# Exact entropy and Rademacher expansion for CHL orbifold black holes

Abhiram M Kidambi
WIP w/ A. Chowdhury, T. Wrase (TU Wien)
& V. Reys (U. Milano Bicocca), S. Murthy (KCL London)

**Institute for Theoretical Physics, TU Wien**

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INSTITUTE for THEORETICAL PHYSICS
Vienna University of Technology

# Overview

Can we recover an exact supergravity - SCFT match for microstate counting in CHL orbifolds on $\mathcal{N} = 4, d = 4$ string theory?

- Dabholkar-Murthy-Zagier (DMZ) for CHL models ("The microscopic approach")
- The Rademacher expansion method and supergravity matching ("The sort of macroscopic approach")
- A note on negative discriminant states

# If you slept through Val's talk

A slide for the non-string theorists. *[DVV, DMVV, DMZ, ...]*

- $\mathcal{N} = 4, d = 4$ string theory on $K3 \times T^2$ - $\frac{1}{2}$-BPS or $\frac{1}{4}$-BPS
- $\frac{1}{2}$-BPS black holes exist everywhere in moduli space, degeneracy independent of contour choice, partition function given by $\dfrac{1}{\eta(\tau)^{24}}$
- $\frac{1}{4}$-BPS black holes are "mortal" i.e. not all of them exist everywhere in moduli space, PF given in terms of $\dfrac{1}{\Phi_{10}(\tau, \sigma, z)}$, i.e. inverse of the Igusa cusp form of wt. 10 (related to the EG of K3)

# If you slept through Val's talk

- $\frac{1}{\Phi_{10}(\tau,\sigma,z)}$ is meromorphic in $z$. Residue of the $\Phi_{10}$ integral at poles represents a jump in the degeneracy of $\frac{1}{4}$-BPS black holes.
- Jump due to when two $\frac{1}{2}$-BPS states become bound and effectively behave as a $\frac{1}{4}$-BPS state. "Wall crossing"

$$\Phi_{10}(\tau,\sigma,z) \sim \eta(\tau)^{24}\eta(\sigma)^{24}$$

$$\frac{1}{\Phi_{10}(\tau,\sigma,z)} = \sum_{m\geq -1} \psi_m(\tau,z)p^m, \; p = e^{2\pi i\sigma}$$

$\psi_m(\tau,z)$ encodes the degeneracies of all $\frac{1}{4}$-BPS black holes.

# If you slept through Val's talk

- The true $\frac{1}{4}$-BPS black holes are the finite part of $\psi_m(\tau, z)$, the bound $\frac{1}{2}$-BPS black holes are the polar part of $\psi_m(\tau, z)$.

$$\psi_m^F(\tau, z) = \psi_m(\tau, z) - \psi_m^P(\tau, z)$$

*[Dabholkar, Murthy, Zagier]*

- Holds for almost all cases of black holes (except for a subtlety regarding negative discriminant states $\Delta = (4mn - l^2) < 0$ (In the string theory picture: if $Q^2 = -2$ or $P^2 = -2$ or both )

# What we're doing

Mathematician:

DMZ concerns the theory of (M)JF's on $SL_2(\mathbb{Z})$ . Can we generalize this to congruence subgroups $\Gamma_0(N) \subset SL_2(\mathbb{Z})$ ?

Physicist :

Can we study the properties of the BH microstate counting functions under a CHL orbifold?

(CHL orbifolds are special supersymmetry preserving orbifolds.)

*[Chaudhuri, Hockney, Lykken]*

# Finite $\frac{1}{4}$-BPS degeneracies via. the product representation

- Under CHL orbifold, explicit formula for the lift of the orbifolded EG is well known for prime values of orbifolds $N = 2, 3, 5, 7$

  *[Sen, Jatkar, David; Volpato, Zimet, Paquette; Pioline, Bossard, Cosnier-Horeau;..]*
- EG is a JF on $\Gamma_0(N)$
- Can write down two equivalent lifts

$$\Phi_k(\tau, \sigma, z) \;\&\; \tilde{\Phi}_k(\tau, \sigma, z), \;\; k = \frac{24}{N+1} - 2$$

  The two lifts are related via $Sp_2(\mathbb{Z})$ transform.
- Can perform a Jacobi form decomposition of the $\tilde{\Phi}_k$ lift, the construction of the polar component (Appell-Lerch sum) and extract the finite component
- Black hole degeneracies are Fourier coefficients of this finite component

# Finite $\frac{1}{4}$-BPS degeneracies under CHL orbifold

- Straightforward to obtain the black hole degeneracies, checked for $N = 2, 3, 5, 7$
- Methods to extend to non-prime orbifolds also exist *[Govindarajan, Gopala Krishna]*
- Subtlety: Post caution about negative discriminant states. ”Bound state metamorphosis”
- Bounds state metamorphosis: Each configuration of a $\frac{1}{4}$-BPS bound state has some contribution to the supersymmetric index. For special values of black hole electric and magnetic charges, different bound states have the same index contribution in a chamber. Identify such bound states to one another. *[Sen, Chowdhury, et,al; Dabholkar, Gaiotto, Nampuri]*
- By accounting for metamorphosis, one can extract black hole degeneracies for all values of charges

# Matching with supergravity

- For an exact match, supergravity must know about these degeneracies.
- With some microscopic assumptions, evaluate the QEF in terms of the K3 prepotential and the worldsheet instanton contributions *[Reys, Murthy]*
- QEF is an infinite convergent sum of std. Bessel functions of the first kind whose coefficients are BH degeneracies *[cf. talk by Val Reys]*
- To compute these coefficients, use the Rademacher circle method with appropriate choice of multiplier systems for the Gen. Kloosterman sums. Some microscopic data is assumed here.
- Matching of coefficients with microscopic case for low values of summation in GKS already evident for low values of $m$ in $\psi_m^F(\tau, z)$. *[Murthy, Reys]*
- Extend this to the CHL cases (Rademacher expansions for (M)MJF on $\Gamma_0(N)$ )

# Rademacher expansion for CHL black holes

- Rademacher expansions for congruence subgroups commensurable with $SL_2(\mathbb{Z})$ is known *[Cheng, Duncan]*
- Subtlety: No S transforms in $\Gamma_0(N)$, use $\Gamma_{0,+}(N)$ which includes the Atkin-Lehner involution. (Atknin-Lehner involution = S-duality transform for $\Gamma_0(N)$). Focus as of now on $\Gamma_{0,+}(N)$
- Rademacher series for modular forms in $\Gamma_{0,+}(N)$ has been studied *[Nally; Sussman]*
- Difference between $\Phi_k$ and $\tilde{\Phi}_k$ is important here.
- $\Phi_k$ transforms as a modular object in $\Gamma^0(N)$ for which there is no AL involution, $\tilde{\Phi}_k$ transforms as a modular object in $\Gamma_{0,+}(N)$. Rademacher series numerically simpler in $\Gamma_{0,+}(N)$.
- Idea/Current status: To recover the coefficients of $\Phi_k$, switch to $\tilde{\Phi}_k$ via $Sp_2(\mathbb{Z})$ transform. Extract coefficients via the circle method and translate back to $\Phi_k$.

# Conclusion and WIP

- Initial impressions: Properties of JF's with regards to polar and finite decomposition seems to extend to JF's on $\Gamma_0(N)$.
- Trying to verify this with known physical calculations by computing the Rademacher series of the relevant (M)MJF's on $\Gamma_0(N)$.
- Extend to Rademacher series on $\Gamma^0(N)$ to compute coefficients of $\Phi_k$ directly.
- The exact supergravity match is as of yet to be solved, glacial progress
- Try to produce a closed form expression to recover ALL finite $\frac{1}{4}$-BPS contributions to the supersymmetric index in moduli space
- Explore number theoretic properties of BH twining by sporadic group elements *[Volpato, Zimet, Paquette; David, Chattopadhyaya]*

Thank you!