\right)_{loc}$$

2) Bianchi identity: $d\tilde{F}_{(5)} = H_{(3)} \wedge F_{(3)} + 2\kappa^2 T_3 \rho_3$

$$\Longrightarrow \quad \frac{1}{2\kappa^2 T_3}\int H_{(3)} \wedge F_{(3)} + N_3 = 0$$

Tadpole cancellation condition

GKP's evasion strategy

[GKP '01]

$$\tilde{\nabla}^2 \left(e^{4A} - \alpha\right) = e^{2A} \frac{|iG_{(3)} - *G_{(3)}|^2}{24\,\mathrm{Im}\,\tau} + e^{-6A}|\partial\left(e^{4A} - \alpha\right)|^2 + 2\kappa^2 e^{2A}\left(\frac{1}{4}T_{loc} - T_3\rho_3\right)$$

BPS-like condition: $T_{loc} \geq 4T_3\rho_3$

1) restricts the choice of local sources (tree level)

- D3, O3, D7 and O7 saturate the inequality
- anti-D3 satisfy
- O5, anti-O3, etc. violate

2) ISD condition: $*G_{(3)} = iG_{(3)}$ $\left(\Longleftrightarrow G_{(3)} \in H^{(2,1)} \oplus H^{(0,3)}\right)$

3) $e^{4A} = \alpha(y) \quad \Longrightarrow \quad \tilde{F}_{(5)}(A)$

If no D7 branes $\Longrightarrow$ $\tilde{R}_{mn} = 0\,, \qquad \partial_m \tau = 0$

i.e. X is conformally CY

A clear strategy:

1) choose X s.t. $\tilde{R}_{mn} = \ldots$ $\tilde{\nabla}^2 \tau = \ldots$

2) consider localized objects that saturate BPS-like bound

3) $\tilde{F}_{(5)}$ with ISD $G_{(3)}$

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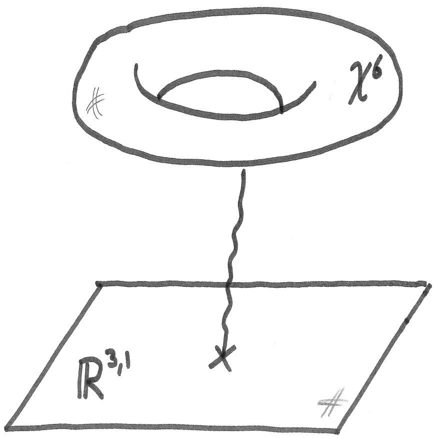


$\mathcal{N} = 1, 2$ SUSY survives compactif. $\Longrightarrow \nabla_{\mathcal{X}} \eta = 0$

- No fluxes: complex Kähler w. $c_1(\mathcal{X}) = 0$ [CHSW '85]

(• Fluxes: generalized complex structures [Hitchin '02, Gualtieri '04])

$$\omega_{ij} = i\eta^\dagger \gamma_{[i}\gamma_{j]}\eta \qquad \Omega \sim \eta^T \gamma_{[i}\gamma_j\gamma_{k]}\eta$$

$\mathcal{X}$ is Calabi-Yau manifold : $c_1 = 0 \Longleftrightarrow \exists \quad R_{mn} = 0$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h_{11} & & 0 & \\ 1 & & h_{21} & & h_{21} & & 1 \\ & 0 & & h_{11} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

Moduli space : $M = \boxed{M_{CS}^{2,1}} \times M_K^{1,1}$

z is $h^{(2,1)}$-dimensional complex coordinate

A set of coordinates of M_{CS} : $\Pi_I = \int_{C_I} \Omega = \int_X C_I \wedge \Omega \qquad \{C_I\} \in H_3(X)$

Periods of *X*

- Periods vector $\Pi(z) = \begin{pmatrix} \Pi_{b_3-1}(z) \\ \vdots \\ \Pi_0(z) \end{pmatrix}$ $b_3 = 2h^{(2,1)} + 2$

- Intersection matrix $Q_{IJ} = \int_{C_I} C_J = \int_X C_I \wedge C_J$ Symplectic structure

$$A^I = -(Q^{-1})^{IJ} \int_{C_J} A_{(3)}$$

Inters. prod. : $\langle A_{(3)}, B_{(3)} \rangle = \int_X A_{(3)} \wedge B_{(3)} = A \cdot Q \cdot B^T$ anti-symm. topological moduli indep.

Scalar prod. : $(A_{(3)}, B_{(3)}) = \int_X A_{(3)} \wedge *B_{(3)} = A \cdot \mathcal{G}_z \cdot B^T$ moduli dep. positive quadratic form on $\mathbb{C}^{b_3}$

- 4-dim effective theory: *N=1* scalar potential (tree-level)

$$V(z,\tau) = e^K \left(g^{i\bar{\jmath}} D_i W D_{\bar{\jmath}} \bar{W} + g^{\tau\bar{\tau}} D_\tau W D_{\bar{\tau}} \bar{W} \right)$$

We know how to express the components in terms of the periods !

Kähler potential: $K_{\mathrm{cs}}(z,\bar{z}) = -\log\left(i \int_X \Omega \wedge \bar{\Omega} \right) = -\log(i\Pi^\dagger \cdot Q^{-1} \cdot \Pi)$

Superpotential: $W = \int_X \Omega \wedge G_{(3)} = G \cdot \Pi$

- Hence the scalar potential can be computed numerically once the periods and their derivatives are known.

D3 tadpole condition revisited: $\int_X F_{(3)} \wedge H_{(3)} = \underbrace{\frac{\chi}{24} + \frac{1}{4} N_{\mathrm{O3}} - N_{\mathrm{D3}}}_{-N_3} \equiv L \leq L_{max}$

G ISD

$$\langle F_{(3)}, H_{(3)} \rangle = \frac{i}{2\mathrm{Im}\tau} \langle \bar{G}_{(3)}, G_{(3)} \rangle = \frac{1}{2\mathrm{Im}\tau} \langle \bar{G}_{(3)}, *G_{(3)} \rangle = \frac{1}{2\mathrm{Im}\tau} \bar{G} \cdot \mathcal{G}_z \cdot G^T > 0$$

Define vector $\hat{N} = (F, H) \in \mathbb{R}^{2b_3}$

Recall: are integer valued !

$$0 < L = \hat{N} \cdot (\mathcal{G}_\tau \otimes \mathcal{G}_z) \cdot \hat{N}^T \leq L_{max}$$

$$\mathcal{G}_\tau = \frac{1}{2\mathrm{Im}\tau} \begin{pmatrix} 1 & -\mathrm{Re}\tau \\ -\mathrm{Re}\tau & |\tau|^2 \end{pmatrix}$$

- $\hat{N}$ must lie within an ellipsoid in $\mathbb{R}^{2b_3}$ whose dimensions are given by the (τ, z)-dependent eigenvalues $\Lambda_i(\tau, z)$ of the matrix $\mathcal{G}_\tau \otimes \mathcal{G}_z$

No-go theorem [Ashok-Douglas '03]

Any region of (τ, z)-space for which $\Lambda_i(\tau, z)$ are bounded from below by some positive number, can support only a finite number of vacua.

$$0 < L = \hat{N} \cdot (\mathcal{G}_\tau \otimes \mathcal{G}_z) \cdot \hat{N}^T \leq L_{max}$$

$$\mathcal{G}_z = 2e^K \mathrm{Re}\left[\Pi\,\Pi^\dagger + g^{i\bar{j}} D_i \Pi\, \bar{D}_{\bar{j}} \Pi^\dagger\right]$$

$$\hat{N} \cdot \mathcal{G}_\tau \otimes \mathcal{G}_t \cdot \hat{N}^T = \sum_{i,j} |\hat{N} \cdot v_i \otimes w_j|^2 \mu_i \lambda_j = \sum_{i,j} |\epsilon_{ij}|^2 \lambda_j \mu_i$$

Evasion from the no-go

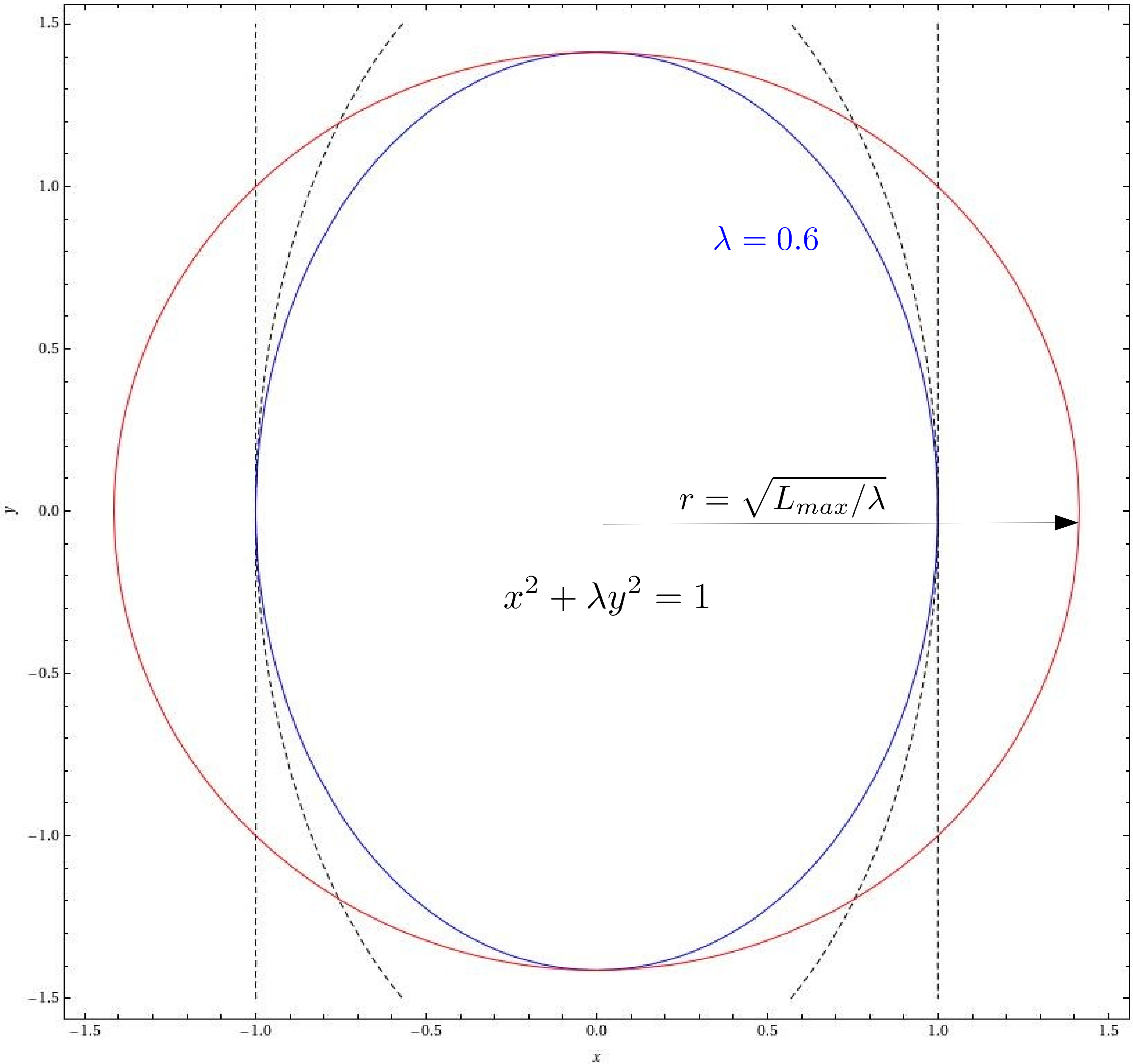
Infinite series of vacua can occur only if their location in the (τ, z)-space approaches a point where the matrix $\mathcal{G}_\tau \otimes \mathcal{G}_z$ develops a null eigenvector.

This can occur in two ways:

1) $\mathcal{G}_\tau$ degenerates, or

2) $\mathcal{G}_z$ degenerates

Points where this happens are referred to as D-limits.



$\lambda = 0.6$
$r = \sqrt{L_{max}/\lambda}$
$x^2 + \lambda y^2 = 1$
x
y

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The Mirror Quintic CY $X^{(101,1)}$

Zero locus of homogeneous polynomial in $\mathbb{P}^4$

$$p = \sum_{i=1}^{5} x_i^5 - 5\psi\, x_1 x_2 x_3 x_4 x_5 = 0 \qquad x_i \in \mathbb{P}^4$$

(more precisely: $X^{(101,1)} = \mathbb{P}^4[p]/\mathbb{Z}_5^3 + \text{ blow-ups}$)

complex structure modulus: $\psi \in \mathbb{C}$

- but $\psi \sim \alpha\psi \quad \text{w/} \quad \alpha = e^{2\pi i/5}$ can be compensated by $x_i \longrightarrow \alpha^{-1} x_i$

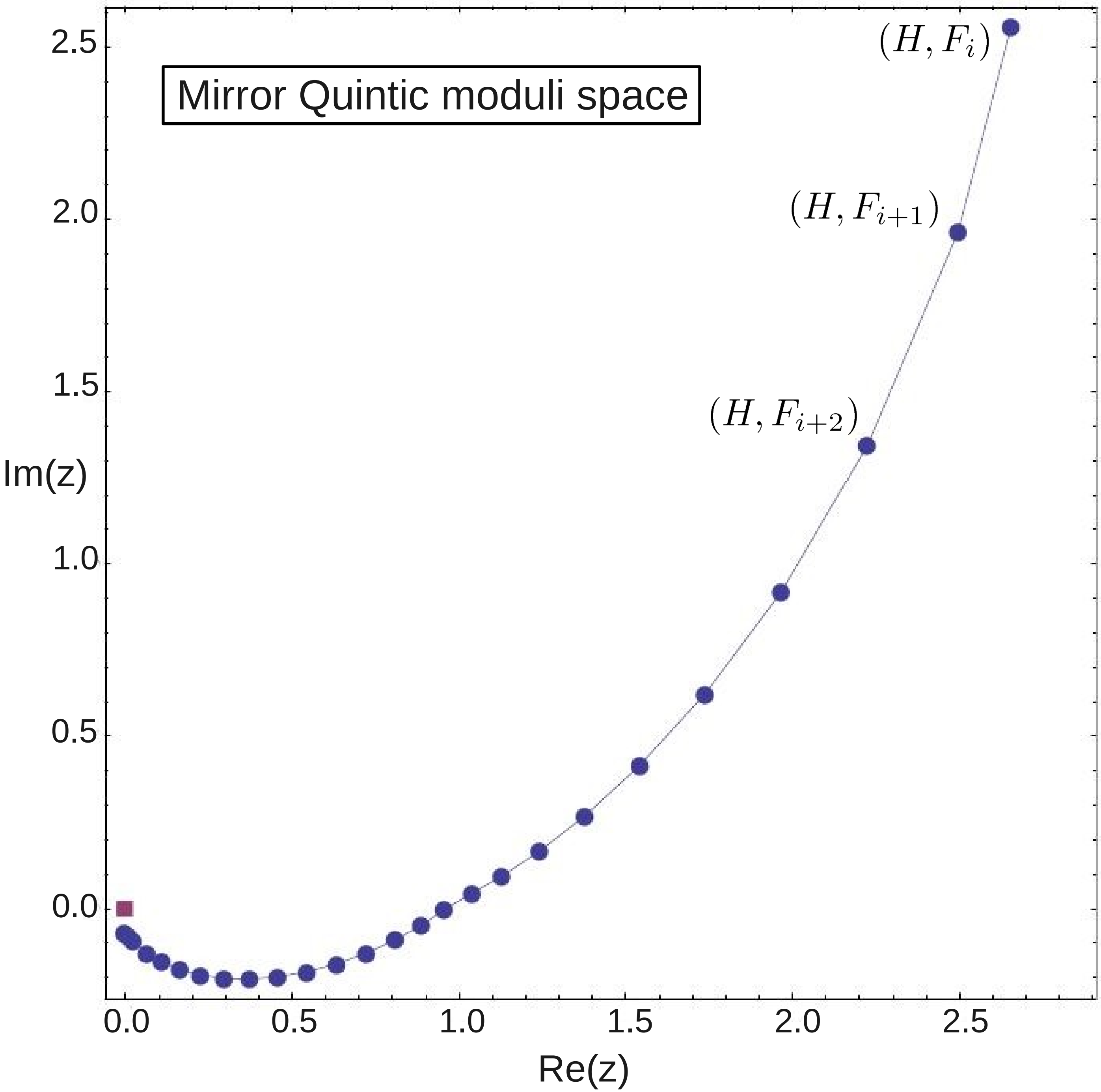
→ moduli space: $\mathbb{C}/\mathbb{Z}_5$ orbifold singularity at $\psi = 0$

- "true" coordinate: $z = 1/\psi^5$

- D-limits
  - $z = 0$ large complex structure limit
  - $z = 1$ conifold singularity

Sequences of SUSY vacua related via monodromy (H, F_n)

[Ahlqvist, Greene et al '10]



NSNS-sector

$$H = (-2, -4, -33, 0)$$

RR-sector

$$\begin{aligned} F_i &= (36, -18, 9, -1) \\ F_{i+1} &= (35, -18, 9, -1) \\ F_{i+2} &= (34, -18, 9, -1) \\ &\ldots \\ F_{i+32} &= (4, -18, 9, -1) \end{aligned}$$

Analytical study the sequences in the vicinity of the LCS point

[Braun, Johansson, Larfors, N-OW '11]

- The period vector takes following general form around LCS point:

$$\begin{pmatrix} \Pi_3 \\ \Pi_2 \\ \Pi_1 \\ \Pi_0 \end{pmatrix} \sim \begin{pmatrix} \alpha t^3 + \beta t + i\gamma \frac{\zeta(3)}{\pi^3} \\ \delta t^2 + \epsilon t + \eta \\ t \\ 1 \end{pmatrix}$$

$t \sim -i \ln z$

$\alpha, \ldots, \eta$ rational coefficients parametrizing family of one-modulus CY

[Doran, Morgan '05]

- Expand $\mathcal{G}_z = 2e^K \mathrm{Re}\left[\Pi\,\Pi^\dagger + g^{i\bar{\jmath}} D_i \Pi\, \bar{D}_{\bar{\jmath}} \Pi^\dagger\right]$

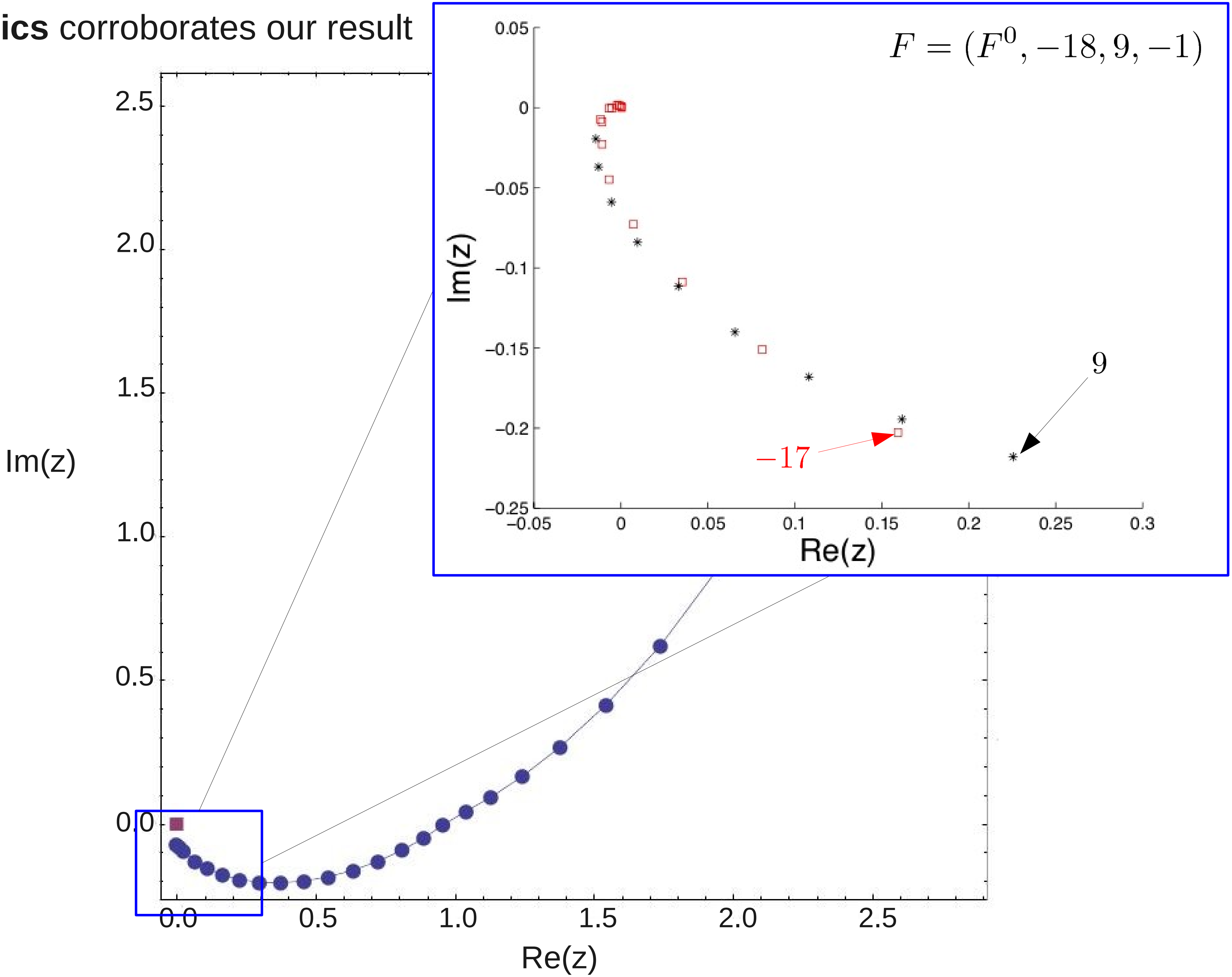
- Determine eigenvalues and eigenvectors of $\mathcal{G}_z$ (and $\mathcal{G}_\tau$) up to $\mathcal{O}(\mathrm{Im}(z)^{-3})$

- Requiring $\lim_{n\to\infty} z_n = 0\,, \qquad \lim_{n\to\infty} N_n \cdot (\mathcal{G}_\tau \otimes \mathcal{G}_{z_n}) \cdot N_n^T \neq \infty$

it follows $F^0 = F^1 = H^0 = H^1 = 0 \implies \int F_{(3)} \wedge H_{(3)} = 0$ the metric degenerates !

> This implies that there is no ISD vacuum, except the singular one
> Located exactly at the LCS point.

Numerics corroborates our result



Summary

- Ashok and Douglas: infinite sequences of type IIB ISD flux vacua can only occur in D-limits.

- We refine this no-go result: there are no infinite sequences accumulating to the LCS point of a class of one-parameter CYs. Most prominent example: Mirror Quintic.

- Similar analysis for conifold points and the decoupling limit obtaining identical results.

- Similar analysis for LCS point of a two-parameter CY.

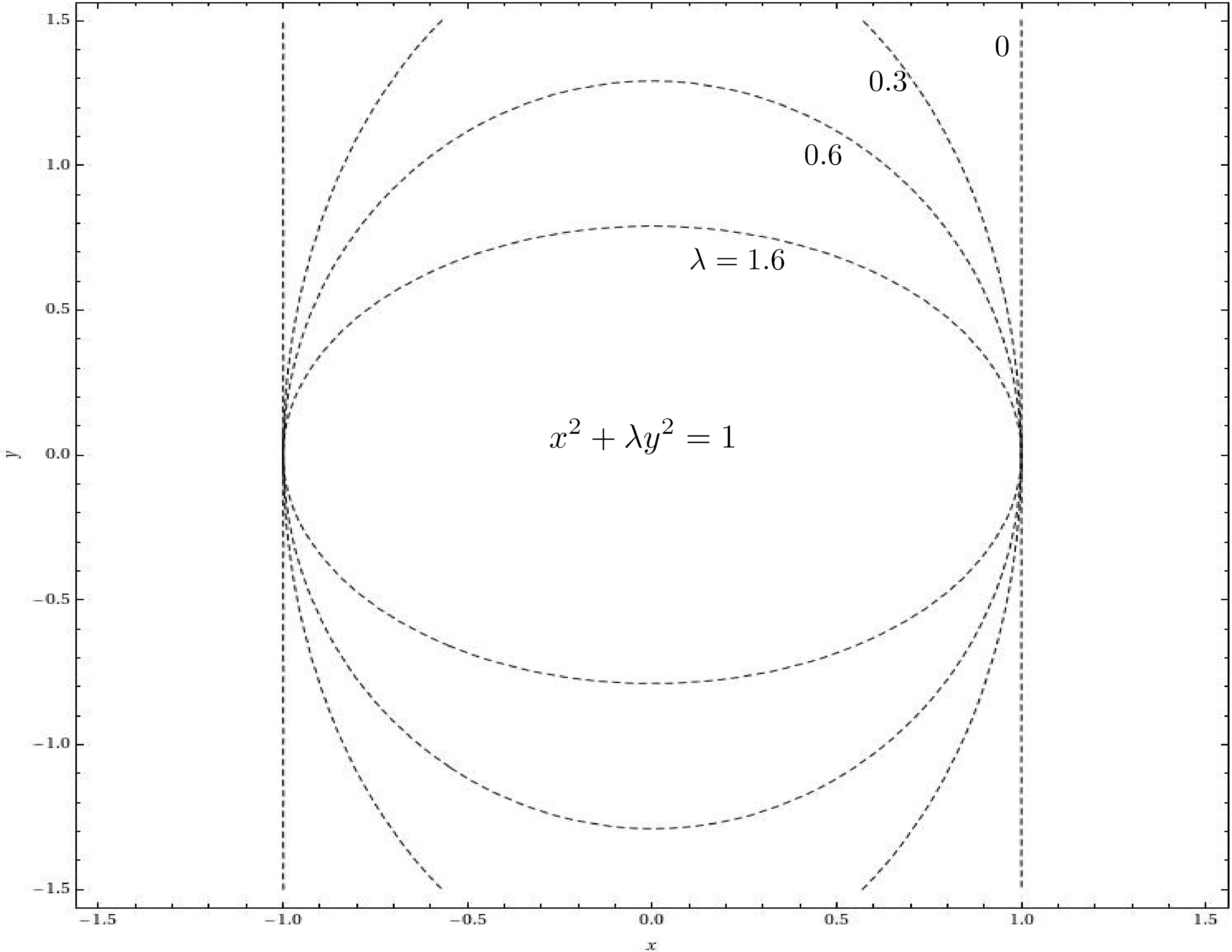
Outlook

- Formulate more general and transparent conditions on the singularity.

  → Statement of a more general finiteness theorem.

- Do similar techniques apply for more general CYs ?

- How do warping corrections affect the results ?


0
0.3
0.6
$\lambda = 1.6$
$x^2 + \lambda y^2 = 1$
y
x
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
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0.0
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