Gromow-Witten-Invariants and Moonshine

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Idea:

- Extend correspondence seen before to CHL-orbifold of $E_8 \times E_8$ Heterotic on $K3 \times T^2 \leftrightarrow$ Type IIA on some CY3 X
- "Guess" X and compare prepotential on both sides
- 1-loop corrections to prepotential on heterotic side are governed by (twisted) elliptic genus of K3 ($\leftrightarrow$ related to M_{24})
- 1-loop corrections to prepotential on TypeIIA side are governed by Gromow-Witten invariants of X

CHL orbifold of $E_8 \times E_8$ heterotic on $K3 \times T^2$

papers by A. Chattopadhyaya, J. R. David (1611.01893, 1712.08791)

- CHL-orbifold: orbifold by $g \in M_{24}$ on $K3 \times$ shift $\frac{1}{N}$ on S^1 ($T^2 = S^1 \times S^1$), ($N = o(g)$)
- holomorphic part of the prepotential corrections at one loop order (no Wilson loop moduli)

$$\bar{F}_0^{\rm hol}(y) = \frac{1}{\pi^2}\sum_{s=0}^{N-1}\left(\frac{1}{2}c_{-1}^{(0,s)}\zeta(3) + \sum_{(n_1,n_2)>0} e^{-2\pi i \frac{n_2 s}{N}} c_{-1}^{(r,s)}(n_1 n_2, 0)\mathrm{Li}_3(e^{2\pi i(n_1 T + n_2 U)})\right)$$

where $n_1 \in \frac{1}{N}\mathbb{Z}$, $n_2 \in \mathbb{Z}$ and $(n_1, n_2) > 0$ means : $(n_1, n_2 \geq 0$ and $(n_1, n_2) \neq (0,0))$ or $(n_2 < 0$ and $n_1|n_2| \leq N$
T, U Kähler and complex structure moduli of T^2 and
$c_{-1}^{(r,s)}(n_1 n_2, 0)$ is determined by twisted elliptic genus

CHL orbifold of $E_8 \times E_8$ heterotic on $K3 \times T^2$

- twisted elliptic genus of $K3$
  $F^{(r,s)}(\tau, z) = \frac{1}{N}\mathrm{Tr}_{RRg^r}\left[(-1)^{F_{K3}+\bar{F}_{K3}} g^s e^{2\pi i z F_{K3}} q^{L_0 - \frac{c}{24}} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}}\right]$
  $F^{(r,s)}(\tau, z) = \alpha_g^{(r,s)} A(\tau, z) + \beta_g^{(r,s)}(\tau) B(\tau, z)$
  $A(\tau, z), B(\tau, z) \ldots$ Jacobi forms of index 1 and weight 0 and -2
  $\alpha_g^{(r,s)} \ldots$ numerical constant, $\beta_g^{(r,s)}(\tau) \ldots$ weight 2 modular form under $\Gamma_0(N)$,$\beta_g^{(0,0)} = 0$
- new supersymmetric index
  $\mathcal{Z}_{\mathrm{new}} = \frac{1}{\eta^2(\tau)}\mathrm{Tr}_R[(-1)^F F q^{L_0 - \frac{c}{24}} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}}]$
  $F = F^{T^2} + F^{K3}$
  Tr is taken over internal sector

Type IIA on CY_3

- $\mathcal{N} = 2, D = 4$ theory with $n_v = h^{1,1} + 1$ vectormultiplets and $n_H = h^{2,1} + 1$ hypermultiplets
- corrections to prepotential given by:
  $F_0^{\mathrm{GV}} = \zeta(3)\frac{\chi(X)}{2} + \sum_{(n_1,n_2)>0} n^0_{(n_1,n_2)} \mathrm{Li}_3(e^{2\pi i(n_1 T + n_2 U)})$
  $n^0_{(n_1,n_2)}$ ... (genus 0) Gromow-Witten invariants
  (Gopakumar-Vafa invariants) $\sim$ "count holomorphic curves of genus g on X"
- This will match with heterotic side for
  $n^0_{(n_1,n_2)} = 2\sum_{s=0}^{N-1} e^{-\frac{2\pi i n_2 s}{N}} c_{-1}^{(r,s)}(n_1 n_2, 0)$.

Comparison of both sides

- massless spectrum of order 2 CHL-orbifolds has been calculated in 1611.01893 (A. Chattopadhyaya, J.R. David)
- after Higgsing as far as possible we find an number of n_h of hypermultiplets and n_v of vectormultiplets (4 additional come from $B_{\mu\nu}, G_{\mu\nu}$ on T^2)
- look up possible CY_3 in the CY database by M. Kreuzer and H. Skarke (given by reflexive polyhedra)
- calculate Gromow-Witten invariants using PALP (Mori cone) and Instaton.m (A. Klemm)
- so far : 2 matching cases ($h^{1,1} = 2$ and $h^{2,1} = 83, 115$)

Comparison of both sides

$N_h - N_v = 84$

Gauge group, Shift $(\gamma; \tilde{\gamma})$	Sector	Matter
$E_6 \times SU(2) \times U(1) \times SO(14) \times U(1)$	g^0	$(\mathbf{27}, \mathbf{2}; \mathbf{1}) + (\mathbf{1}, \mathbf{2}; \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \mathbf{64})$ $+2(\mathbf{1}, \mathbf{1}; \mathbf{1})$
$(2, 1, 1, 0^5; 2, 0^7)$	$g^1 + g^3$	$6(\mathbf{1}, \mathbf{1}; \mathbf{1}) + 4(\mathbf{1}, \mathbf{2}; \mathbf{1})$ $+2(\mathbf{27}, \mathbf{1}; \mathbf{1}) + 2(\mathbf{1}, \mathbf{1}; \mathbf{14})$
	g^2	$(\mathbf{1}, \mathbf{2}; \mathbf{14}) + 6(\mathbf{1}, \mathbf{2}; \mathbf{1})$

$N_H - N_V = 116$

Gauge group, Shift $(\gamma; \tilde{\gamma})$	Sector	Matter
$SU(8) \times U(1) \times SO(12) \times SU(2) \times U(1)$	g^0	$(\mathbf{8}; \mathbf{1}, \mathbf{1}) + (\mathbf{56}; \mathbf{1}, \mathbf{1}) + (\mathbf{1}; \mathbf{12}, \mathbf{1})$ $(\mathbf{1}; \mathbf{32}, \mathbf{1}) + 2(\mathbf{1}; \mathbf{1}, \mathbf{1})$
$(1^7, -1; 3, 1, 0)$	$g^1 + g^3$	$4(\mathbf{1}; \mathbf{1}, \mathbf{2}) + 2(\mathbf{1}; \mathbf{12}, \mathbf{1}) + 2(\mathbf{8}; \mathbf{1}, \mathbf{2})$
	g^2	$6(\mathbf{8}; \mathbf{1}, \mathbf{1}) + 2(\mathbf{8}; \mathbf{1}, \mathbf{1})$