First Results for the Growth of Collective Instabilities in the Melting Color Glass Condensate

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Outline

1. Introduction
   - Motivation
   - Melting the Color Glass Condensate
2. Preliminary Results
   - Effects of Rapidity-Fluctuations

Outline

Dynamics of Heavy-Ion Collisions

- In the limit of high collision energy one assumes boost-invariance to be a reasonable approximation at central rapidities (Bjorken, 1982)
- Immediately after the collision, gluons have large occupation number and thus interact strongly
- At earliest times, dynamics of the system is appropriately described by nonlinear gluonic fields
- Because of longitudinal expansion, gluon occupation number becomes smaller with time
- Once occupation number is small enough, nonlinearity effects become weak and one can describe gluons as on-shell particles

Saturation Scenario

- Saturation scenario: hard scale Q_s with $Q_s \sim 1$ GeV at RHIC energies
- At earliest times $\tau Q_s \leq 1$, one can describe system evolution in terms of classical gluonic fields (Krasnitz, Nara, Venugopalan 2000,2001,2003)
- Because of assumption of boost-invariance of the fields, there is no longitudinal dynamics
- Gluon distribution function at $\tau Q_s \sim 1$ is very anisotropic in momentum space ($\sim \delta(k_z)$)
- For thermalization, one needs isotropic system
- Baier, Müller, Schiff, Son (2001): Elastic scatterings increase longitudinal momentum of gluons

Plasma instabilities

- At times $\tau Q_s \geq 1$, dynamics of the system is in terms of hard ($k \sim Q_s$) particles coupled to soft fields
- Description via kinetic theory
- Anisotropic gluon distribution function generate Weibel instabilities (Arnold, Dumitru, Lenaghan, Manuel, Moore, Mrowczynski, Nara, Rebhan, PR, Strickland, Yaffe)
- These instabilities lead to exponentially growing magnetic fields in the transverse plane
- Growth rate of these instabilities $\Gamma \sim gQ_s$

Mini-Summary

- Early times: $\tau Q_s \leq 1$: system described in terms of classical gluon fields
- “Late” times: $\tau Q_s \geq 1$: system described in terms of hard particles coupled to soft ($k \sim gQ_s$) classical fields
- What happens near $\tau Q_s \sim 1$?

Dynamics near $\tau Q_s \sim 1$

- At times $\tau Q_s \sim 1$, system may be described either via transport equations or via classical field theory
- Since kinetic theory predicts exponentially growing modes with growth rate $\Gamma \sim g Q_s$, one would expect a similar phenomenon to occur in the classical field theory description (Arnold, Lenaghan, Moore, Yaffe, PRL 2005)
- To test this expectation, one needs to generalize existing simulations to include longitudinal dynamics
- Specifically, one needs to give up exactly boost-invariant initial conditions

Outline

Initial Conditions for Heavy-Ion Collisions

Boost-Invariant Case

- Color source for a large nucleus moving (nearly) with $v \sim c$

$$J_a^\mu = \delta_+^\mu \rho_a(x_\perp)\delta(x^-)$$

  where $x^\pm = (t \pm z)/\sqrt{2}$.
- Color charges ρ_a are modeled as classical random sources with Gaussian distribution (McLerran and Venugopalan, PRD49 &PRD50, 1994)

$$< \rho_a(x_\perp)\rho_b(y_\perp) >= g^2\mu^2\delta_{ab}\delta^2(x_\perp - y_\perp)$$

- The classical color field is obtained from $D_\mu F^{\mu\nu} = J^\nu$

Model of a Heavy-Ion Collision

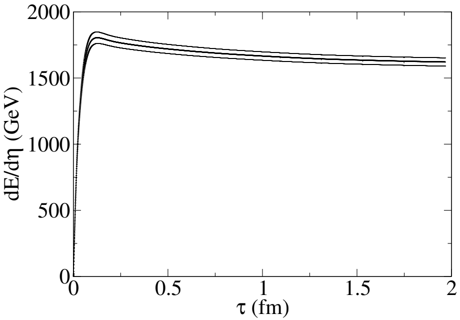
- Consider two infinitely large nuclei

$$J^\mu_a = \delta^\mu_+ \rho^{(1)}_a(x_\perp)\delta(x^-) + \delta^\mu_- \rho^{(2)}_a(x_\perp)\delta(x^+)$$

- *Before* the collision, $F^{\mu\nu}$ is a pure gauge solution
- Nuclei interact only at $\tau = \sqrt{2x^+x^-} = 0$
- Can obtain $F^{\mu\nu}$ at $\tau = 0$ by *matching* to pure gauge solutions before the collision
- Property of $A^\mu(\tau = 0)$: independent of rapidity $\eta = \mathrm{arctanh}\frac{z}{t}$

Lattice simulations

Subsequent evolution of $A^\mu(x_\perp, \tau)$ can be calculated by numerically solving $D_\mu F^{\mu\nu} = 0$ on a $2+1$ lattice (Krasnitz, Nara, Venugopalan 2000,2001,2003)



T.Lappi, PRC 2003

Initial Conditions for a Heavy-Ion Collision

With Boost-Invariance Violated

- Use boost-invariant initial conditions+add random perturbations in rapidity of size $\delta\mu$,

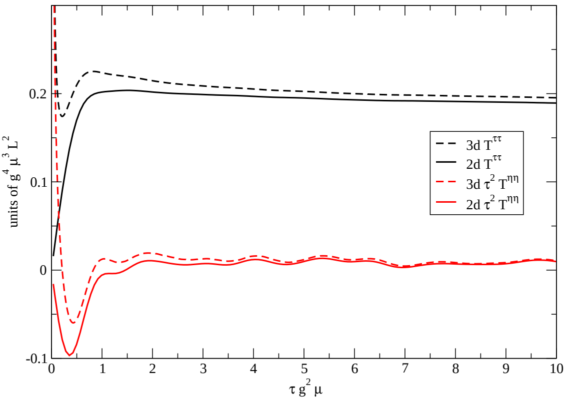
$$E_i(x_\perp,\eta) = \delta E_i(x_\perp,\eta), \qquad E_\eta(x_\perp,\eta) = \tilde{E}_\eta(x_\perp)+\delta E_\eta(x_\perp,\eta),$$

  which have to respect Gauss's law $D_i E_i + D_\eta E_\eta = 0$
- Have to start simulation at *finite* time τ_0, with $\tau_0 \ll 1/(g^2\mu)$
- Follow evolution by solving $D_\mu F^{\mu\nu} = 0$ on a 3+1 lattice

Outline

Convergence to Boost-Invariant Result

Energy Density in 3+1d Simulation



Real-time Correlators

Define correlators

$$C_x(\tau, k_\eta) = \int d\eta e^{ik_\eta \eta} < \mathrm{Tr}\, A_x(\tau, x_\perp, 0) A_x(\tau, x_\perp, \eta) >_\perp$$

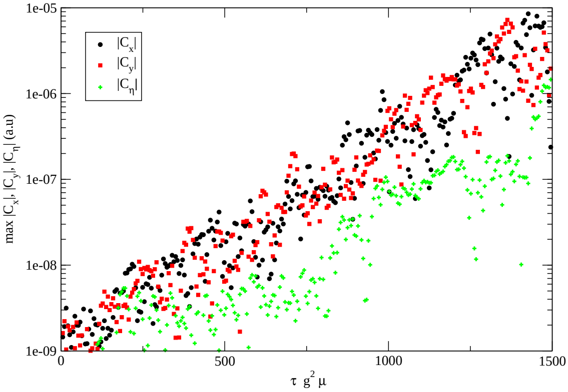
$$C_y(\tau, k_\eta) = \int d\eta e^{ik_\eta \eta} < \mathrm{Tr}\, A_y(\tau, x_\perp, 0) A_y(\tau, x_\perp, \eta) >_\perp$$

$$C_\eta(\tau, k_\eta) = \tau^{-2} \int d\eta e^{ik_\eta \eta} < \mathrm{Tr}\, A_\eta(\tau, x_\perp, 0) A_\eta(\tau, x_\perp, \eta) >_\perp$$

which for $k_\eta \neq 0$ are strictly zero in the boost-invariant case.

Real-time Correlators (2)

Preliminary Results



Determination of Growth Rates

Preliminary results suggest $\Gamma \sim 0.005 g^2 \mu$

Growth rate unchanged when...

- ...reducing temporal time step
- ...reducing lattice spacing a_η while $N_\eta a_\eta$ fixed
- ...increasing N_η at fixed a_η
- ...changing τ_0

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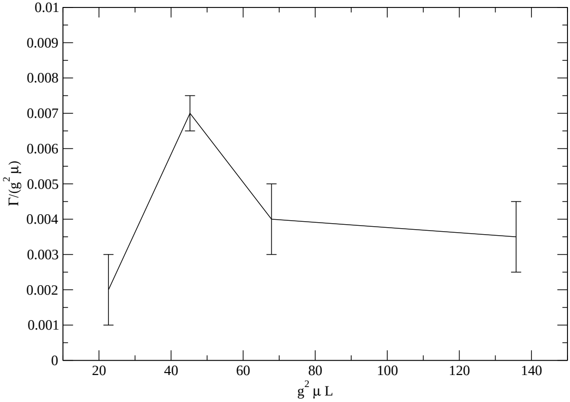
Dependence on $g^2\mu L$

Preliminary Results



Summary

- Motivated by results for $\tau Q_s \geq 1$, we are looking for fast isotropization mechanisms at $\tau Q_s \leq 1$
- Preliminary results suggest that there are exponentially growing rapidity fluctuations not unlike a Weibel instability
- So far, we found growth rates $\Gamma \sim 0.005 g^2 \mu$