M_{24} Moonshine in IIA and IIB string theory

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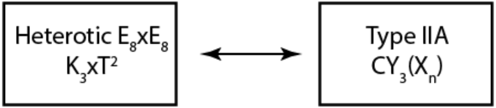
Moonshine workshop

Heterotic side



- X_n are elliptic fibrations over Hirzebruch surfaces F_n
- In order to fulfill the Bianchi Identity for H_3
  $\Rightarrow$ turn on non trivial gauge bundle in one or both the E_8 gauge groups
- Embed 24 instantons into the two E_8's
- 13 different cases $(12 - n, 12 + n) \qquad n = 0, 1, .., 12$

Heterotic side

- For $n = 0, 1, 2$ the instantons break the $E_8 \times E_8$ gauge symmetry completely
- 3 vector multiplets: scalar comp. S, T, U

  S....axio-dilaton
  T....Kähler modulus of the torus T^2
  U....Complex structure modulus of T^2
- $n > 2 \Rightarrow$ Wilson line moduli V^i, set $V^i = 0$
- the prepotential of the 13 four dim. $\mathcal{N} = 2$ theories is always the same

Heterotic side

$$F = STU + \frac{1}{3}U^3 + \frac{1}{(2\pi i)^3}c(0)\xi(3) - \frac{2}{(2\pi i)^3}\sum_{k>0,l\epsilon\mathbb{Z};k=0,l>0} c(kl)Li_3(q_T^k q_U^l) + O(e^{2\pi i S})$$

$\zeta(3) \approx 1.2...$ Riemann zeta function, $q_U = e^{2\pi i U}, \quad q_T = e^{2\pi i T}$,

$Li_3 = \sum_{n=1}^{\infty} \frac{x^n}{n^3}$

Heterotic side

- The coefficients $c(m)$ of are given by the series

$$\frac{E_4(q)\,E_6(q)}{\eta(q)^{24}} = \sum_{m\geq -1} c(m)q^m = \frac{1}{q} - 240 - 141444q - \ldots$$

- $E_i(q)$ are specific modular forms of weight i, the Eisenstein series
- $E_6(q)$ is related to the elliptic genus of $K3$

$$\frac{-4E_6(q)}{\eta(q)^{12}} \propto \mathcal{Z}^{\text{ell.}}_{\text{K3}}(q,y)$$

$$= 24 g_{h=\frac{1}{4},l=0}(q) + g_{h=\frac{1}{4},l=\frac{1}{2}}(q) \sum_{n=0}^{\infty} A_n q^n$$

- Via the Mathieu Moonshine paper of 2010 by Eguchi, Ooguri and Tachikawa

Mathieu group M_{24}

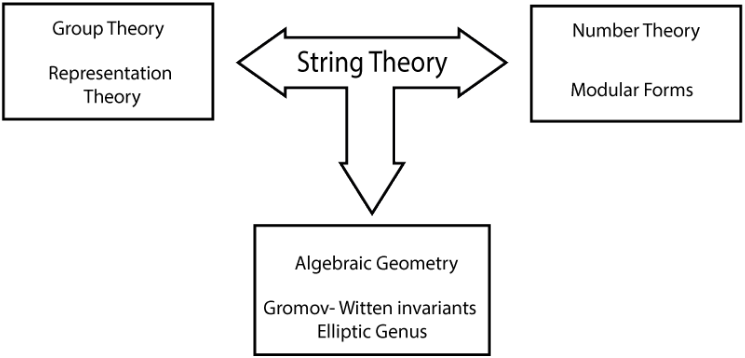
The A_n are irreducible representations of the Mathieu group M_{24}

$$\begin{aligned}
A_0 &= -2 = -1 - 1 \\
A_1 &= 90 = 45 + \underline{45} \\
A_2 &= 462 = 231 + \underline{231} \\
A_3 &= 1540 = 770 + \underline{770} \\
A_4 &= 4554 = 2277 + \underline{2277} \\
&\dots \quad .
\end{aligned}$$

Moonshine heterotic side - IIA side

- Hence the prepotential of the four dimensional $\mathcal{N} = 2$ theory directly relates to the elliptic genus of $K3$ at torsion points
- In the dual type IIA picture sum in prepotential from instanton corrections
- $c(m)$ related to GW-invariants $\Rightarrow$ GW-invariants are related to M_{24}

Math Connection



Mirror symmetry

- type IIA(X_n)$\longleftrightarrow$type IIB(Y_n)
- The topologies are related as follows $h_{11}\,(X_n) = h_{21}(Y_n)$ and $h_{21}\,(X_n) = h_{11}(Y_n)$
- In IIA vectormultiplets are connected to M_{24}
- In IIA the vectormultipelts come from the Kähler moduli sector.
- In IIB the vectormultiplets come from the complex structure moduli sector

type IIB side

- The period vector is related to the holomorphic prepotential and hence to M_{24}.

$$\Pi = \begin{pmatrix} 1 \\ t^i \\ \frac{\partial}{\partial t^i} F \\ 2F - t^i \frac{\partial}{\partial t^i} F \end{pmatrix}$$

The $t^i = \frac{z^i}{z^0}$ are three moduli on the IIA side dual to S, T, U on the heterotic side.