Chemical Nonequilibrium in QGP and The Phase Boundary to Hadron Matter

Vienna Equilibration Workshop, August 12, 2005

Jean Letessier and JR, nucl-th/0504028, other works Is there a chemical nonequilibrium in deconfined and/or confined phase? Can chemical nonequilibrium change the phase transition properties? What is strangeness content in RHIC-200 CERN-SPS? Is it consistent with deconfinement? Where as function of volume and energy is a threshold of deconfinement? What is the nature of the phase created at low energies?

We propose that the chemically over-saturated 2+1 flavor hadron matter system undergoes a 1st order phase transition.

Supported by a grant from the U.S. Department of Energy, DE-FG02-04ER41318

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Phase boundary considering Fermi degrees of freedom



adapted from: THE THREE FLAVOR CHIRAL PHASE TRANSITION WITH AN IMPROVED QUARK AND GLUON ACTION IN LATTICE QCD. By A. Peikert, F. Karsch, E. Laermann, B. Sturm, (LATTICE 98), Boulder, CO, 13-18 Jul 1998. in Nucl.Phys.Proc.Suppl.73:468-470,1999.

....and considering the baryochemical potential



adapted from: CRITICAL POINT OF QCD AT FINITE T AND MU, LATTICE RESULTS FOR PHYSICAL QUARK MASSES. By Z. Fodor, S.D. Katz (Wuppertal U.), JHEP 0404:050,2004; hep-lat/0402006 Maybe the cross-over T is TODAY at 180 MeV, this is of no relevance to the point made.

Chemical Equilibrium Phase Boundary

Temperature of phase transition depends on available degrees of freedom (up to systematic errors):

- For 0 flavor theory T > 200 MeV
- For 2 flavors: $T \rightarrow 170 \text{ MeV}$
- For 2+1 flavors: $T = 162 \pm 3$ and appearance of minimum μ_B we need extra quarks to reach a 1st order transition
- For 3, 4 flavors further drop in T.

Heavy Ions Collision Situation

Experiments are carried out in a nonequilibrium environment. What can we expect?

- Chemical non-equilibrium can increase or decrease quark 'occupancy', favoring/disfavoring presence of a real phase transition, and thus help/hinder phase transition. What $\mu_{\rm B}$ can do, γ_i can do better as both quark and antiquark number increases.
- Dynamical expansion is enhancing the deconfined phase pressure, expect decrease of transition temperature, no change of the nature of the phase transition expected.

FOUR QUARKS: $s, \overline{s}, q, \overline{q} \rightarrow$ FOUR CHEMICAL PARAMETERS

γ_i	controls overall abundance	Absolute chemical
	of quark $(i = q, s)$ pairs	equilibrium
λ_i	$=e^{\mu_i/T}$ controls difference between	Relative chemical
	strange and non-strange quarks $(i = q, s)$	equilibrium

HG-EXAMPLE: redistribution, Relative chemical equilibrium

production of strangeness Absolute chemical equilibrium



See Physics Reports 1986 Koch, Müller, JR

Particle yields in chemical (non)equilibrium

The counting of hadrons is conveniently done by counting the valence quark content $(u, d, s, ..., \lambda_q^2 = \lambda_u \lambda_d, \lambda_{I3} = \lambda_u / \lambda_d)$:

$$\Upsilon_i \equiv \Pi_i \gamma_i^{n_i} \lambda_i^{k_i} = e^{\sigma_i/T}$$

There is a natural relation of quark fugacities with hadron fugacities, for particle 'i' but for one complication: for historical reasons hyperon number is opposite to strangeness, thus $\mu_S = \frac{\mu_b}{3} - \mu_s$, where $\lambda_q^3 = e^{\mu_b/T}$, and

Example of NUCLEONS
$$\gamma_N = \gamma_q^3$$
:
 $\Upsilon_N = \gamma_N e^{\mu_b/T}, \qquad \qquad \Upsilon_{\overline{N}} = \gamma_N e^{-\mu_b/T};$
 $\sigma_N \equiv \mu_b + T \ln \gamma_N, \qquad \sigma_{\overline{N}} \equiv -\mu_b + T \ln \gamma_N$

Meaning of parameters from e.g. the first law of thermodynamics:

$$dE + P \, dV - T \, dS = \sigma_N \, dN + \sigma_{\overline{N}} \, d\overline{N}$$
$$= \mu_b (dN - d\overline{N}) + T \ln \gamma_N (dN + d\overline{N}).$$

 μ_b controls the particle difference = baryon number.

 γ regulates the number of particle-antiparticle pairs present.

DISTINGUISH HG and QGP parameters: γ_i are discontinuous so the entropy, etc preserved despite change in nature of the phase, μ_i continuous.

IS OVERPOPULATION OF PHASE SPACE POSSIBLE?

• production of strangeness in gluon fusion $\overline{GG \rightarrow s\bar{s}}$ strangeness linked to gluons from QGP;



- $\overline{s} \simeq \overline{q} \rightarrow$ strange antibaryon enhancement at RHIC (anti)hyperon dominance of (anti)baryons.
- at LHC $\gamma_s^{\text{QGP}}|_{\text{Had}} >> 1$ Phase transition for $\mu_{\text{B}} = 0$?

Strangeness relaxation to/BEYOND chemical equilibrium Strangeness density time evolution in local rest frame:

$$\frac{d\rho_s}{d\tau} = \frac{d\rho_{\bar{s}}}{d\tau} = \frac{1}{2}\rho_g^2(t) \left\langle \sigma v \right\rangle_T^{gg \to s\bar{s}} + \rho_q(t)\rho_{\bar{q}}(t) \left\langle \sigma v \right\rangle_T^{q\bar{q} \to s\bar{s}} - \rho_s(t) \rho_{\bar{s}}(t) \left\langle \sigma v \right\rangle_T^{s\bar{s} \to gg, q\bar{q}}$$

Evolution for s and \bar{s} identical, which allows to set $\rho_s(t) = \rho_{\bar{s}}(t)$. characteristic time constant τ_s :



Dotted line: 1981 estimate. Dashed area: m_s uncertainty. Thick line: running α_s .

Include ENTROPY CONSERVING EXPANSION: The volume expansion and temperature change such that $\delta(T^3V) = 0$. We introduce phase space occupancy:

$$\gamma_s(t) \equiv \frac{n_s(t)}{n_s^{\infty}(T(t))}, \quad n_s(t) = \gamma_s(t)T(t)^3 \frac{3}{\pi^2} z^2 K_2(z), \quad z = \frac{m_s}{T(t)}, \quad K_i : \text{Bessel f.}$$

Strangeness has a mass scale, its time evolution follows:

$$2\tau_s \frac{d\gamma_s}{d\tau} = 1 - \gamma_s^2 - \gamma_s 2\tau_s \frac{d\ln z^2 K_2(z)}{d\tau} = 1 - \gamma_s^2 + \gamma_s 2\tau_s \frac{dz}{d\tau} \frac{K_1(z)}{K_2(z)}.$$

Last term presents the residual effect of expansion. Without scale $(m \rightarrow 0)$ it disappears, and $\gamma_s \leq 1$, but its importance grows with mass of the quark, z = m/T. Since the volume expansion reduces temperature, $dz/d\tau > 0$, early on produced strangeness can overpopulate the smaller final phase space. This effect is more significant for more massive particles. Pivotal role for strangeness due to $T_{\rm cr} \simeq m_s$: strangeness can rise well above chemical equilibrium near to $T_{\rm cr}$. This may facilitate presence of a real phase transition at zero baryon density. Requirement: initial state hot, and expansion time $\tau_{\rm QGP} > \tau_s$.

RHIC EXAMPLE



HOW TO MEASURE γ_s^{QGP}

STRANGENESS / ENTROPY CONTENT s/S

Strangeness s and entropy S produced predominantly in early hot parton phase. Yield ratio eliminates dependence on reaction geometry. Strangeness and entropy could increase slightly in hadronization. s/S relation to K^+/π^+ is not trivial when precision better than 25% needed.

CONFIRM BY: STRANGENESS / NET BARYON NUMBER s/b

Baryon number b is conserved, strangeness could increase slightly in hadronization. s/b ratio probes the mechanism of primordial fireball baryon deposition and strangeness production. Ratio eliminates dependence on reaction geometry.

Strangeness / Entropy

Relative s/S yield measures the number of active degrees of freedom and degree of relaxation when strangeness production freezes-out. Perturbative expression in chemical equilibrium:

$$\frac{s}{S} = \frac{(3/\pi^2)T^3(m_s/T)^2 K_2(m_s/T)}{(32\pi^2/45)T^3 + n_{\rm f}[(7\pi^2/15)T^3 + \mu_q^2T]} \simeq 0.027$$

assumption: $\mathcal{O}(\alpha_s)$ interaction effects cancel out between S, s

Allow for chemical equilibrium of strangeness n γ_s^{QGP} , and possible quark-gluon pre-equilibrium:

$$\frac{s}{S} = \frac{0.027\gamma_s^{\text{QGP}}}{0.38\gamma_{\text{G}} + 0.12\gamma_s^{\text{QGP}} + 0.5\gamma_q^{\text{QGP}} + 0.054\gamma_q^{\text{QGP}}(\ln\lambda_q)^2} \to 0.027.$$

We expect the yield of gluons and light quarks to approach chemical equilibrium first: $\gamma_{\rm G} \rightarrow 1$ and $\gamma_q^{\rm QGP} \rightarrow 1$, thus $s/S \propto \gamma_s^{\rm QGP}$. HOW TO USE: FIT YIELDS OF PARTICLES, EVALUATE STRANGENESS AND ENTROPY CONTENT AND COMPARE WITH EXPECTED RATIO

CAN WE ESTIMATE THE EXPECTED $\gamma_s^{ m HG}$?

In fast breakup of expanding QGP, $V^{\text{HG}} \simeq V^{\text{QGP}}$, $T^{\text{QGP}} \simeq T^{\text{HG}}$, the chemical occupancy factors accommodate the different magnitude of particle phase space. Chemical equilibrium in one phase means non-equilibrium in the the other.

Compare phase spaces to obtain $\gamma_{\rm s}^{\rm HG}/\gamma_{\rm s}^{\rm QGP}$



Solid lines $\gamma_q^{HG} = 1$, short dashed $\gamma_q^{HG} = 1.6$ Thin lines for T = 170 and thick lines T = 150 MeV, T common to both phases. m_s relevant.

 $\boxed{\gamma_s^{\text{HG}} \simeq 2 - 3\gamma_s^{\text{QGP}}}$ Most people TACITLY assume $\gamma_q = 1$ and fit γ_s/γ_q which they call γ_s , which ranges $0.5 < \gamma_s/\gamma_q < 1$

ESTIMATE THE EXPECTED γ_{q}^{HG}

QGP has excess of entropy, maximize entropy density at hadronization: $\gamma_q^2 \rightarrow e^{m_\pi/T}$ Example:maximization of entropy density in pion gas $E_\pi = \sqrt{m_\pi^2 + p^2}$



SPECIFIC STRANGENESS YIELD IN QGP MEASURES $\gamma_s^{QGP}/\gamma_q^{QGP}$

$$\frac{\rho_{\rm s}}{\rho_{\rm b}} = \frac{s}{q/3} = \frac{\gamma_{\rm s}^{\rm QGP} \frac{3}{\pi^2} T^3(m_{\rm s}/T)^2 K_2(m_{\rm s}/T)}{\gamma_{\rm q}^{\rm QGP} \frac{2}{3} \left(\mu_{\rm q} T^2 + \mu_{\rm q}^3/\pi^2\right)}, \rightarrow \frac{s}{b} \simeq \frac{\gamma_{\rm s}^{\rm QGP}}{\gamma_{\rm q}^{\rm QGP}} \frac{0.7}{\ln \lambda_{\rm q} + (\ln \lambda_{\rm q})^3/\pi^2}.$$

assumption: $\mathcal{O}(\alpha_s)$ interaction effects cancel out between b, sWe consider $m_s = 200$ MeV and hadronization T = 150 MeV,



EXAMPLE: SPS Pb–Pb 158 *A* GeV λ_q =1.5–1.6, implies $s/b \simeq 1.5$. Observation: $s/b \simeq 0.75 \rightarrow \gamma_s^{\text{QGP}} / \gamma_q^{\text{QGP}} = 0.5$.

DATA ANALYSIS WITHIN STATISTICAL HADRONIZATION Hypothesis (Fermi, Hagedorn): particle production can be described by evaluating the accessible phase space.

Verification of statistical hadronization:

Particle yields with same valance quark content are in relative chemical equilibrium, e.g. the relative yield of $\Delta(1230)/N$ as of K^*/K , $\Sigma^*(1385)/\Lambda$, etc, is controlled by chemical freeze-out i.e. Hagedorn Temperature $T_{\rm H}$:



$$\frac{N^*}{N} = \frac{g^* (m^* T_{\rm H})^{3/2} e^{-m^*/T_{\rm H}}}{g(m T_{\rm H})^{3/2} e^{-m/T_{\rm H}}}$$

Resonances decay rapidly into 'stable' hadrons and dominate the yield of most stable hadronic particles.

Resonance yields test statistical hadronization principles.

Resonances reconstructed by invariant mass; important to consider potential for loss of observability.

HADRONIZATION GLOBAL FIT: \rightarrow

Statistical Hadronization fits of hadron yields

Chemical nonequilibrium implies phase space with additional γ -parameters: The phase space density is in general different in the two phases. To preserve entropy (the valance quark pair number) across the phase boundary there must be a jump in the phase space occupancy parameters γ_i .

This replaces the increase in volume in a slow re-equilibration with mixed phase which accommodates transformation of entropy dense phase into dilute phase.

Full analysis of experimental hadron yield results requires a significant numerical effort in order to allow for resonances, particle widths, full decay trees, isospin multiplet sub-states.

Kraków-Tucson NATO supported collaboration produced a public package SHARE Statistical Hadronization with Resonances which is available e.g. at http://www.physics.arizona.edu/~torrieri/SHARE/share.html

Lead author: Giorgio Torrieri nucl-th/0404083 Comp. Phys. Com. 167, 229 (2005) Online SHARE: Steve Steinke No fitting online (server too small)

http://www.physics.arizona.edu/~steinke/shareonline.html

Aside of particle yields, also PHYSICAL PROPERTIES of the source are available, both in SHARE and ONLINE. Several papers use this tool: nucl-th/0412072 (PRC in press) and nucl-th/0506044 [address impact parameter], nucl-th/0504028 [E-dependence], hep-ph/0506140 [LHC]

Centrality dependence of dN/dy for π^{\pm} , K^{\pm} , p and \bar{p} . The errors are systematic only. The statistical errors are negligible. PHENIX data

N_{part}	π^+	π^-	K^+	K^{-}	p	$ar{p}$				
351.4	$\textbf{286.4} \pm \textbf{24.2}$	$\textbf{281.8} \pm \textbf{22.8}$	$\textbf{48.9} \pm \textbf{6.3}$	45.7 ± 5.2	18.4 ± 2.6	13.5 ± 1.8				
299.0	$\textbf{239.6} \pm \textbf{20.5}$	$\textbf{238.9} \pm \textbf{19.8}$	40.1 ± 5.1	$\textbf{37.8} \pm \textbf{4.3}$	15.3 ± 2.1	11.4 ± 1.5				
253.9	$\textbf{204.6} \pm \textbf{18.0}$	198.2 ± 16.7	$\textbf{33.7} \pm \textbf{4.3}$	31.1 ± 3.5	12.8 ± 1.8	9.5 ± 1.3				
215.3	173.8 ± 15.6	$\textbf{167.4} \pm \textbf{14.4}$	$\textbf{27.9} \pm \textbf{3.6}$	25.8 ± 2.9	10.6 ± 1.5	$\textbf{7.9} \pm \textbf{1.1}$				
166.6	130.3 ± 12.4	127.3 ± 11.6	$\textbf{20.6} \pm \textbf{2.6}$	19.1 ± 2.2	$\textbf{8.1}\pm\textbf{1.1}$	$\boldsymbol{5.9 \pm 0.8}$				
114.2	87.0 ± 8.6	$\textbf{84.4} \pm \textbf{8.0}$	13.2 ± 1.7	12.3 ± 1.4	5.3 ± 0.7	$\textbf{3.9}\pm\textbf{0.5}$				
74.4	54.9 ± 5.6	52.9 ± 5.2	$\textbf{8.0} \pm \textbf{0.8}$	$\textbf{7.4} \pm \textbf{0.6}$	$\textbf{3.2}\pm\textbf{0.5}$	$\boldsymbol{2.4\pm0.3}$				
45.5	$\textbf{32.4} \pm \textbf{3.4}$	$\textbf{31.3} \pm \textbf{3.1}$	$\textbf{4.5} \pm \textbf{0.4}$	$\textbf{4.1} \pm \textbf{0.4}$	1.8 ± 0.3	1.4 ± 0.2				
25.7	17.0 ± 1.8	$\textbf{16.3} \pm \textbf{1.6}$	$\boldsymbol{2.2\pm0.2}$	2.0 ± 0.1	0.93 ± 0.15	0.71 ± 0.12				
13.4	$\textbf{7.9} \pm \textbf{0.8}$	$\textbf{7.7} \pm \textbf{0.7}$	$\boldsymbol{0.89 \pm 0.09}$	0.88 ± 0.09	0.40 ± 0.07	0.29 ± 0.05				
6.3	$\textbf{4.0} \pm \textbf{0.4}$	$\textbf{3.9} \pm \textbf{0.3}$	0.44 ± 0.04	0.42 ± 0.04	0.21 ± 0.04	0.15 ± 0.02				
include STAR data on K [*] and ϕ yields.										

s/b and s/S rise with increasing centrality $A \propto V$; E/s falls



Showing results for both $\gamma_q, \gamma_s \neq 1$ and when $\gamma_q = 1$ is assumed. REASON: there is some hesitance to accept a $T \simeq 140$ when $\gamma_q \rightarrow 1.6$. No difference in this result:

 $s/S \rightarrow 0.027$, as function of Vno saturation for largest volumes available. Result consistent with QGP expectation. $\gamma_s^{\text{QGP}} \simeq 1$, confirmed by s/B. Indication that physics is different for most two central reaction bins.

REMARK ASIDE: The rapidity density of entropy $dS/dy \simeq 5000 \pm 10\%$. This implies an initial thermally equilibrated parton state with rapidity desntiy $dN/dy \simeq 1250$.



LINES: $\gamma_s, \gamma_q \neq 1$ and $\gamma_s \neq 1, \gamma_q = 1$, γ_q changes with $A \propto V$ from under-saturated to over-saturated value, γ_s^{HG} increases steadily to 2.4, implying near saturation in QGP. P, σ, ϵ increase by factor 2–3, at A > 20 (onset of new physics?), E/TS decreases with A.

Statistical + fit errors are seen in fluctuations, systematic error impacts absolute normalization by $\pm 10\%$.



RHIC200 PREDICTION OF dependence on centrality



STAR $\sqrt{s_{\text{NN}}} = 200 \text{ GeV}$ yields of hyperons $d\Lambda/dy$ and $d\Xi^{-}/dy$, (a), and antihyperons $d\overline{\Lambda}/dy$ and $d\overline{\Xi}^{+}/dy$, (b), normalized with, and as function of, A, relative to these yields in pp reactions: $d(\Lambda + \overline{\Lambda})/dy = 0.066 \pm 0.006$, $d(\Xi^{-}+\overline{\Xi}^{+})/dy = 0.0036 \pm 0.0012$, $\overline{\Lambda}/\Lambda = 0.88 \pm 0.09$ and $\overline{\Xi}^{+}/\Xi^{-} = 0.90 \pm 0.09$. Solid lines, chemical non-equilibrium, dashed chemical equilibrium, dotted lines, semi-equilibrium. On right, the predicted hyperons per π^{-} yields (blue for hyperons and for antihyperons).



Full 4π and central rapidity results. We again find $s/S \rightarrow 0.027$, as function of $\sqrt{s_{\rm NN}}$ and V: no saturation, consistent with QGP expectation and $\gamma_s^{\text{QGP}} \simeq 1$, confirmed by s/B. Energy/strangeness E/s cost drop at $\sqrt{s_{NN}^{cr}}$, suggests appearance of a new (e.g. $GG \rightarrow s\bar{s}$) production mechanism.

COMPARE $\sqrt{s_{\text{NN}}}$ and V dependence of s/b and s/S, E/s



SUMMARY OF $\sqrt{s_{NN}}$ **FIT RESULTS:** Statistical parameters



Note that behavior is the same as we saw as function of A: the large jumps by factor 2–3 in densities (to left) and pressure (on right) as the collision energy changes from 20 GeV to 30 GeV. There is clear evidence of change in reaction mechanism. There no difference between top SPS and RHIC energy range.

Why low/high PHASE BOUNDARY Temperature?

- Dynamical effects of expansion: colored partons like a wind, displace the boundary
- Degrees of freedom
 - Temperature of phase transition depends on available degrees of freedom.
 - For 2+1 flavors: $T = 162 \pm 3$, for $\gamma_s \rightarrow 0$
 - $2+1 \rightarrow 2$ flavor theory with $T \rightarrow 170$ MeV,
 - what happens when $\gamma_s \rightarrow 1.5?$
 - The nature of phase transition/transformation changes when number of flavors rises from 2+1 to 3 is effect of $\gamma_i > 1$ creating a real phase transition?
- at high μ_B we encounter
 - either conventional hadrons (contradiction with continuity of quark related variables: strangeness, strange antibaryons).
 - or more likely, a new heavy (valon) quark phases. Under saturation of phase space compatible with higher T.

Questions with answers

Is there chemical **non**equilibrium?

In QGP: strangeness sector. HG: light and strange sector fast nonequilibrium transformation

Can chemical nonequilibrium impact phase transition properties? Behavior as function of N_f suggests that $\gamma_s^{QGP} > 1$ helps establish a true 1st order phase transition for $\mu_B \to 0$.

What is strangeness content from CERN-SPS to RHIC-200?

Gradual rise as function of collision energy of the yield s/S (per entropy), saturating the QGP phase space at RHIC, expected further increase at LHC. Is it consistent with deconfinement? Other strangeness evidence for deconfinement?

Threshold seen in s/S, s/b and E/s.

Where as function of volume and energy is a PHASE threshold ? $6.26 \text{GeV} < \sqrt{s_{\text{NN}}^{\text{cr}}} < 7.61 \text{GeV}$. Bulk properties also respond at that threshold. Softer threshold at $A \simeq 20$.

What is the nature of the phase created at low energies?

Phase under-saturates phase space, probably involves effectively massive quarks. To understand E/TS one can invoke thermal quarks with $m \simeq 2-4T$. **Do we describe the particle production as function of energy**? Turn page



The horn requires a shift in γ_q



Looking a the fit χ^2 we see that between 20 and 30GeV results favor that γ_q jumps from highly unsaturated to fully saturated: from $\gamma_q < 0.5$ to $\gamma_q > 1.5$. This produces the horn (below). The individual fits relevant to understanding how the horn is created have good quality - see P%.



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	$N_{4\pi}$ 5%			$dN/dy _{u=0}$ 5%			
$\sqrt{s_{ m NN}} ~[{ m GeV}]$	62.4	130	200	62.4	130	200	
b	350.2	350.2	350.1	32.64	19.79	14.8	
π^+	1001	1282	1470	225.8	236.6	237.4	
π^-	1072	1368	1558	236.7	246.8	247.2	
K^+	194.5	289.9	297.9	43.3	49.5	50.7	
K^-	139.4	222.5	236.3	37.5	45.5	47.6	
K_S	162.3	248.2	259.2	39.2	45.9	47.5	
$\check{\phi}$	18.6	34.6	32.9	4.96	6.58	7.06	
p	156.5	163.9	177.5	21.56	18.91	18.02	
$\bar{\bar{p}}.$	25.9	40.7	50.6	9.77	12.05	12.95	
$\overline{\Lambda}$	68.6	89.3	89.0	12.3	11.4	11.4	
$\overline{\Lambda}$	16.0	29.1	32.2	5.91	7.94	8.7	
Ξ	11.3	18.1	16.5	2.18	2.60	2.70	
	3.7	7.85	7.67	1.34	1.97	2.21	
Ω	1.13	2.37	1.97	0.27	0.38	0.42	
$\overline{\Omega}$	0.56	1.40	1.21	0.20	0.32	0.37	
$K^{0}(892)$	47.9	70.1	80.0	19.5	11.8	12.1	
Δ^0	28.8	28.5	31.3	3.76	3.22	3.05	
Δ^{++}	27.2	27.8	30.6	3.71	3.19	3.03	
$\Lambda(1520)$	4.43	5.73	5.76	0.72	0.73	0.73	
$\Sigma^{+}(1385)$	8.50	10.94	10.93	1.37	1.38	1.37	
$\Xi0(1530)$	2.98	4.90	4.45	0.59	0.71	0.74	
η	110.2	158.7	172.7	26.3	29.6	30.3	
η'_{a}	8.45	13.03	13.75	2.08	2.44	2.54	
ρ^0	84.4	106	125	18.9	19.5	19.6	
$\omega(782)$	75.5	94.9	112.2	17.1	17.6	17.6	
$f_0(980)$	7.08	10.79	11.47	1.74	2.02	2.09	