Hints of incomplete thermalization in RHIC data

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in collaboration with

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RHIC Au–Au results: the fashionable view

BROOKHAVEN
NATIONAL LABORATORY

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

Ideal fluid dynamics reproduce both p_t spectra and elliptic flow $v_2(p_t)$ of soft ($p_t \lesssim 2$ GeV/c) identified particles for minimum bias collisions, near central rapidity.

This agreement necessitates a soft equation of state, and very short thermalization times: $\tau_{\text{thermalization}} < 0.6$ fm/c.

$\Rightarrow$ **strongly interacting Quark-Gluon Plasma**

Ideal fluid dynamics in heavy-ion collisions

- A few reminders on fluid dynamics
- Fluid dynamics in heavy ion collisions: theory
  - Overall scenario
  - General predictions of ideal fluid dynamics
    - Anisotropic flow
- Fluid dynamics and heavy ion collisions: theory vs. data
- Reconciling data and theory (?)
  (including predictions for Cu–Cu@RHIC and Pb–Pb@LHC)

R.S. Bhalerao, J.-P. Blaizot, N.B., J.-Y. Ollitrault, **nucl-th/0508009**

Fluid dynamics: various types of flow

- **Thermodynamic equilibrium?** ☞ Knudsen number $Kn = \frac{\lambda}{L}$ (mean free path → λ; system size → L)
  - $Kn \gg 1$: Free-streaming limit
  - $Kn \ll 1$: Thermalization: Fluid (hydro) limit
- **Viscous or Ideal?** ☞ Reynolds number $Re = \frac{L v_{\rm fluid}}{\eta}$ (viscosity → η; $\eta \sim \lambda c_s$)
  - $Re \gg 1$: Ideal (non-viscous) flow
  - $Re \leq 1$: Viscous flow
- **Compressible or Incompressible?** ☞ Mach number $Ma = \frac{v_{\rm fluid}}{c_s}$ (speed of sound → c_s)
  - $Ma \ll 1$: Incompressible flow
  - $Ma > 1$: Compressible (supersonic) flow

Fluid dynamics: various types of flow

Three numbers:

$$Kn = \frac{\lambda}{L}, \qquad Re = \frac{L v_{\rm fluid}}{\eta}, \qquad Ma = \frac{v_{\rm fluid}}{c_s}$$

$\Rightarrow$ **an important relation:**

$$Kn \times Re = \frac{\lambda\, v_{\rm fluid}}{\eta} \sim \frac{v_{\rm fluid}}{c_s} = Ma$$

Compressible fluid: Thermalized means Ideal

Viscosity $\equiv$ departure from equilibrium

Ideal fluid picture of a heavy-ion collision

⓪ Creation of a dense gas of particles

① At some time τ_0, the mean free path λ is much smaller than *all* dimensions in the system

$\Rightarrow$ thermalization (T_0), ideal fluid dynamics applies

② The fluid expands: density decreases, λ increases (system size also)

③ At some time, the mean free path is of the same order as the system size: ideal fluid dynamics is no longer valid

"(kinetic) freeze-out"

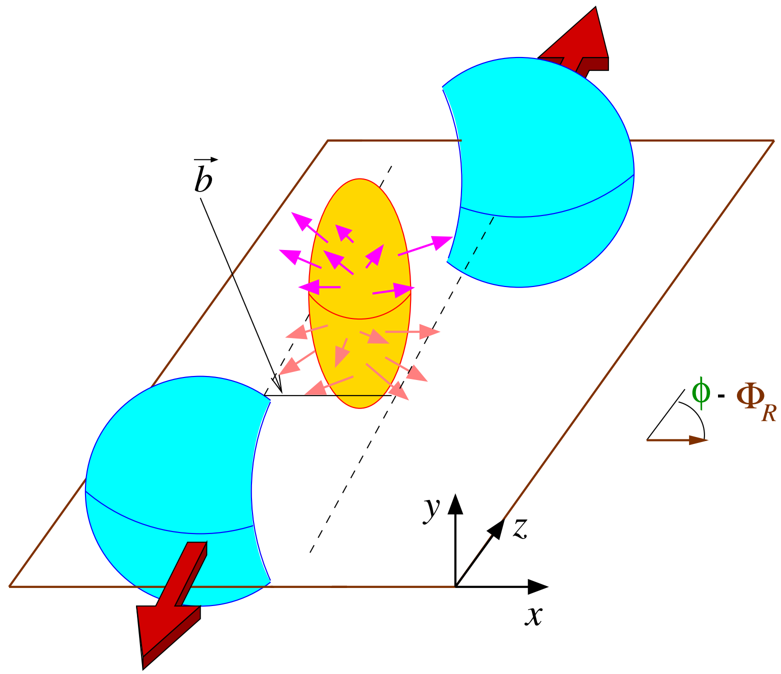
Freeze-out usually parameterized in terms of a temperature $T_{\rm f.o.}$

If λ varies smoothly with temperature, consistency requires $T_{\rm f.o.} \ll T_0$

☞ analytical predictions, see N.B. & J.-Y. Ollitrault, **nucl-th/0506045**

Heavy-ion observable: Anisotropic flow

Non-central collision:



Initial anisotropy of the source (in the transverse plane)

⇒ anisotropic pressure gradients, larger along the impact parameter $\vec{b}$

⇒ anisotropic emission of particles:

anisotropic (collective) flow

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_t\,\mathrm{d}p_t\,\mathrm{d}y}\left[1 + 2v_1\cos(\phi - \Phi_R) + 2v_2\cos 2(\phi - \Phi_R) + \ldots\right]$$

"directed" (v_1) "elliptic" (v_2)

"Flow": misleading terminology; does NOT imply fluid dynamics!

Non-central collisions: parameters

Initial conditions in non-central collisions, will be characterized by

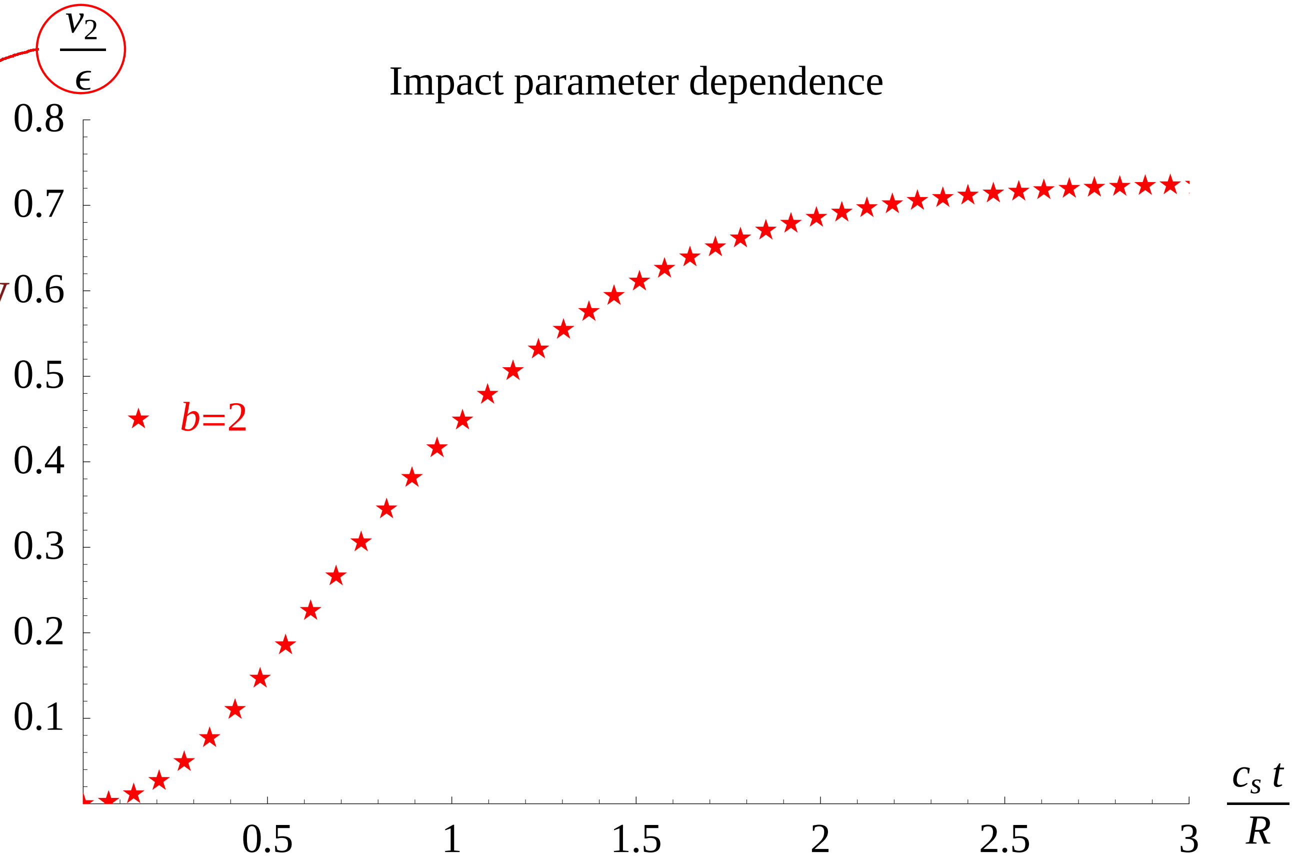
- a parameter measuring the shape of the overlap region:
  - spatial eccentricity $\epsilon = \dfrac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$
- two numbers measuring the size of the overlap region:
  - “reduced” radius $\dfrac{1}{\bar{R}} = \sqrt{\dfrac{1}{\langle x^2 \rangle} + \dfrac{1}{\langle y^2 \rangle}}$

    (anisotropic flow caused by pressure *gradients*)
  - transverse area of the collision zone $S = 2\pi\sqrt{\langle x^2 \rangle \langle y^2 \rangle}$

Dependence of v_2 on centrality

The natural time scale for v_2 is $\bar{R}/c_s$: massless particles $c_s = \frac{1}{\sqrt{3}}$

v_2 scaled by initial excentricity $\frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$



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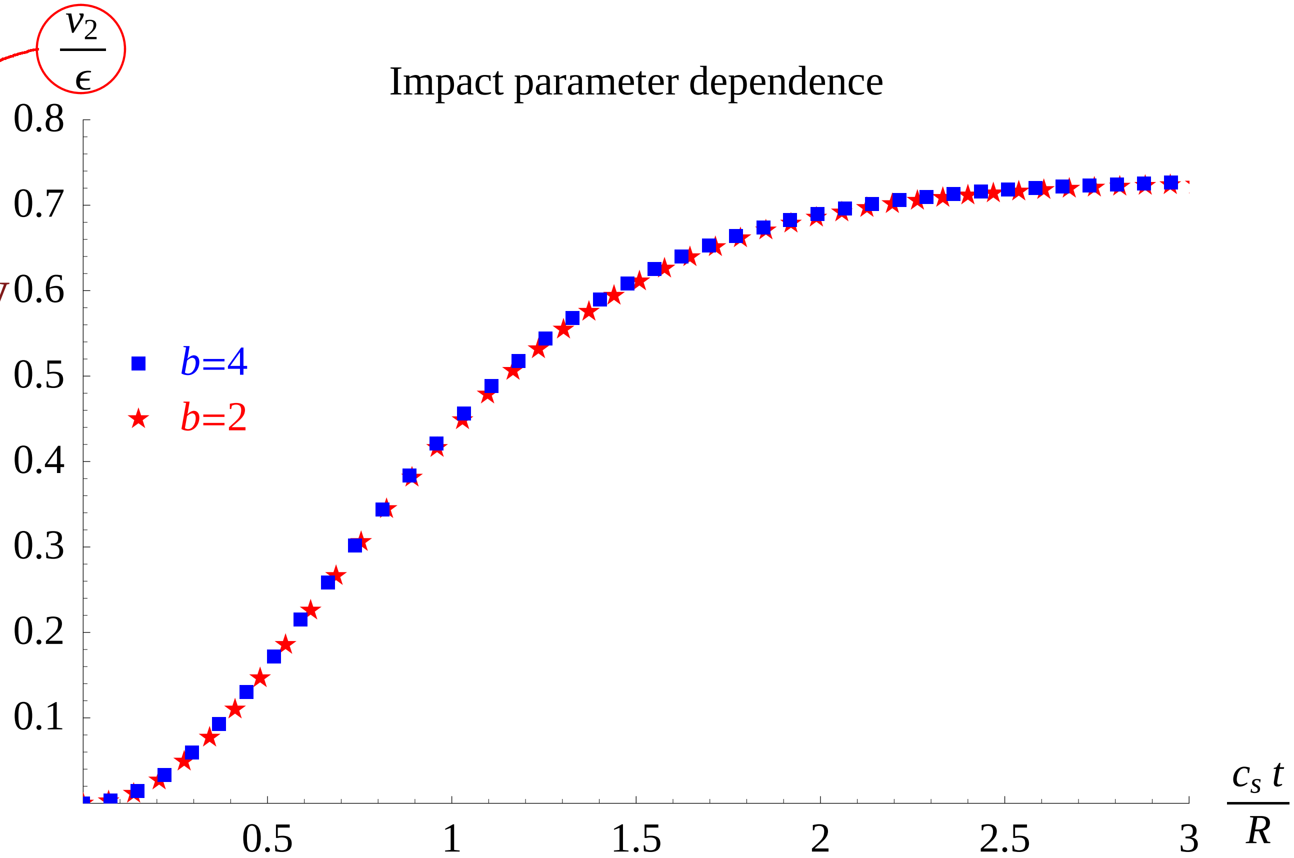

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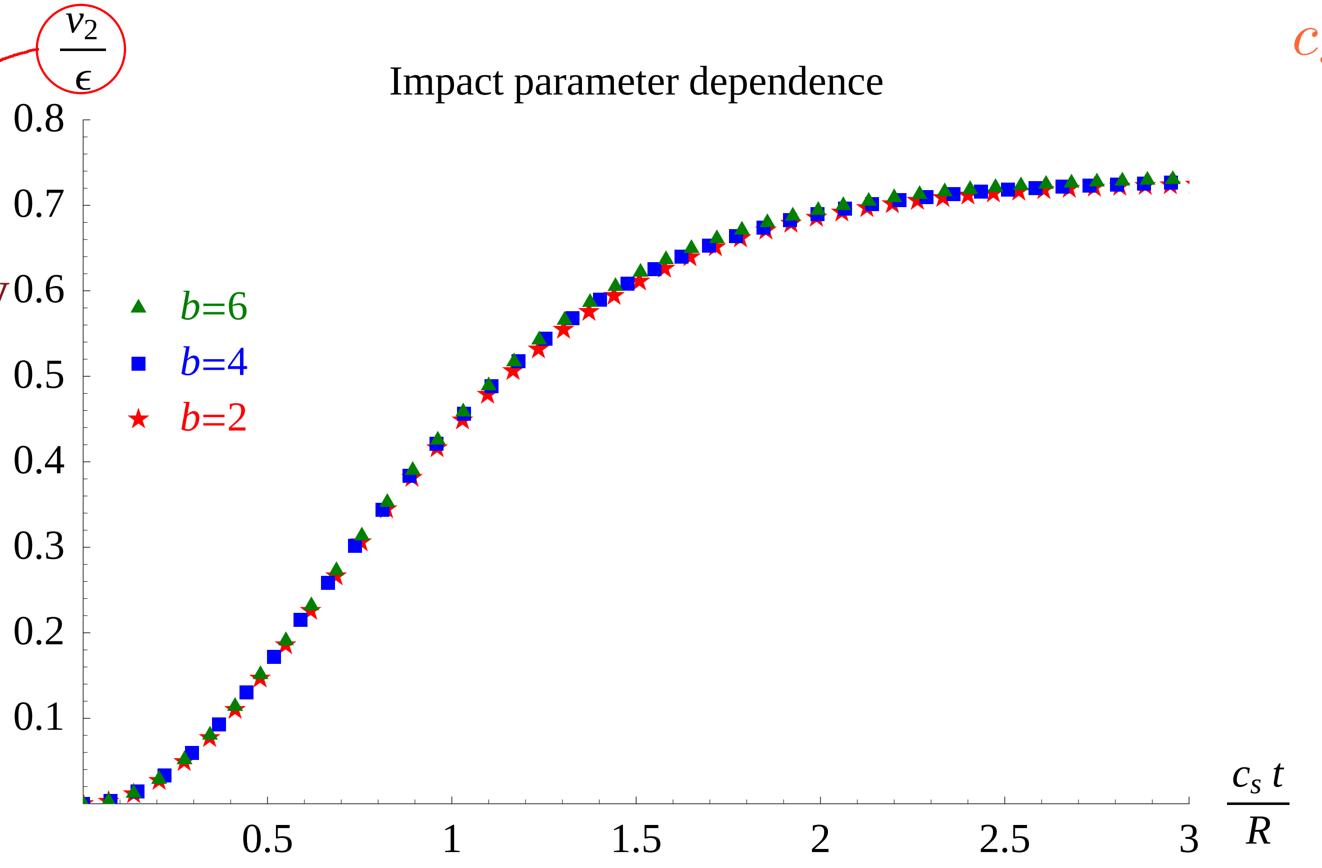

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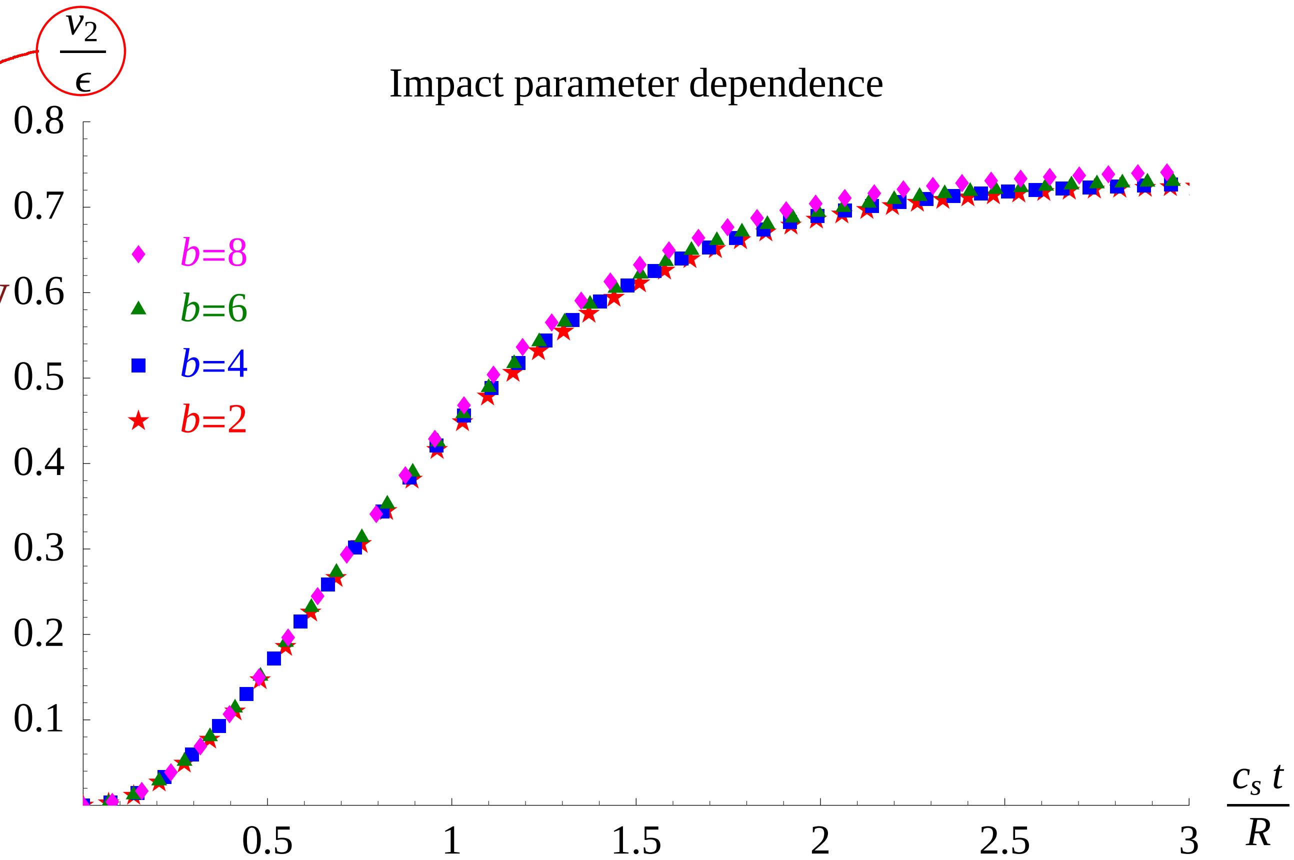

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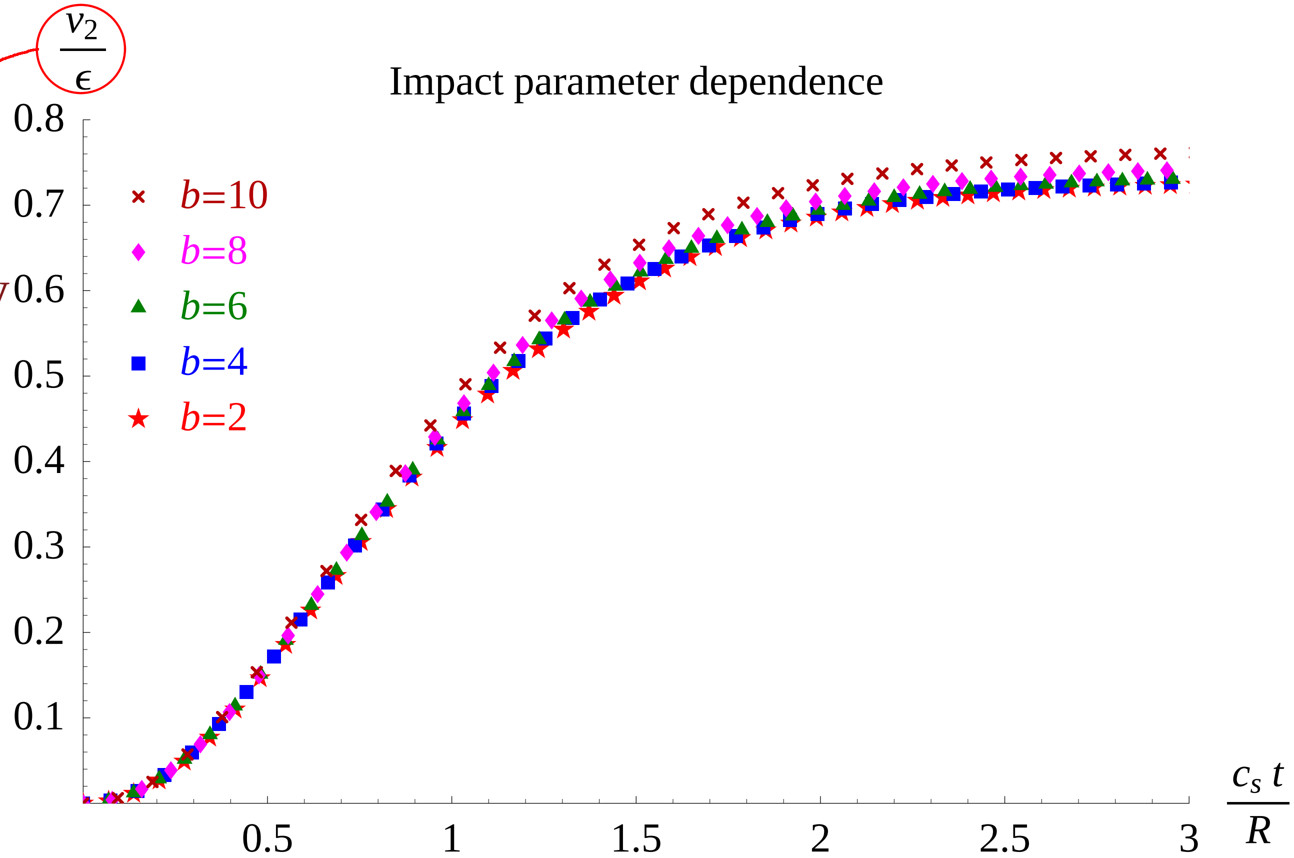

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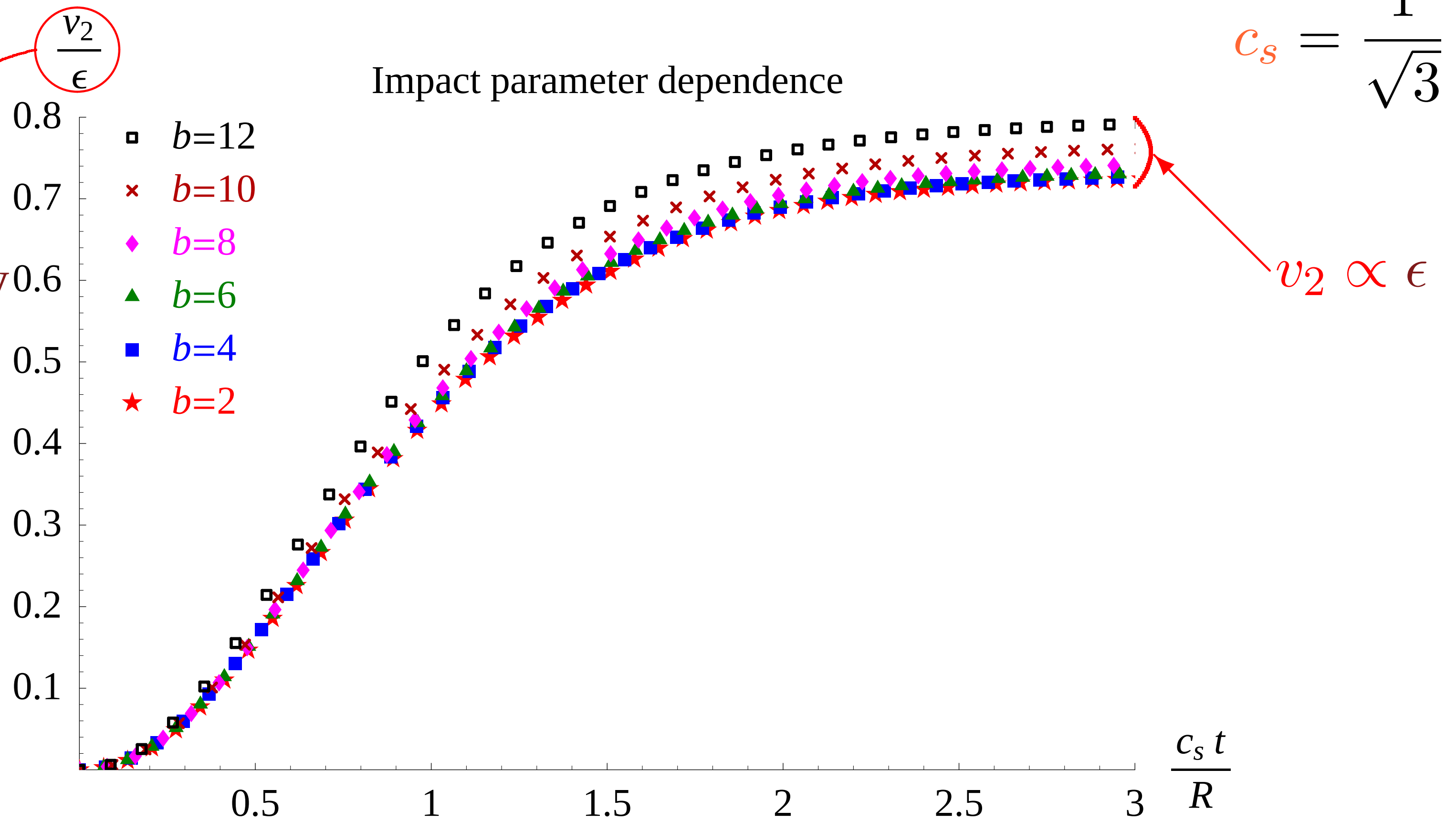
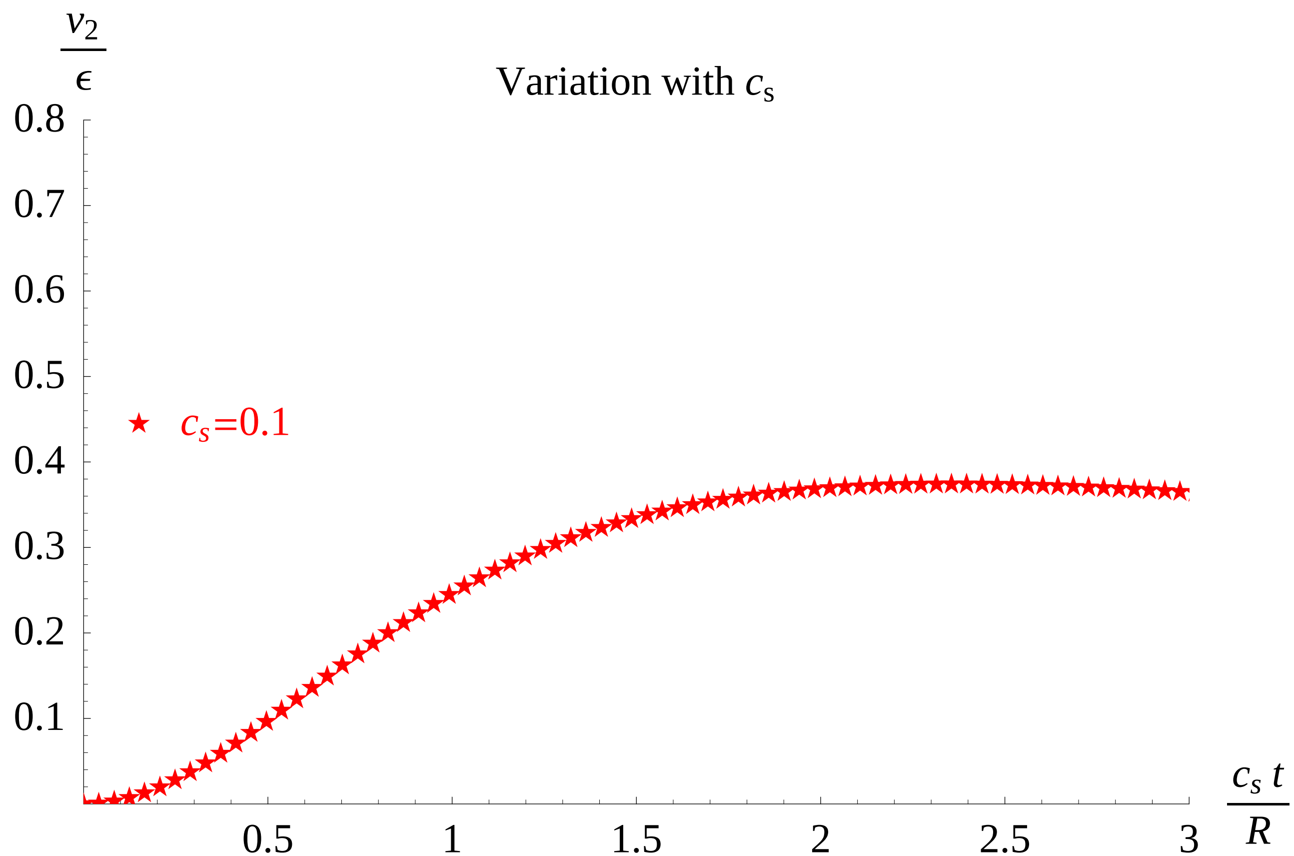


v_2 knows nothing about early times!

Dependence of v_2 on the speed of sound



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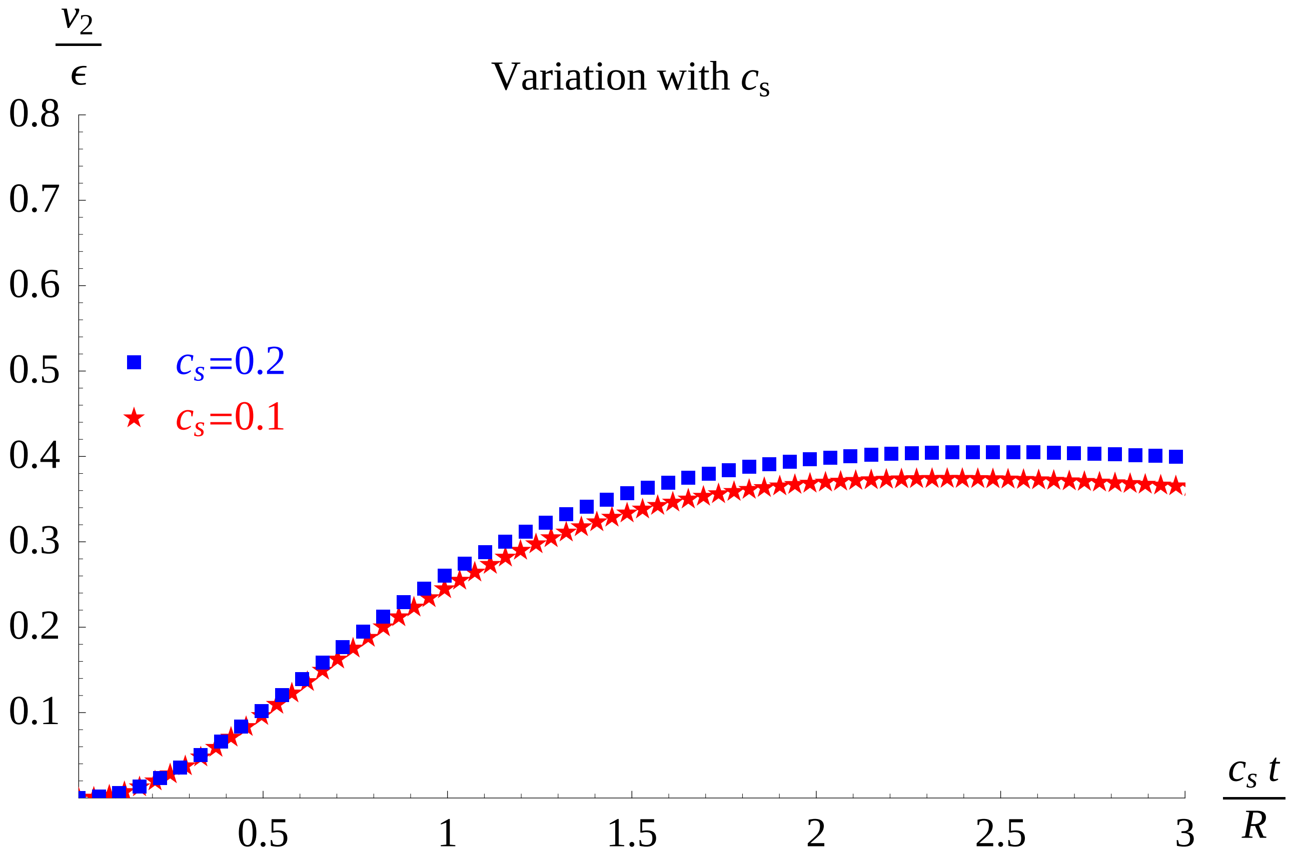

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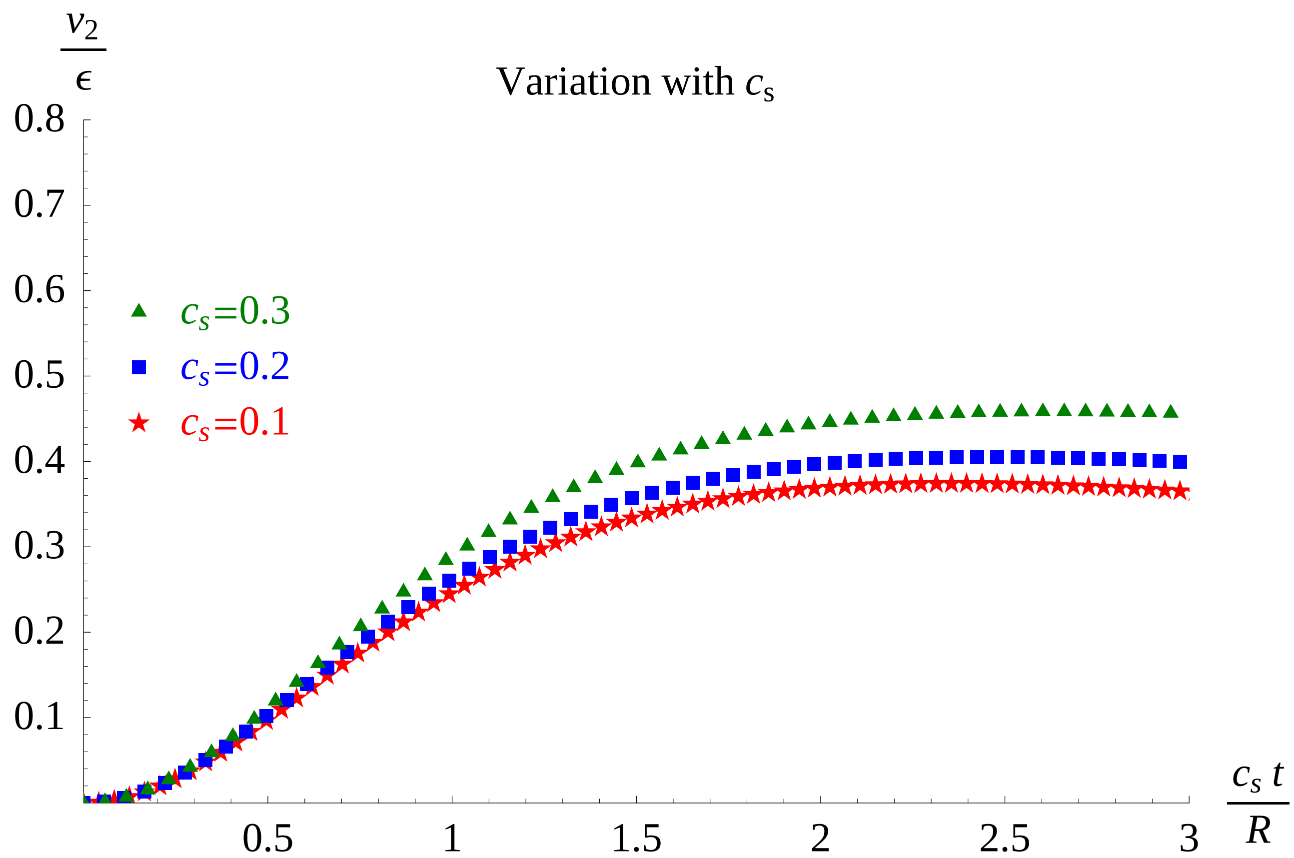

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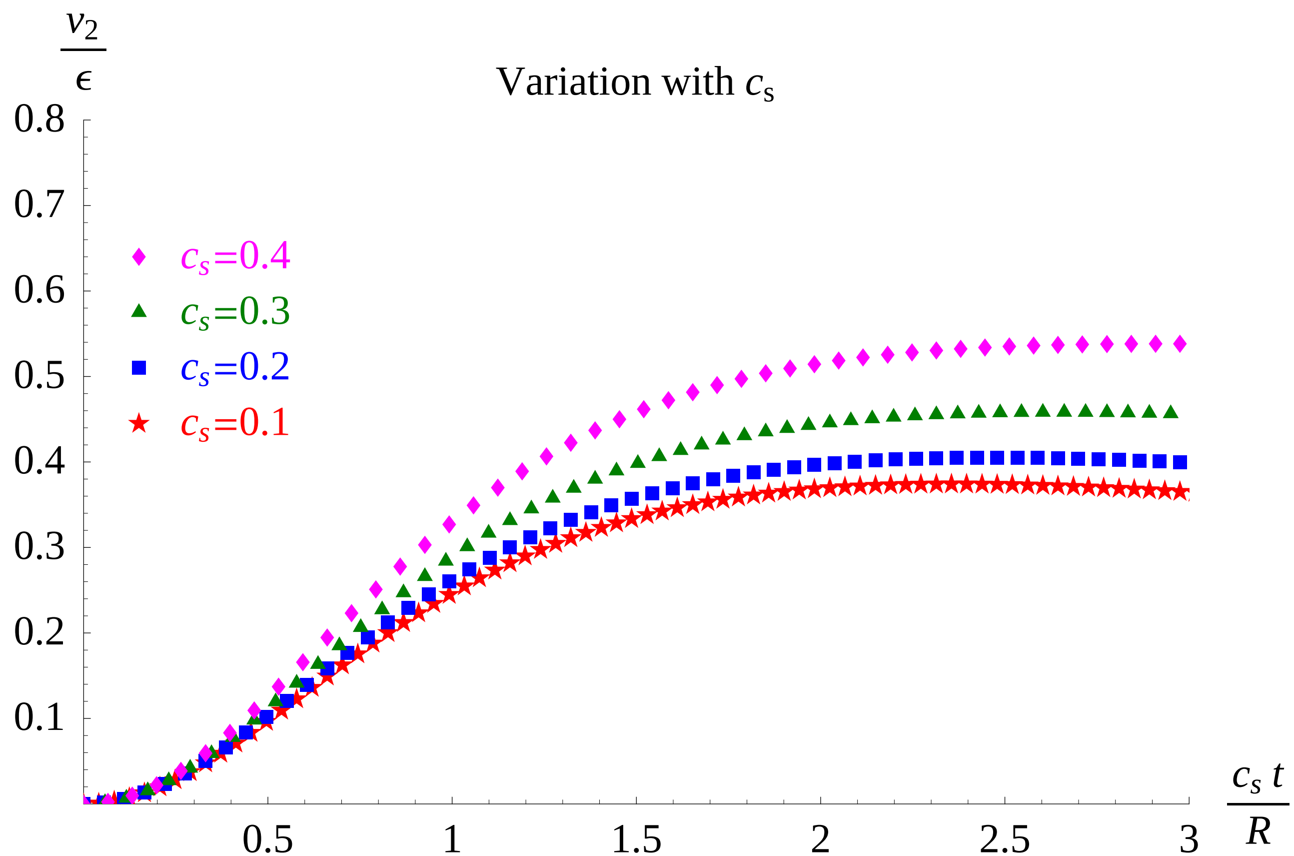

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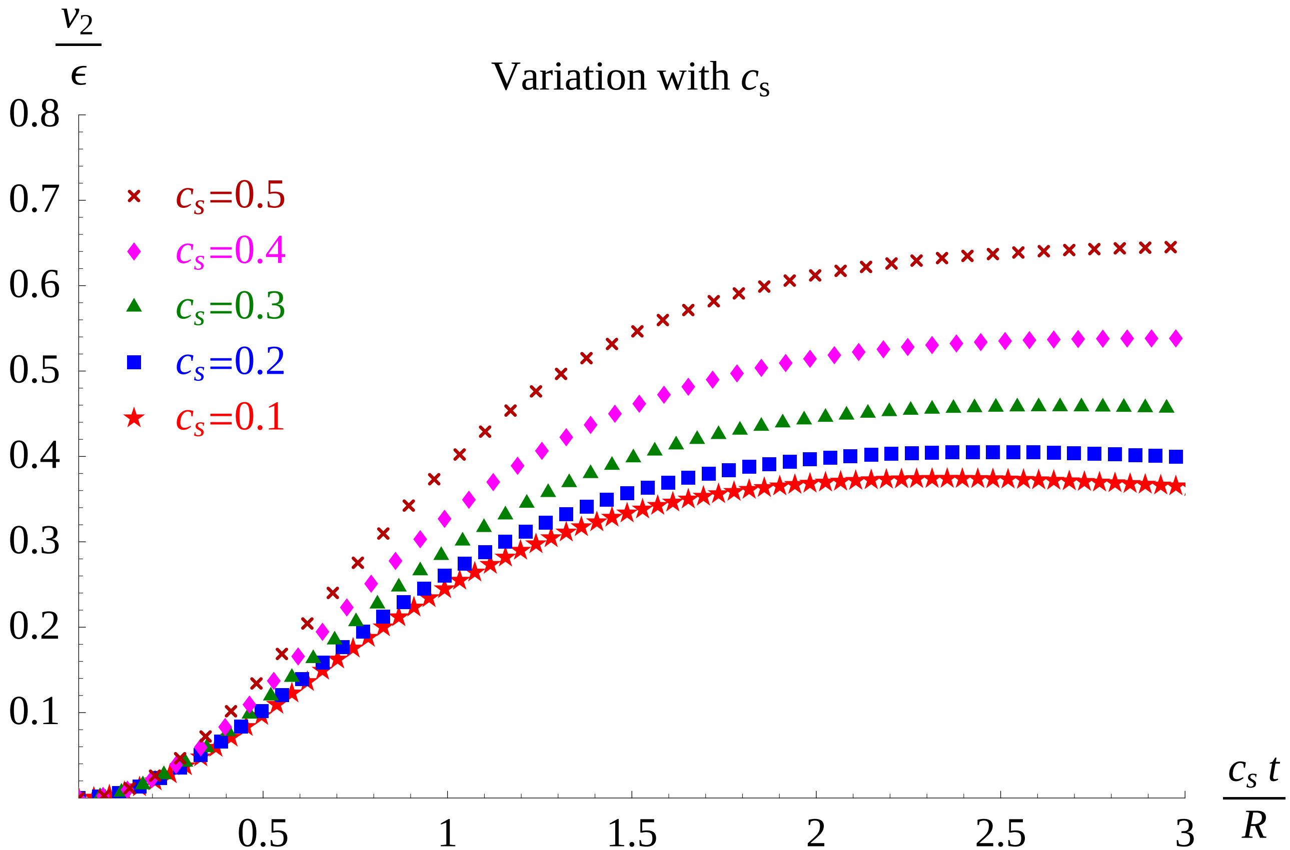

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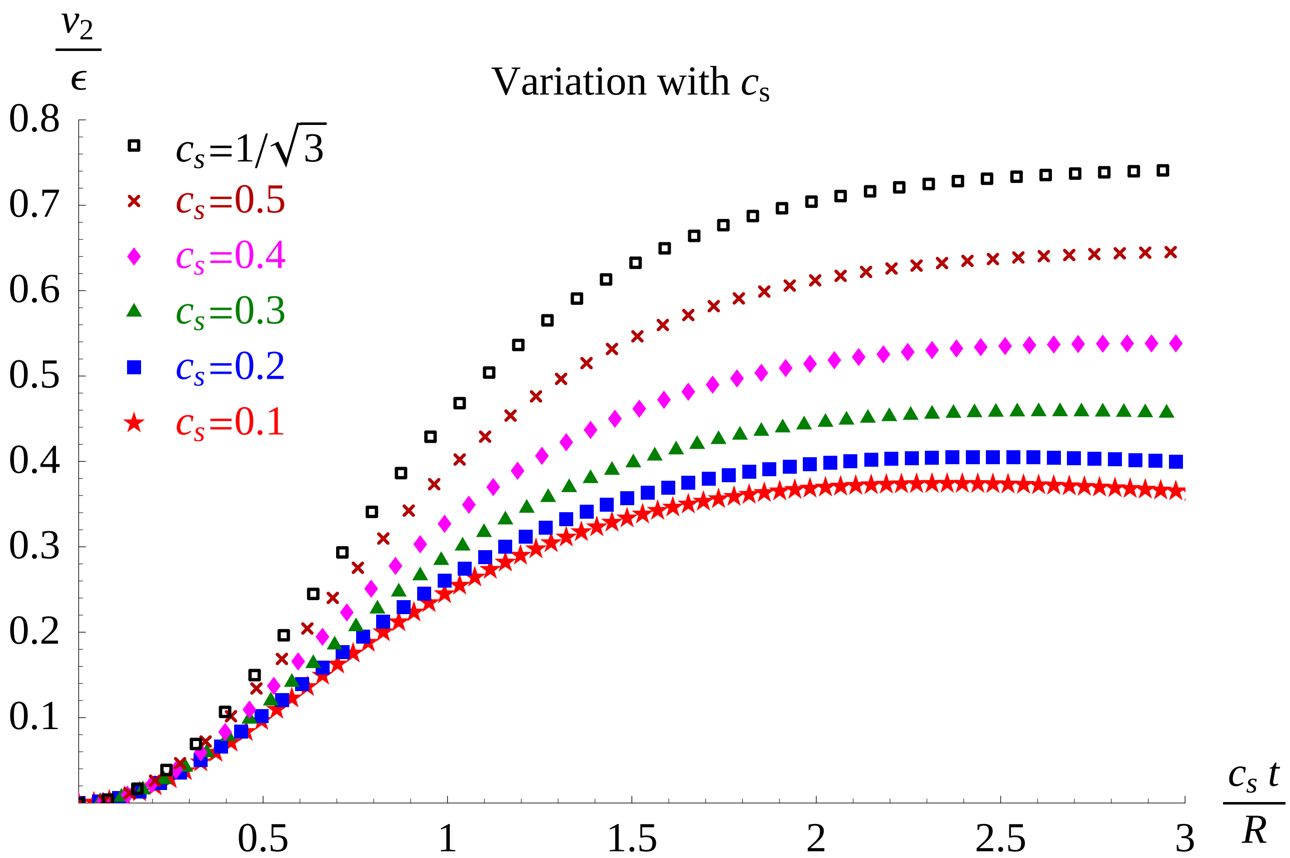


☞ one can increase v_2 by increasing c_s

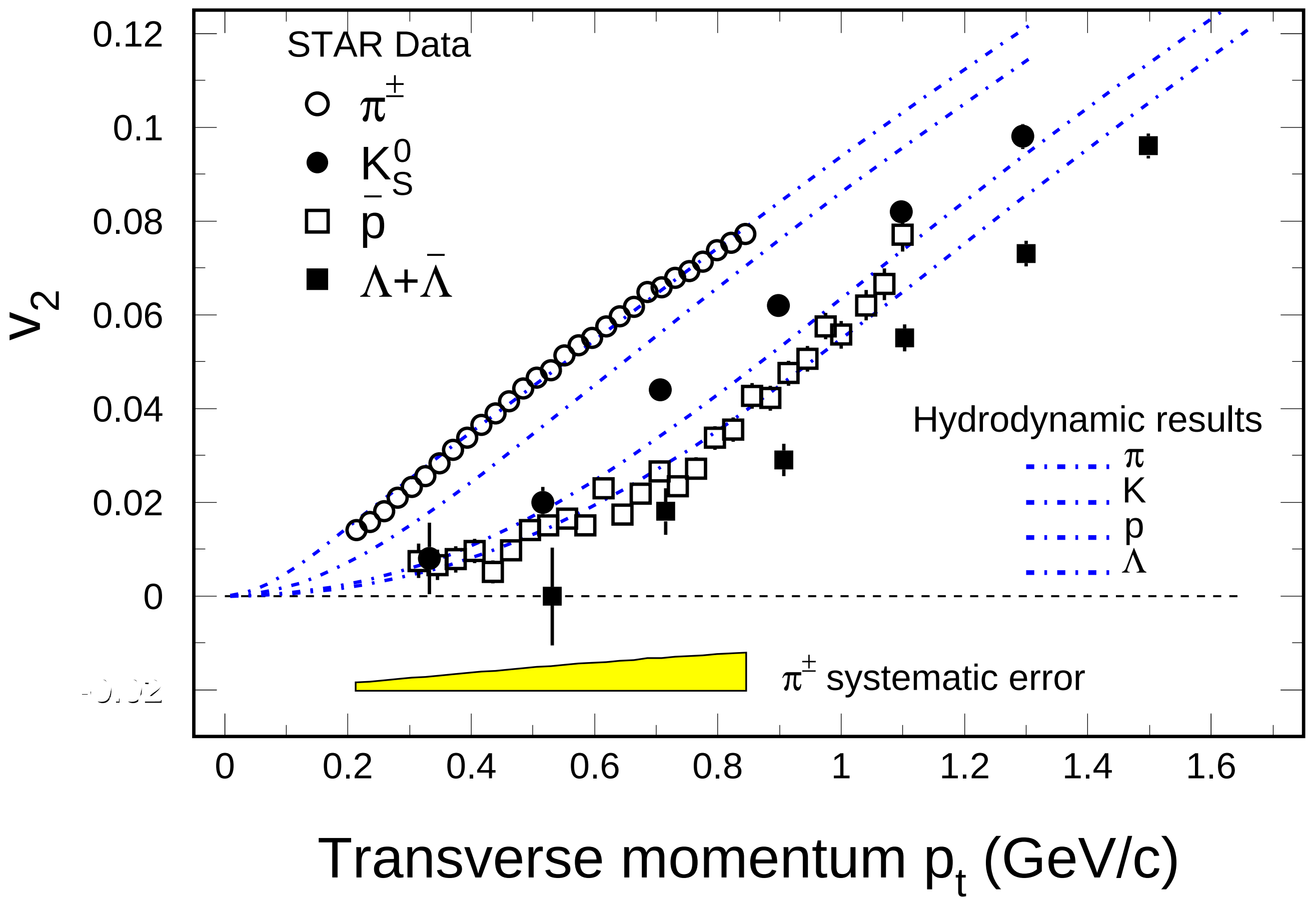
Anisotropic flow: predictions of hydro

- Characteristic build-up time of v_2 is $\bar{R}/c_s$
- v_2/ϵ constant across different centralities
- v_2 roughly independent of the system size (Au–Au vs. Cu–Cu)
- v_2 increases with increasing speed of sound c_s
- Mass-ordering of the $v_2(p_T)$ of different particles
  (the heavier the particle, the smaller its v_2 at a given momentum)
- Relationship between different harmonics: $\frac{v_4}{(v_2)^2} = \frac{1}{2}$

RHIC Au–Au data: a personal choice [1/4]

$v_2(p_t)$ at midrapidity, minimum bias collisions:

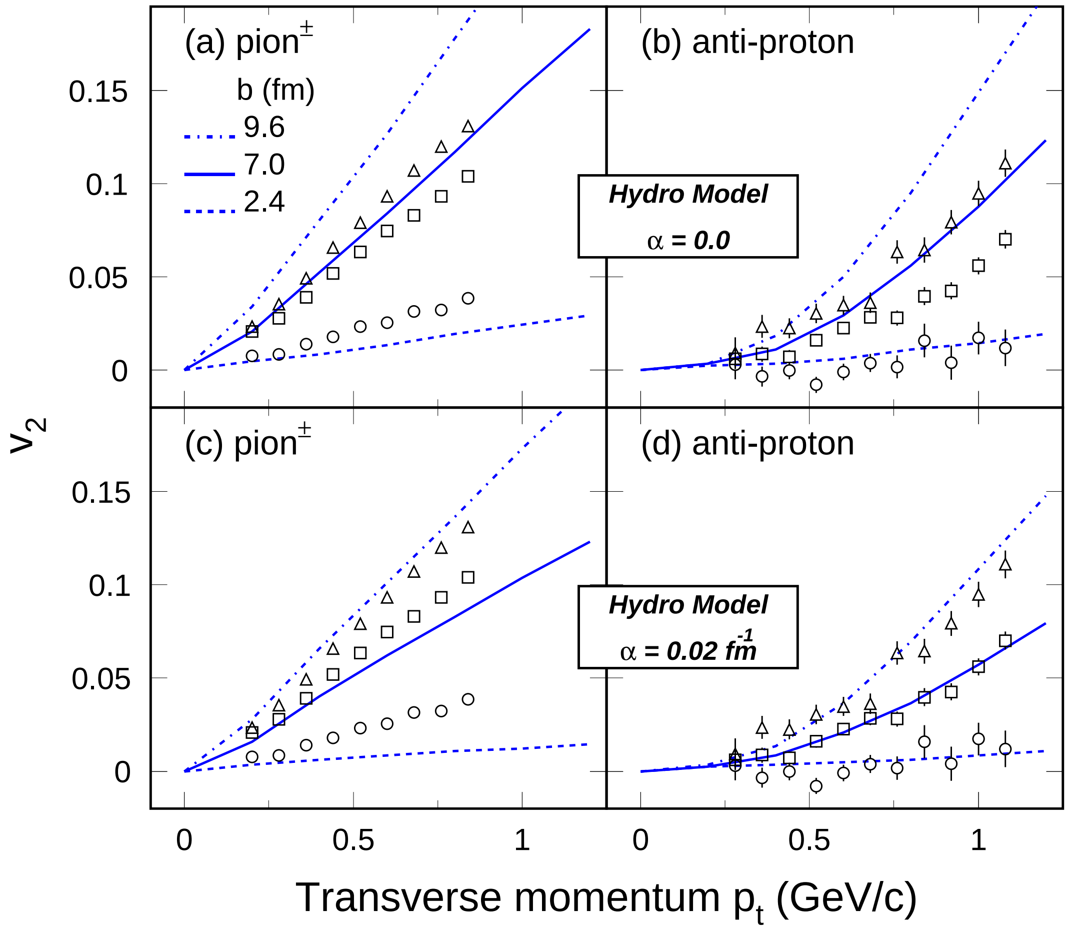
STAR Collaboration, Phys. Rev. C **72** (2005) 014904



RHIC Au–Au data: a personal choice [2/4]

$v_2(p_t)$ for various centralities (impact parameters):

STAR Collaboration, Phys. Rev. C **72** (2005) 014904



RHIC Au–Au data: a personal choice [2/4]

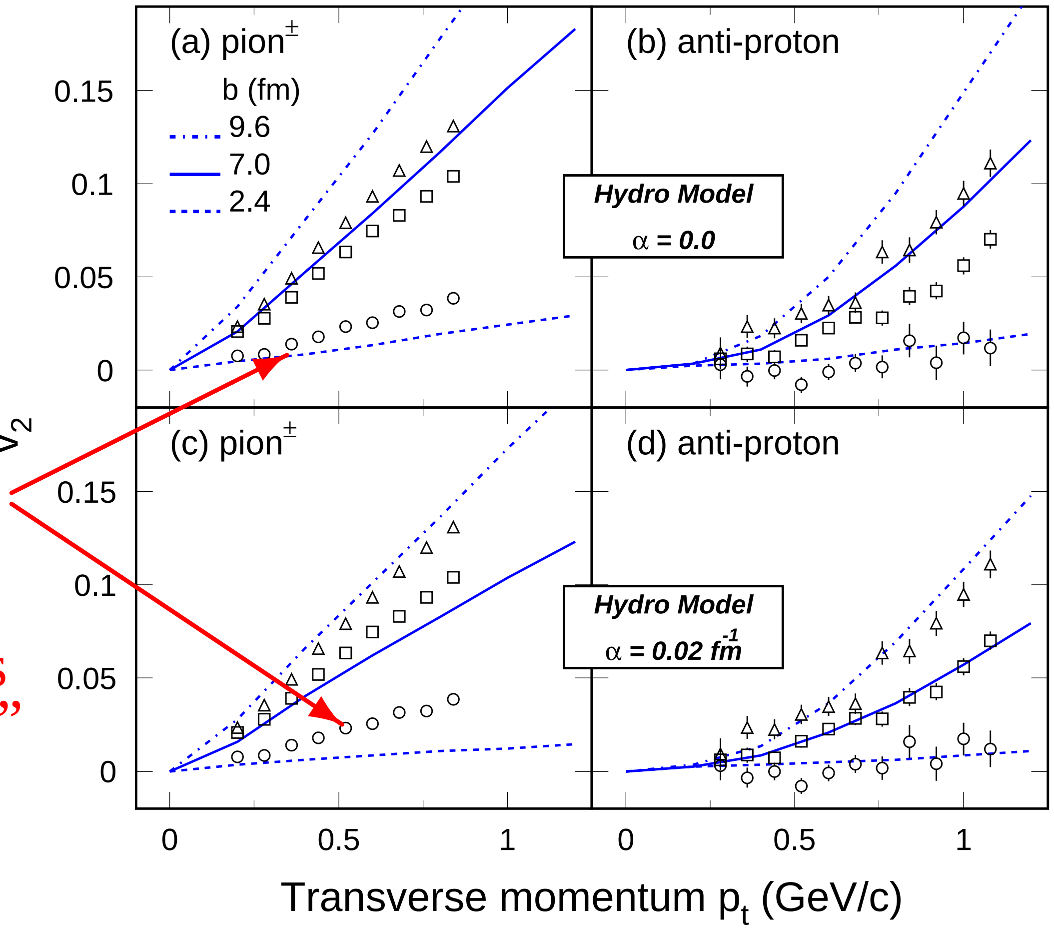
$v_2(p_t)$ for various centralities (impact parameters):

STAR Collaboration, Phys. Rev. C **72** (2005) 014904

data above ideal fluid

$\neq$

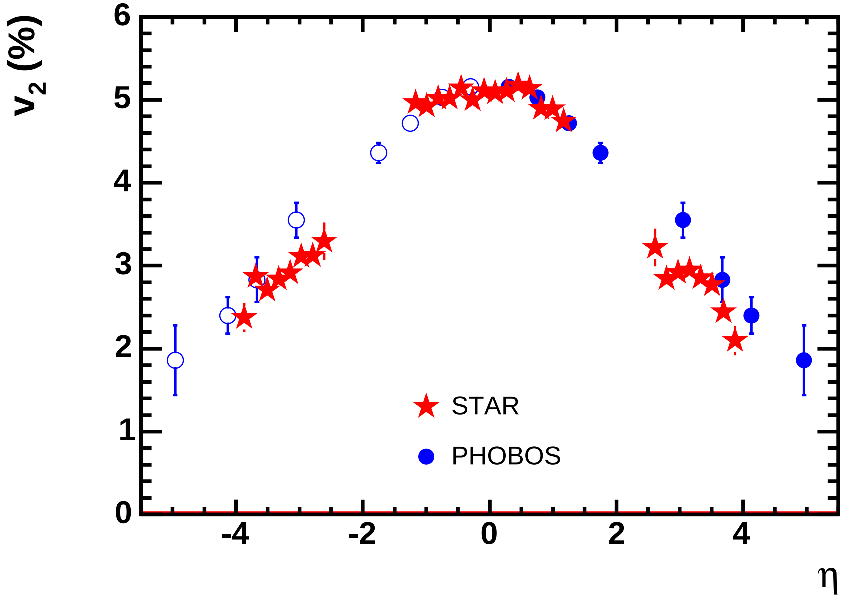
"hydro yields maximum v_2"



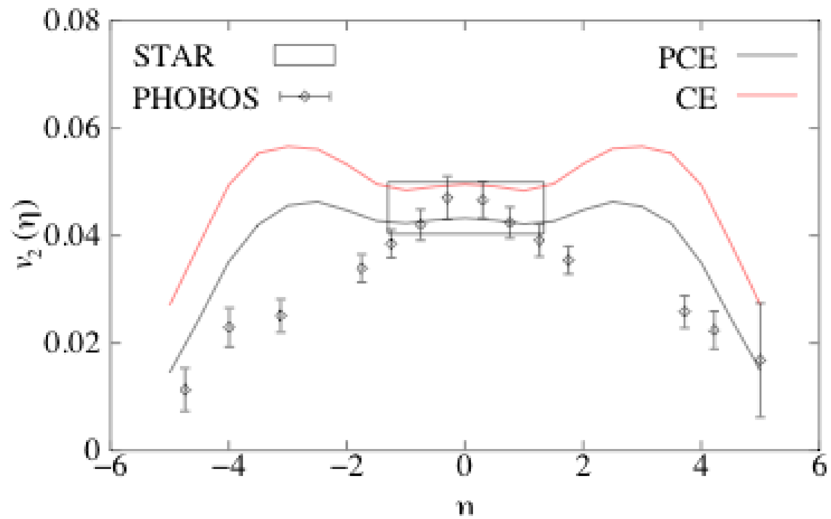
RHIC Au–Au data: a personal choice [3/4]

(Pseudo)rapidity dependence of v_2

STAR Collaboration, Phys. Rev. C **72** (2005) 014904



Hirano & Tsuda, Phys. Rev. C **66** (2002) 054905

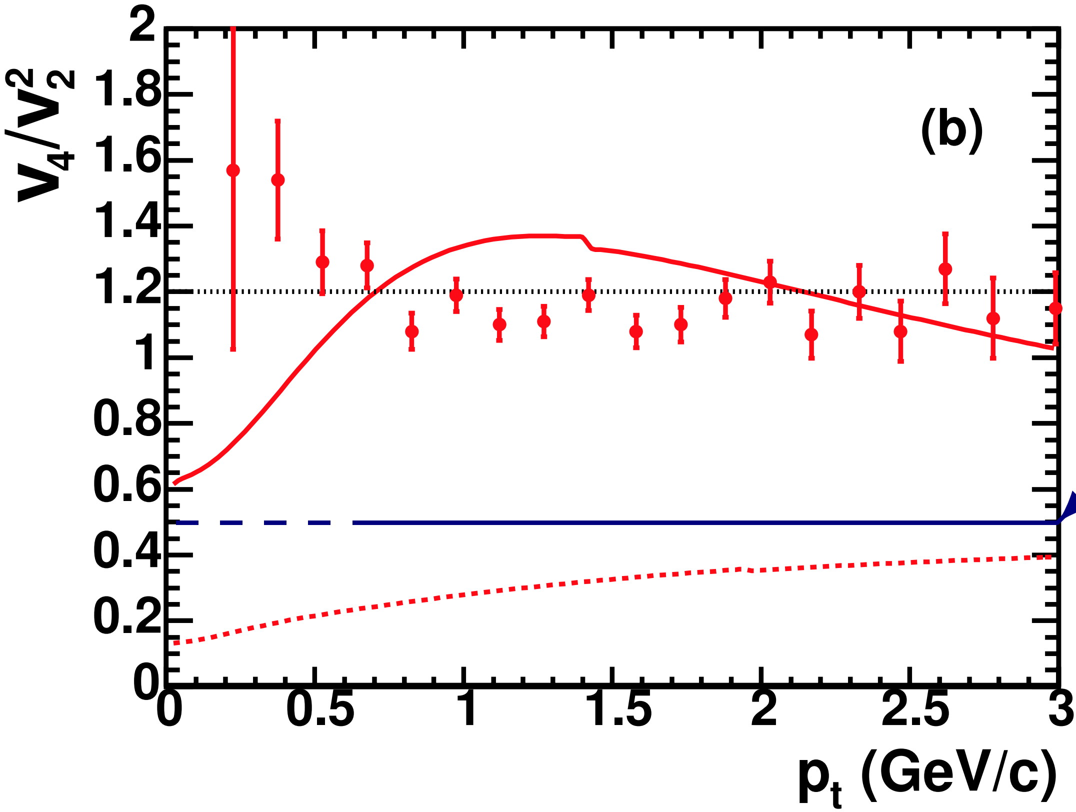


v_2(hydro) flatter than data

RHIC Au–Au data: a personal choice [4/4]

Transverse momentum dependence of $\dfrac{v_4}{(v_2)^2}$

STAR Collaboration, Phys. Rev. C **72** (2005) 014904



ideal fluid prediction

Ideal fluid dynamics vs. RHIC data

- ♠ $v_2(p_t)$ hydro $<$ data
- ♠ $v_2(y)$ hydro $\neq$ data
- ♠ $\dfrac{v_4}{(v_2)^2}$ hydro $<$ data

} what is wrong with ideal fluid scenario?

Ideal fluid dynamics vs. RHIC data

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} what is wrong with ideal fluid scenario?

---

0. Creation of a dense gas of particles

1. At some time τ_0 (~ 0.6 fm/c in hydro models), the mean free path λ is much smaller than *all* dimensions in the system ($Kn \ll 1$)
$\Rightarrow$ thermalization, ideal fluid dynamics applies

2. The fluid expands: density decreases, λ increases (system size also)

3. At some time, the mean free path is of the same order as the system size: ideal fluid dynamics is no longer valid

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① At some time τ_0 (~ 0.6 fm/c in hydro models), the mean free path λ is much smaller than *all* dimensions in the system ($Kn \ll 1$)

Is this really true?

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⓪ Creation of a dense gas of particles

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Is this really true?

☞ Can we estimate the number of collisions per particle Kn^{-1}?

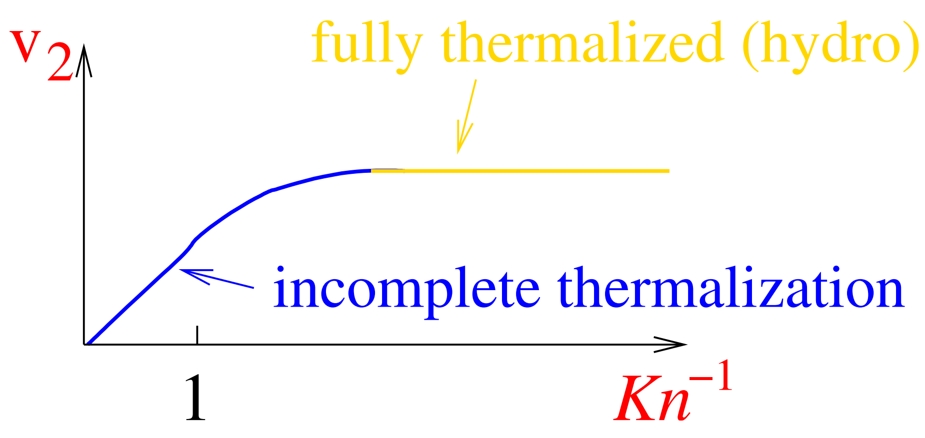
☞ How do v_2, v_4 depend on Kn^{-1}?

Anisotropic flow
vs. number of collisions Kn^{-1}

An exact computation of the dependence of v_2, v_4 on the number of collisions per particle Kn^{-1} requires some cascade model…

…but we can guess the general tendency!

- in the absence of reinteractions ($Kn^{-1} = 0$), no flow develops
- the more collisions, the larger the anisotropic flow
- for a given number of collisions, the system thermalizes: further collisions no longer increase v_2



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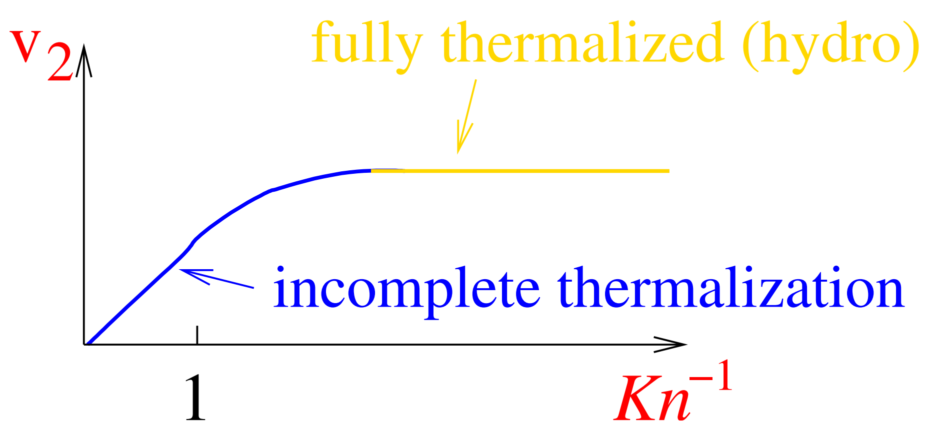
should be quantified!

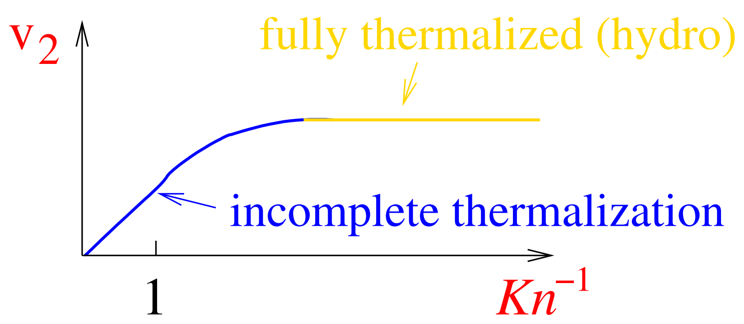


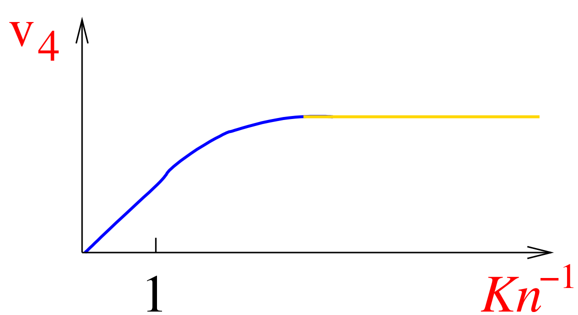
Incomplete equilibration & RHIC data [1]

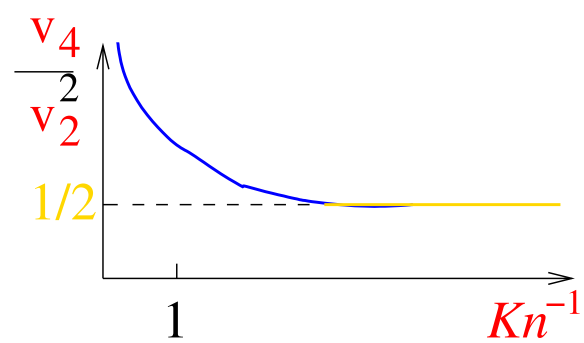
Ideal fluid dynamics predicts $\frac{v_4}{(v_2)^2} = \frac{1}{2}$, RHIC data are above ($\sim 1.2$)

☞ increase can be explained by incomplete equilibration naturally:

v_n proportional to the number of collisions Kn^{-1} $\Rightarrow$ $\frac{v_4}{(v_2)^2} \propto \frac{1}{Kn^{-1}}$







Number of collisions Kn^{-1}: a control parameter

The natural time (resp. length) scale for v_2 is $\bar{R}/c_s$ (resp. $\bar{R}$)

$\Rightarrow$ number of collisions per particle to build up v_2:

$$Kn^{-1} \simeq \frac{\bar{R}}{\lambda} = \bar{R}\,\sigma\,n\left(\frac{\bar{R}}{c_s}\right) \simeq \frac{c_s}{c}\frac{\sigma}{S}\frac{\mathrm{d}N}{\mathrm{d}y}$$

σ interaction cross section, $n(\tau)$ particle density, S transverse surface

☞ $\dfrac{1}{S}\dfrac{\mathrm{d}N}{\mathrm{d}y}$ control parameter for v_2: to vary Kn^{-1}, one can study

- centrality dependence (using the universality of v_2/ϵ)
- beam-energy dependence
- system-size dependence $\longrightarrow$ importance of lighter systems!
- rapidity dependence
- transverse momentum dependence ($p_T \nearrow \Rightarrow \sigma \searrow \Rightarrow Kn^{-1} \searrow$)

Control parameter: centrality dependence

The number of collisions to build up v_2 is $Kn^{-1} = \frac{\bar{R}}{\lambda} \propto \frac{\sigma}{S}\frac{dN}{dy}$

In Au–Au collisions at RHIC:

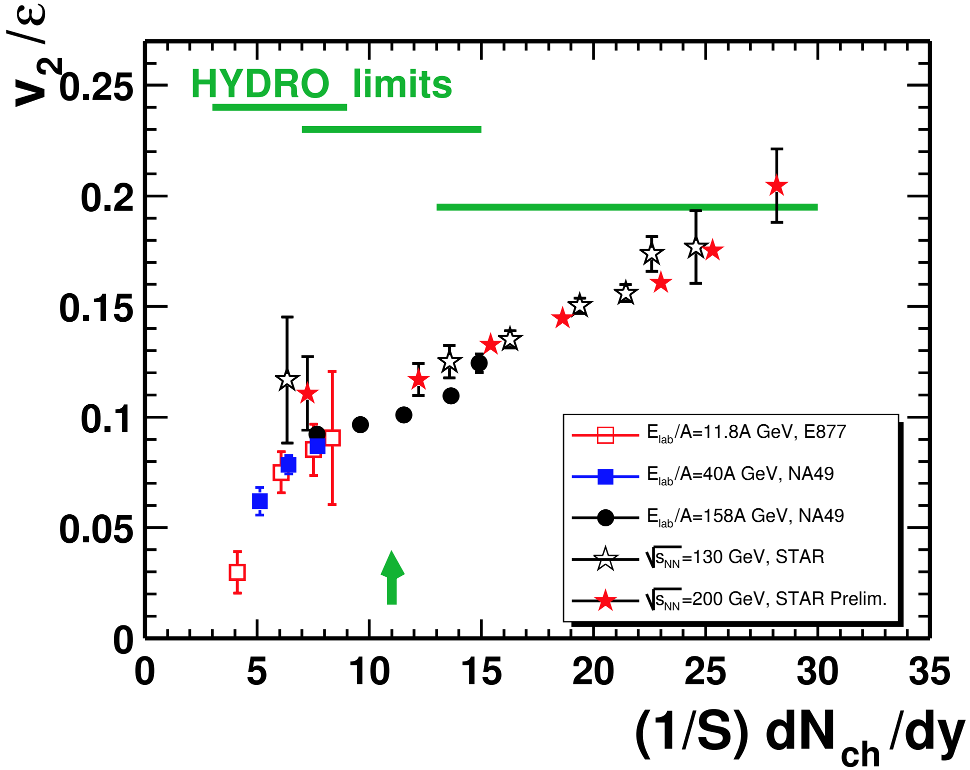
b (fm)	$\bar{R}$ (fm)	$\frac{dN}{dy}$	$n\left(\frac{\bar{R}}{c_s}\right)$ (fm^{-3})
0	2.07	1050	5.4
2	2.02	975	5.4
4	1.89	790	5.5
6	1.68	562	5.3
8	1.45	344	4.9
10	1.22	167	3.8

$n\left(\frac{\bar{R}}{c_s}\right)$, hence λ, varies little for $b = 0$–8 fm, while $\bar{R}$ varies by 30%

☞ centrality-dependence of $\frac{v_2}{\epsilon} \Leftrightarrow \frac{1}{S}\frac{dN}{dy}$-dependence

Incomplete equilibration & RHIC data [2]

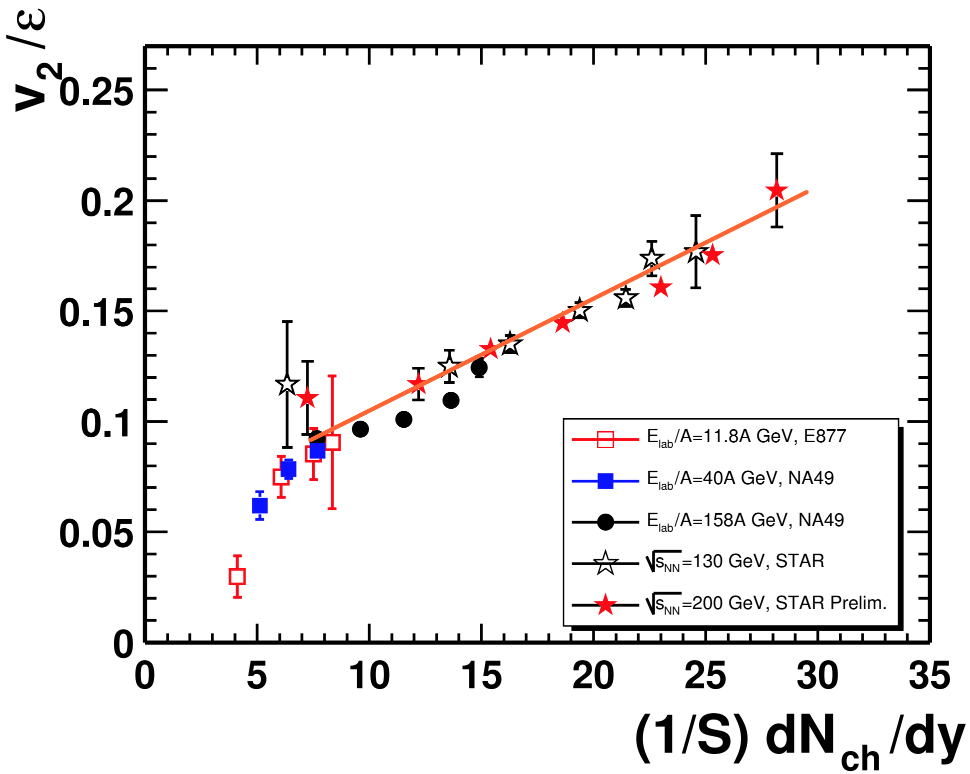
Centrality and beam-energy dependence:



NA49 Collaboration, Phys. Rev. C **68** (2003) 034903

Incomplete equilibration & RHIC data [2]

Centrality and beam-energy dependence:



NA49 Collaboration, Phys. Rev. C **68** (2003) 034903

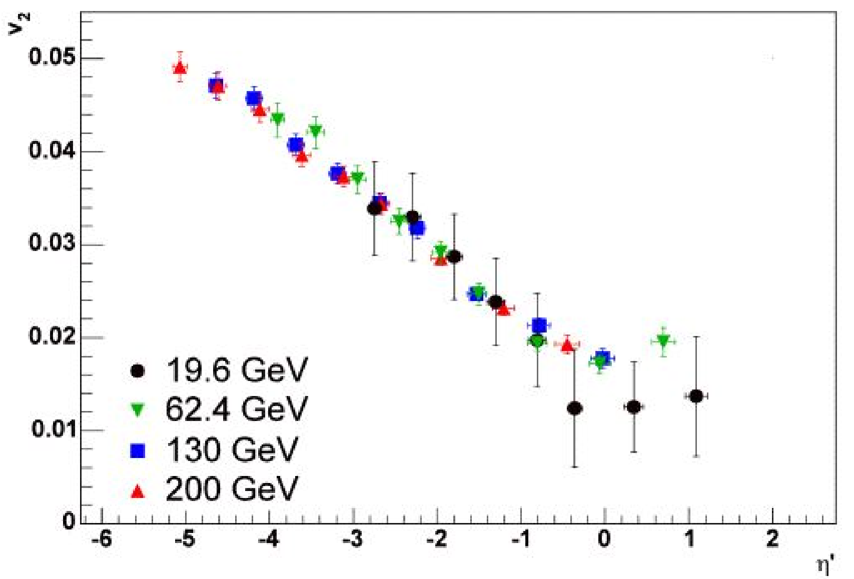
Scaling law seems to work for RHIC data (+ matching with SPS)

$v_2(Kn^{-1})$ increases steadily (no hint at hydro saturation in the data)

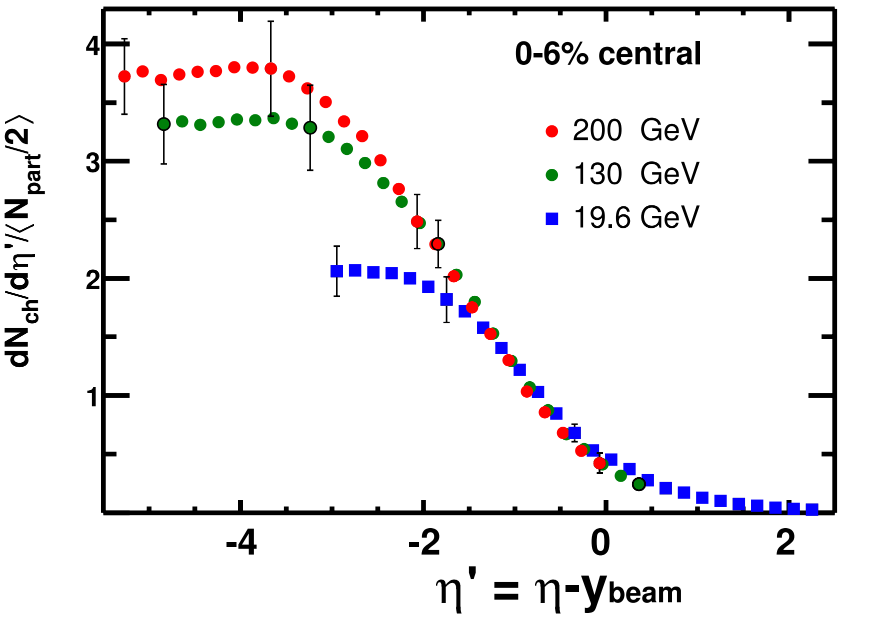
Incomplete equilibration & RHIC data [3]

(Pseudo)rapidity dependence of v_2

Steve Manly (PHOBOS Coll.)
QM'05



PHOBOS Collaboration
Phys. Rev. Lett. **91** (2003) 052303



☞ $v_2(\eta)$ and $\dfrac{\mathrm{d}N}{\mathrm{d}y}$ approximately proportional $\Leftrightarrow$ $v_2 \propto Kn^{-1}$

Hirano, Phys. Rev. C **65** (2002) 011901

Reconciling data and theory

In hydrodynamical fits, the speed of sound is constrained by p_t spectra, which require a soft equation of state

$\longrightarrow$ with a hard equation of state, the energy per particle is too high

All relies on the assumption that the energy per particle is related to the density, i.e., that chemical equilibrium is maintained

- chemical equilibrium is more fragile than kinetic equilibrium
- the only experimental indication of chemical equilibrium is in the particle-abundance ratios (cf. however $e^+e^-\ldots$)

If there is no chemical equilibrium, energy per particle and density are independent variables, as in ordinary thermodynamics

☞ there is no constraint on the equation of state from p_t spectra:

one can consider a larger c_s to increase v_2 in central collisions

Incomplete equilibration: predictions for Cu–Cu flow

- The matching between central SPS and peripheral RHIC suggests that we can even compare systems with different densities, i.e., different σ (and c_s)

  ☞ compare Au–Au at $b = 8$ fm with Cu–Cu at $b = 5.5$ fm (similar centrality)

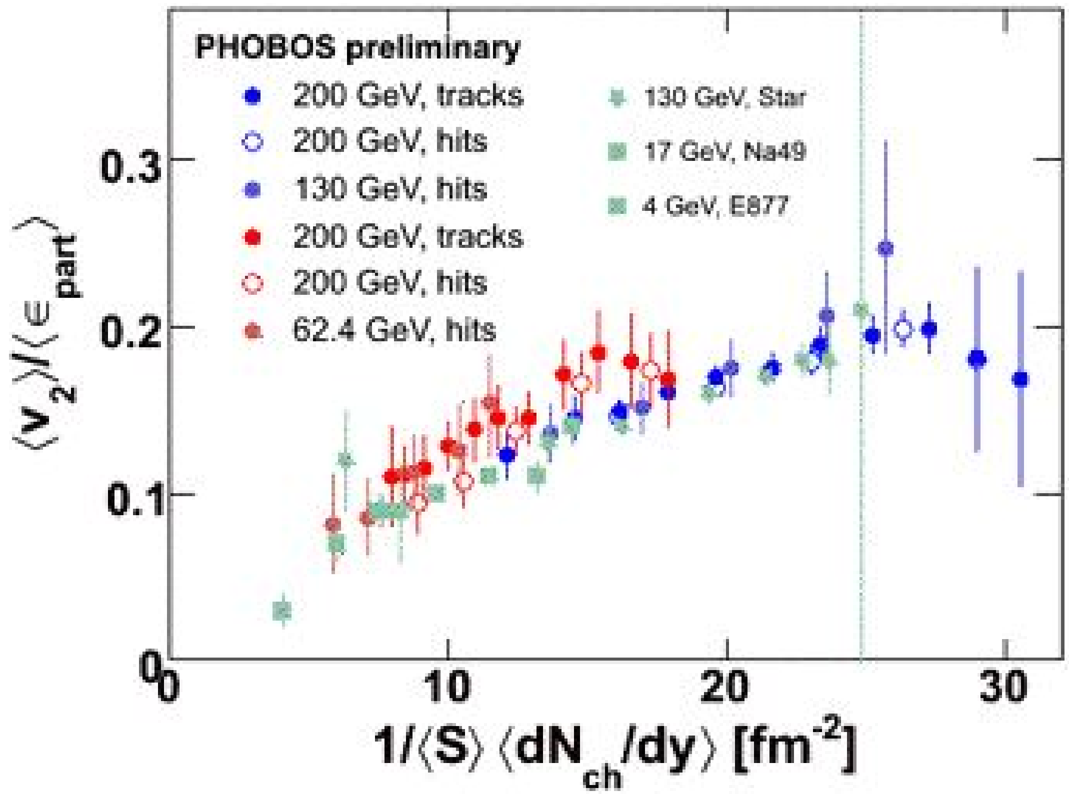
  - If hydro holds, v_2 should scale like ϵ: $v_2(\mathrm{Cu}) = 0.69\, v_2(\mathrm{Au})$
  - If thermalization is incomplete, $\dfrac{v_2}{\epsilon} \propto \dfrac{1}{S}\dfrac{\mathrm{d}N}{\mathrm{d}y} \propto Kn^{-1}$, i.e.

$$v_2(\mathrm{Cu}) = 0.34\, v_2(\mathrm{Au})$$

- Cu–Cu further from equilibrium than Au–Au $\Rightarrow$ $\dfrac{v_4}{(v_2)^2} > 1.2$

Cu–Cu collisions at RHIC: anisotropic flow [1/2]

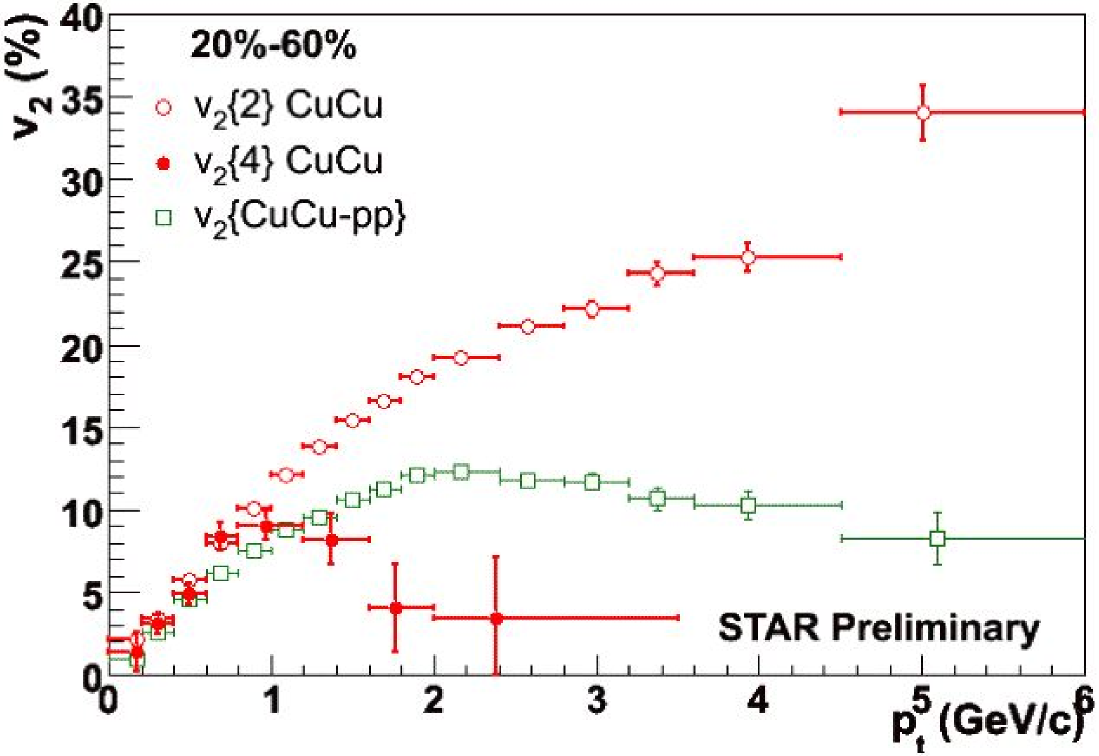
Steve Manly (PHOBOS Collaboration) @ QM'05:



☞ Cu–Cu results seem to be compatible with Kn^{-1}-scaling

Cu–Cu collisions at RHIC: anisotropic flow [2/2]

Gang Wang (STAR Collaboration) @ QM'05:



Measurements with different methods give very different v_2 values (not a surprise…)

Wait and see!

Hints of incomplete equilibration in RHIC data

R.S. Bhalerao, J.-P. Blaizot, N.B., J.-Y. Ollitrault, **nucl-th/0508009**

- A reminder: the natural time scale for anisotropic flow is $\frac{\bar{R}}{c_s}$
  - no knowledge about early times
  - anisotropic flow cannot conclude on *early* thermalization
- Size of v_2 controlled by $Kn^{-1} \propto \frac{1}{S}\frac{\mathrm{d}N}{\mathrm{d}y}$, but data do not saturate:

incomplete equilibration

- v_2 overshoots the hydrodynamical prediction... because the latter is over-constrained by a non-existent chemical equilibrium
- Predictions for Cu–Cu collisions at RHIC…

data shown at QM'05 too preliminary

- …and for Pb–Pb at LHC!

Predictions for LHC

Measuring anisotropic flow at LHC, you will find

- $\frac{v_2}{\epsilon}$ larger than at RHIC (getting closer to thermalization)

  larger signal, larger statistics ☞ easier measurement ☺

- $\frac{v_4}{(v_2)^2}$ smaller than at RHIC (closer to the ideal fluid value $\frac{1}{2}$)

  Well... that definitely means a smaller signal...

- Smaller systems yield complementary values of $\frac{1}{S}\frac{\mathrm{d}N}{\mathrm{d}y} \propto Kn^{-1}$, allowing checks (thermalization or not? onset of equilibration?)

Hints of incomplete equilibration in RHIC data

Extra slide

Methods of flow analysis

Anisotropic flow is usually measured using two-particle correlations:

$$\langle\cos 2(\phi_1 - \phi_2)\rangle \approx \langle\cos 2(\phi_1 - \Phi_R)\rangle\langle\cos 2(\Phi_R - \phi_2)\rangle = (v_2)^2$$

Assumption: all two-particle correlations are due to flow...

...which is obviously wrong!

"Non-flow" sources of correlations: jets, decays of short-lived particles, global momentum conservation, quantum effects between identical particles, etc. can bias the "standard" flow analysis

The bias is comparatively larger for smaller systems

☞ New methods for measuring flow have been developed

cumulants of multiparticle correlations, Lee–Yang zeroes

(N.B., P.M. Dinh, J.-Y. Ollitrault, R.S. Bhalerao, 2000–2004)