Strongly and Weakly Unstable Anisotropic QGP

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based on hep-ph/0504156, to be published in PRD
and Phys. Rev. D **67**, 014015 (2003)

August, 2005

A short introduction

- Appearance of unstable gauge modes for anisotropic parton distribution functions
  $\Rightarrow$ they may speed up thermalization.
- Gauge fields have an initial stage of exponential growth $A(t) \sim e^{\gamma t} a$. For how long?
- The conjecture: Abelianization of the process
  Arnold and Lenaghan
  Recently tested in numerical simulations:
  $\Rightarrow$ ok in $1 + 1$ d
  Arnold and Lenaghan, 04; Rebhan, Romatschke, Strickland, 04; Dumitru and Nara, 05
  $\Rightarrow$ but not ok in $1 + 3$ d.
  Arnold, Moore and Yaffe, 05; Rebhan, Romatschke, Strickland, 05

Dynamical Evolution

Dynamics studied with

$$S_{\text{eff}} = S_{YM} + S_{HL}$$

the Hard Loop effective action obtained from the transport equation, in a (covariant) linear expansion around the anisotropic state (or equivalently in a diagrammatic expansion)

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Is it legitimate to remain at the HL level?
Instability growth is "faster" than collisions, so it is ok to neglect the collision term in the studies.

Dynamical Evolution: Abelianization ?

If the system is such that only depends in one spatial direction z
(Arnold and Lenaghan)
The static effective potential

$$V_{\rm eff}[A] = V_{YM} + V_{HL} = \frac{g^2}{4} f^{abc} f^{ade} (\mathbf{A}^b \cdot \mathbf{A}^d)(\mathbf{A}^c \cdot \mathbf{A}^e) - \mu^2 \mathbf{A}^a_T \cdot \mathbf{A}^a_T$$

suggest that the system Abelianizes, as the Abelian directions correspond to the steepest decrease in $V_{\rm eff}$.

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  Reason: existence of constants of motion.

Toy Example: Electromagnetic charged particle

Suppose an invariance in the system

$$A_\mu(x^\alpha + \epsilon n^\alpha) = A_\mu(x^\alpha)$$

Then it is easy to check that

$$n_\mu(mu^\mu + eA^\mu) = \mathrm{ct.}\ , \qquad u^\mu = \frac{dx^\mu}{d\tau}$$

From the Hamiltonian equations

$$\frac{dx^\mu}{d\tau} = \frac{\partial H}{\partial p_\mu}\ , \qquad \frac{dp^\mu}{d\tau} = -\frac{\partial H}{\partial x_\mu} = eu_\lambda \partial^\mu A^\lambda$$

$$\Rightarrow \qquad \frac{d(n \cdot p)}{d\tau} = 0$$

$$\frac{d}{dt} f(x,u) = \{f, H\}_{\mathrm{P.B.}} = 0$$

Classical colored charged particles

The same philosophy works for classical colored particles.
M. Laine and C.M. 02
Wong equations

$$m\frac{dx^\mu}{d\tau} = p^\mu \ , \qquad m\frac{dp^\mu}{d\tau} = gQ^a\, F_a^{\mu\nu}\, p_\nu \ , \qquad m\frac{dQ^a}{d\tau} = -gf^{abc}\, p^\mu\, A_\mu^b Q^c$$

When there is an translational invariance in the system then a solution of

$$\frac{d}{d\tau} f(x, p, Q) = \{f, H\}_{\mathrm{P.B.}} = 0$$

is given by

$$f\left(n_\mu(p^\mu + gQ_a A_\mu^a)\right)$$

Quantum color transport equations

Transport equation for quarks (anal. for antiquarks and gluons)
Q : a matrix in the fundamental representation of $SU(N_c)$

$$p^\mu D_\mu Q(p,x) + \frac{g}{2}\, p^\mu \left\{ F_{\mu\nu}(x), \frac{\partial Q(p,x)}{\partial p_\nu} \right\} = 0$$

Color current

$$j^\mu(x) = -\frac{g}{2} \int dP\; p^\mu \left[Q(p,x) - \frac{1}{N_c} \mathrm{Tr}\, Q(p,x) \right]$$

Generated by the effective action

$$j_a^\mu(x) = -\frac{\delta S}{\delta A_\mu^a(x)}$$

Fast way to get the effective action and effective potential

Exact Solutions

Suppose an invariance in the direction of the index(ces) α_i.

$$Q(p,x) = f(p_{\alpha_i} - gA_{\alpha_i}(x))$$

$$= \sum_{n=0}^{\infty} \frac{(-g)^n}{n!} A_{\alpha_1}(x)\, A_{\alpha_2}(x) \cdots A_{\alpha_n}(x)\, \frac{\partial^n f(p_{\alpha_i})}{\partial p_{\alpha_1}\, \partial p_{\alpha_2} \ldots \partial p_{\alpha_n}}$$

is a solution iff $\quad [D_\mu A_{\alpha_i}, A_{\alpha_j}] = 0$

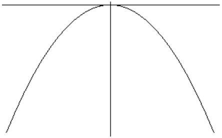
the associated effective action is then given by

$$\mathcal{L}_{\text{eff}} = -\sum_{n=0}^{\infty} \frac{(-g)^{n+1}}{(n+1)!} \mathrm{Tr}[A_{\alpha_1}(x) \cdots A_{\alpha_{n+1}}(x)] \int dP\, p^{\alpha_1}\, \frac{\partial^n f(p_{\alpha_i})}{\partial p_{\alpha_2} \cdots \partial p_{\alpha_{n+1}}}$$

Unstable

With an anisotropic f, at the HL level, one always gets a first negative term

In the Abelian direction



Strongly and Weakly Unstable

But higher order terms can change the shape of V!

- If V remains always negative and unbound ⇒ strongly unstable
- V may get positive contributions and develop local minima ⇒ weakly unstable

The final shape of V depends on the parton distribution function. (general criteria ??)

Note for strongly unstable solutions, the Abelianization should work perfectly ok.

Example: Strongly Stable

Gaussian function for a system with only z dependence

$$f(p_x, p_y, p_0) = 2^3\pi^{3/2}\sqrt{\beta(\beta-\alpha_x)(\beta-\alpha_y)}\,\rho\exp\Big(\alpha_x p_x^2 + \alpha_y p_y^2 - \beta\, p_0^2\Big)$$

$$V_{\text{eff}} = -g^2\Big\{\alpha_x\Big\langle\frac{p_x^2}{E_p}\Big\rangle\,\mathrm{Tr}[A_x^2] + \alpha_y\Big\langle\frac{p_y^2}{E_p}\Big\rangle\,\mathrm{Tr}[A_y^2]\Big\}$$

$$-g^4\Big\{\Big(\frac{1}{3}\alpha_x^3\Big\langle\frac{p_x^4}{E_p}\Big\rangle + \frac{1}{2}\alpha_x^2\Big\langle\frac{p_x^2}{E_p}\Big\rangle\Big)\,\mathrm{Tr}[A_x^4] + (x\leftrightarrow y)$$

$$\Big((\alpha_x\alpha_y^2 + \alpha_x^2\alpha_y)\Big\langle\frac{p_x^2 p_y^2}{E_p}\Big\rangle + \frac{1}{2}\alpha_x\alpha_y\Big\langle\frac{p_x^2+p_y^2}{E_p}\Big\rangle\Big)\,\mathrm{Tr}[A_x^2A_y^2]\Big\} + \mathcal{O}(g^6)$$

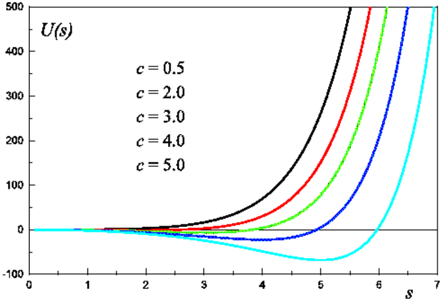
Example: Weakly Unstable

$$f(p_x, p_y, p_0) = \frac{2\pi^2\beta^2}{1+\beta\mathcal{P}}\, e^{\beta\mathcal{P}}\, \rho \left[\delta(p_x - \mathcal{P}) + \delta(p_x + \mathcal{P})\right] e^{-\beta p_0}$$

$$V_{\mathrm{eff}} = \frac{g^2}{2!}\rho\,\beta\,\frac{1-\beta\mathcal{P}}{1+\beta\mathcal{P}}\,\mathrm{Tr}[A_x^2] + \frac{g^4}{4!}\rho\,\beta^3\,\frac{3-\beta\mathcal{P}}{1+\beta\mathcal{P}}\,\mathrm{Tr}[A_x^4]$$

$$+\cdots+\frac{g^{2n}}{(2n)!}\rho\,\beta^{2n-1}\,\frac{2n-1-\beta\mathcal{P}}{1+\beta\mathcal{P}}\,\mathrm{Tr}[A_x^{2n}]+\cdots$$

$$c = \beta\mathcal{P} \ , \qquad s = g\beta A_x$$



Estimates

When are quartic terms as important as quadratic terms?

$$V_{\text{eff}} \sim -\mu^2 A^2 + \lambda A^4$$

$$A^2 \sim \frac{\mu^2}{\lambda}$$

$$\mu^2 \sim g^2 \frac{\rho}{p_{\text{hard}}} \,, \qquad \lambda \sim g^4 \frac{\rho}{p_{\text{hard}}^3}$$

$$\Rightarrow A \sim p_{\text{hard}}/g$$

assuming exponential growth, this happens at a time

$$t \sim \frac{1}{g p_{\text{hard}}} \ln 1/g$$

This was an estimate for a 1+1 d system, assuming Abelianization.

Outlook

According to numerical simulations in 1+3 d, saturation of the non-Abelian instabilities occur before.

$$A \sim \frac{p_{\rm soft}}{g}$$

and $p_{\rm soft} < p_{\rm hard}$

Can terms beyond the HL modify that conclusion?