Isotropization and QCD Plasma Instabilities

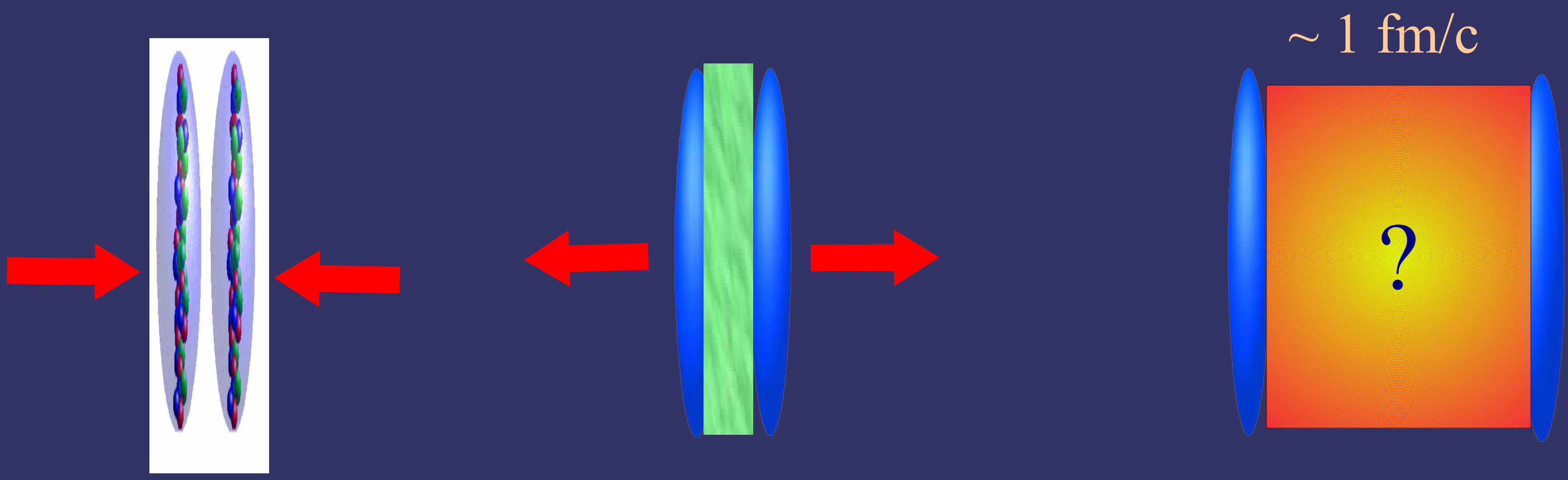
Adrian Dumitru
Johann Wolfgang Goethe University, Frankfurt am Main

based on the collaboration with
Yasushi Nara

Phys. Lett. B621, 89 (2005)

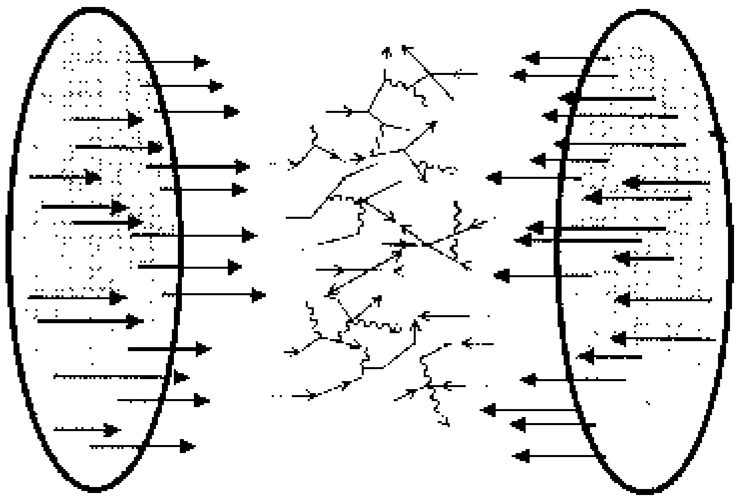
Goal

- Understanding the pre-equilibrium dynamics in high energy heavy ion collisions.
- Do we have thermalized deconfined matter (QGP) at RHIC/LHC?
- If so, How? When? Which T?



High-Energy Heavy-Ion Collisions

Space-Time Parton Cascade
in High-Energy Collisions :
(Geiger, Müller, 1992)



Boltzmann Equation :

$$p_\mu \partial^\mu f(x,p) = C[f]$$
$$C[f] = gg \to gg + \ldots$$

➣ Thermalization ?

Non-equilibrium dynamics within kinetic theory

Traditional plasma physics L.M.Lifshitz and L.P.Pitaevskii, Physical kinetics

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla_x f + g\left(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}\right)\cdot\nabla_p f = C[f]$$

$$\dot{\boldsymbol{E}} = \nabla\times\boldsymbol{B} - \boldsymbol{J} \qquad \boldsymbol{J} = g\int\frac{d^3p}{(2\pi)^3}\boldsymbol{v}\, f$$

$$\dot{\boldsymbol{B}} = -\nabla\times\boldsymbol{E}$$

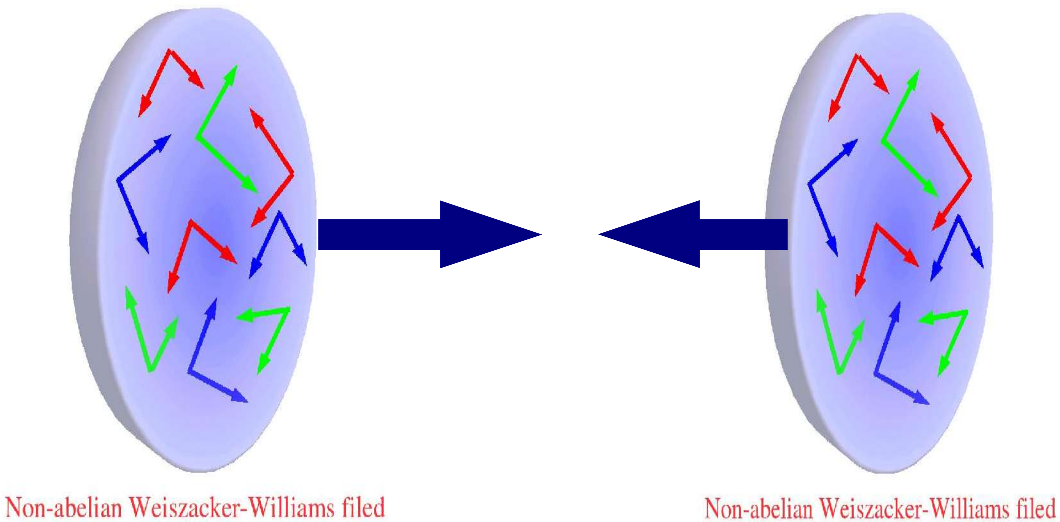
$f(\boldsymbol{x}, \boldsymbol{p})$ One-particle distribution function

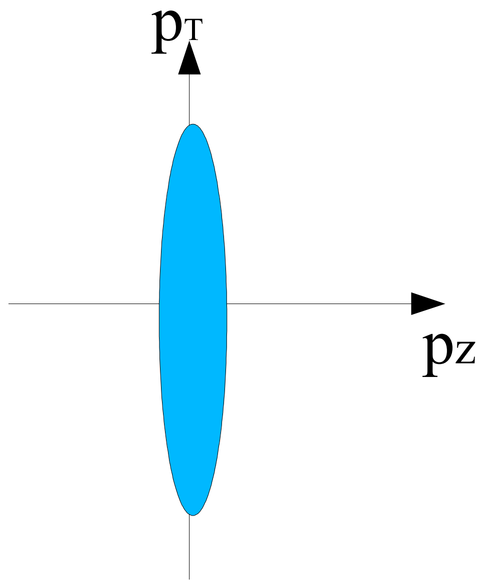
- Collisions only: Boltzmann equation (PCM: Geiger, Bass, Müller; Zhang, Molnar, Gyulassy; Xu, Greiner; ...)
- Soft field only: Vlasov equation.

“Bottom-Up” Scenario

Phys. Lett. B502 (2001) 51

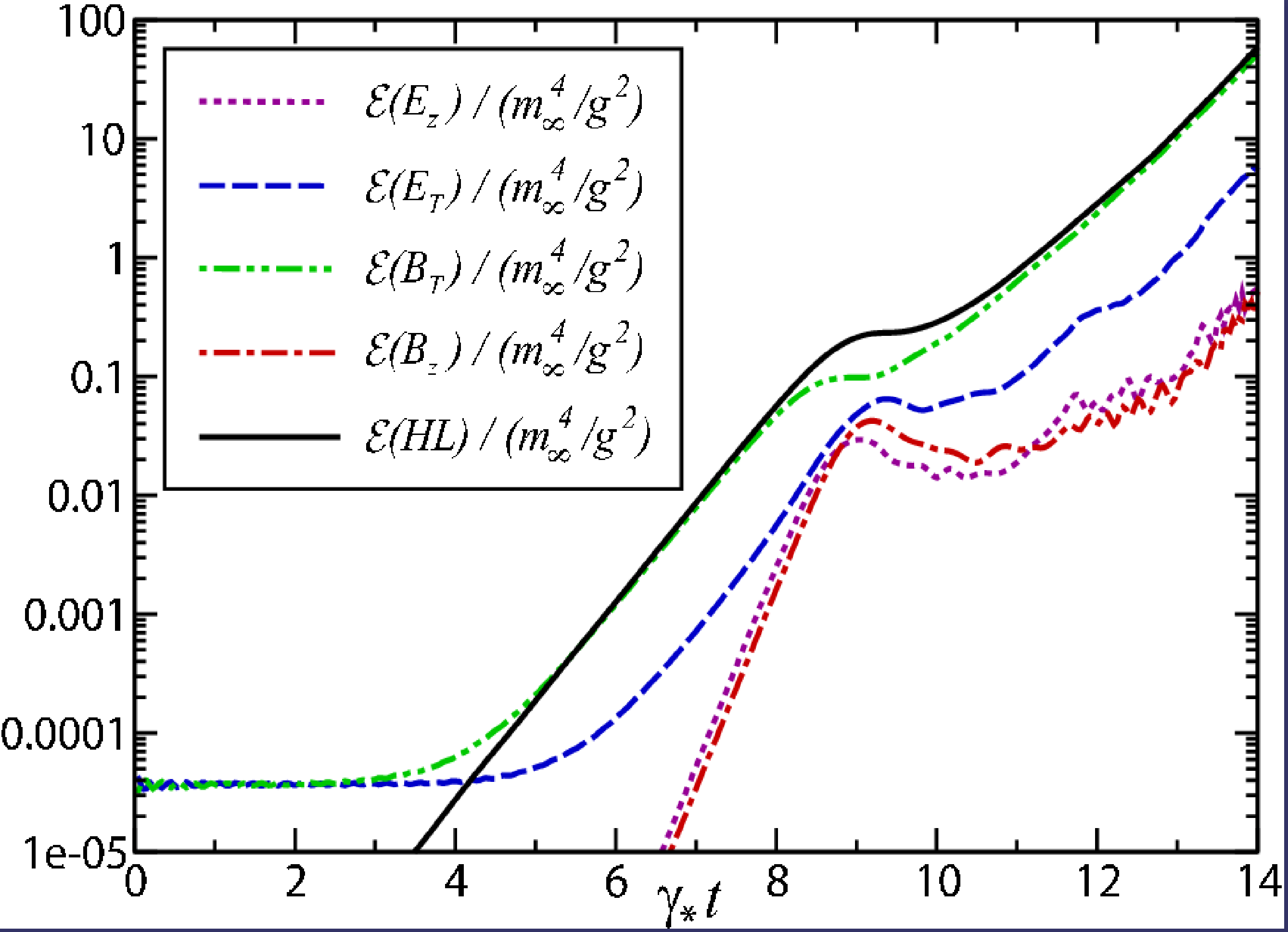
- Separation scale Q_s (saturation momentum) between soft and hard modes
- Hard gluons on-shell at $\tau \sim 1/Q_s$,
  $f \sim 1/\alpha_s$, typical momentum $p \sim Q_s$
- Due to longitudinal expansion, in the absence of collisions : $p_z \sim 1/\tau$, $p_x \sim p_y \sim Q_s$ at $\tau > 1/Q_s$





1+1D hard loop field simulations

A. Rebhan, P. Romatschke and M. Strickland, hep-ph/0412016.



Linear Vlasov $f \simeq f_0 + \delta f$

First confirmation of instabilities in SU(2) gauge theory predicted by

Mrowczynski,
Arnold, Lenaghan, Moore
Romatschke, Strickland

Initial Conditions:

- random fields
- squeezed particle momentum distr.

Role of Instabilities in Heavy-Ion Coll.

Context "Bottom-Up Scenario", discussion by ALM03

- Redshift of long. particle momenta leads to anisotropy for times $\tau Q_s >> 1, \; f(p) \sim \delta(p_x)$
- Instab. Growth time $\tau / \sqrt{\tau Q_s} > \tau$ (expansion time)
- Time btwn collisions $(Q_s \tau)^{2/3} / Q_s$ also larger

Instability is fastest process, "bottom-up" must be modified

- Quantitative studies not yet available

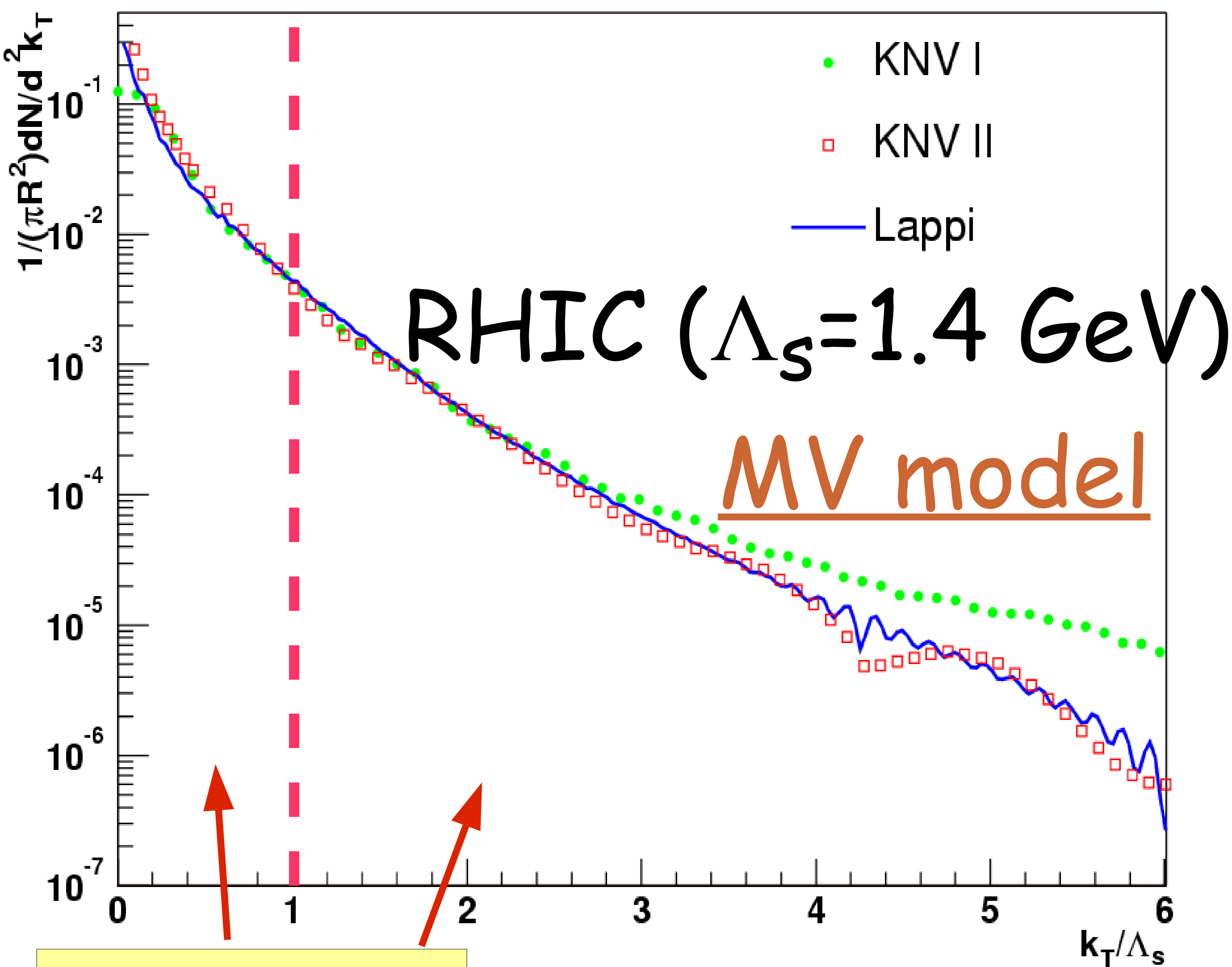
Purpose of our Study

solving full Vlasov-YM equations for gluons,
i.e. coupled dynamics of fields and particles.

- non-linear effects: saturation of instabilities
- isotropization of soft and hard modes (particles and field)
- weak and strong initial fields

Energy density of fields vs. particles

Krasnitz, Nara, Venugopalan, hep-ph/0305112



$\varepsilon = 3 : 1$

Particle in cell simulation for colored particles (Wong-YM)

$$\frac{d\,\boldsymbol{x}_i}{dt}=\boldsymbol{v}_i,\quad \frac{d\,\boldsymbol{p}_i}{dt}=gQ_i^a(\boldsymbol{E}_i^a+v_i\times\boldsymbol{B}_i^a),\quad \frac{d\,\boldsymbol{Q}_i}{dt}=igv_i^\mu[A_\mu,Q_i].$$

$$D_\mu F^{\mu\nu}=J^\nu=g\sum Q\,v^\mu\,\delta(x-x_i(t))$$

Non-Abelian version of the charge conservation algorithm
C. R. Hu and B. Muller, Phys. Lett. B409 (1997)377
G. D. Moore, C.R. Hu and B. Muller, Phys. Rev. D58 (1998)045001.

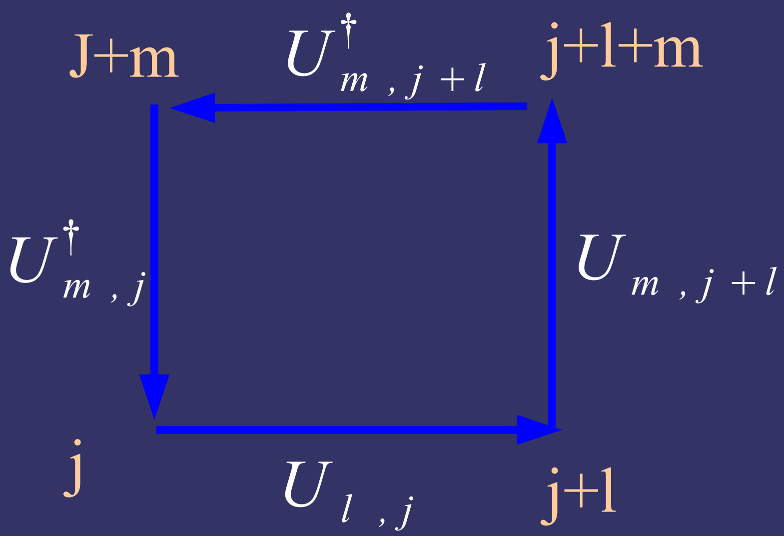
Yang-Mills Hamiltonian on the lattice

Numerical implementation of real time simulation in classical YM is well established. Kogut-Susskind Hamiltonian:

$$H_{YM} = \frac{1}{2}\sum_{l} E_l^2 + \frac{\tau}{2}\sum_{\square}\left(N_C - \Re\, Tr\, U_{\square}\right)$$

Link $\quad U_{j,i} = \exp[-iga\, A_j(i)]$

Plaquette $\quad U_{\square} \equiv U_{l,j} U_{m,j+l} U^{\dagger}_{l,j+m} U^{\dagger}_{m,j}$



Equations of motion for dynamical variable *v*

$$\frac{dv}{d\tau} = \{H_L, v\}$$

Initial conditions for simulations

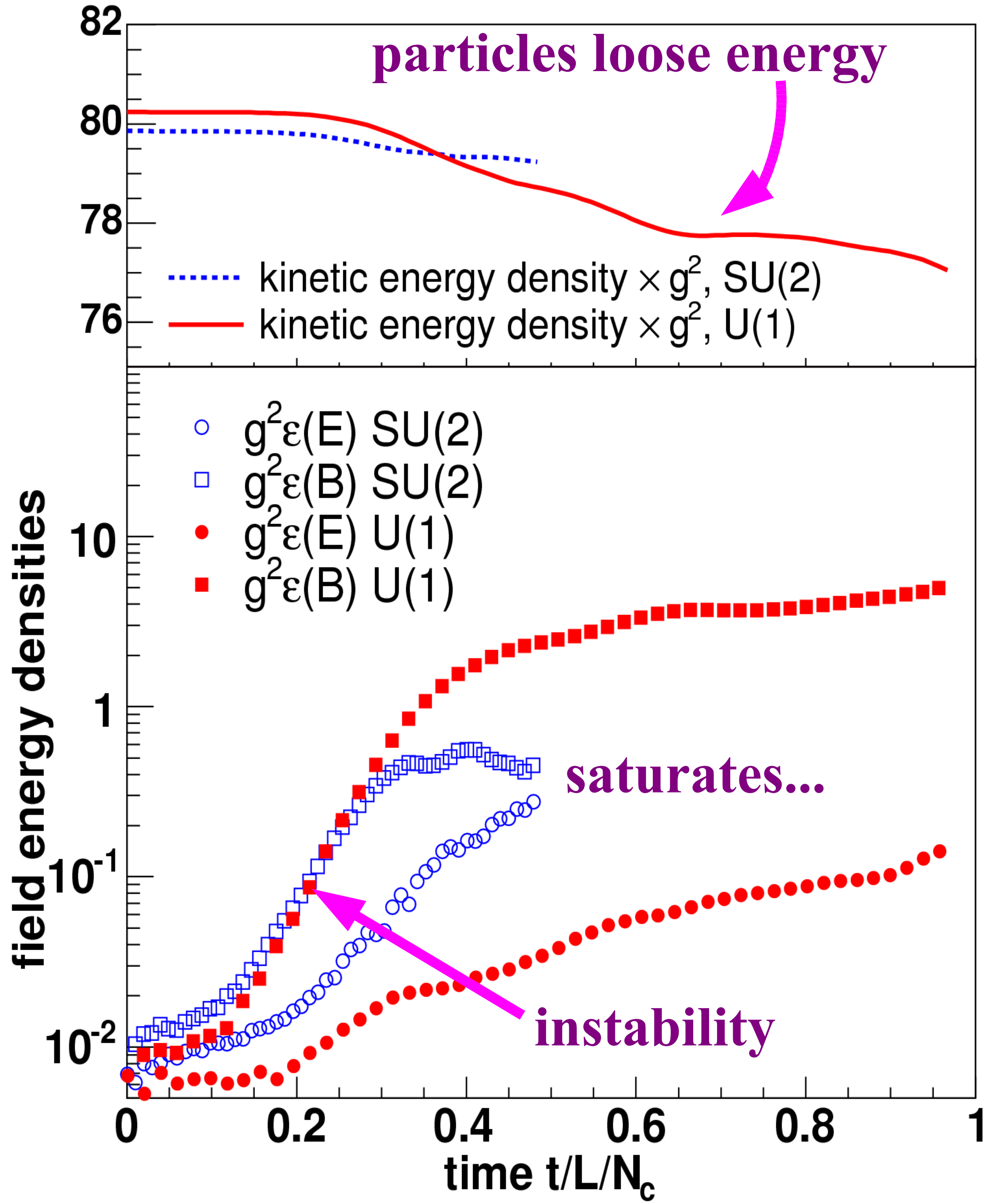
- Fields depend only on t, x (1+1D), but particle motion is in 3+1D
- Initial field configuration: random white noise.

  - Weak field: i.e. large separation between soft and hard modes.
    $P_{hard} = 10$ GeV
  - Strong field: small separation between soft and hard modes.
    $P_{hard} = 1$ GeV

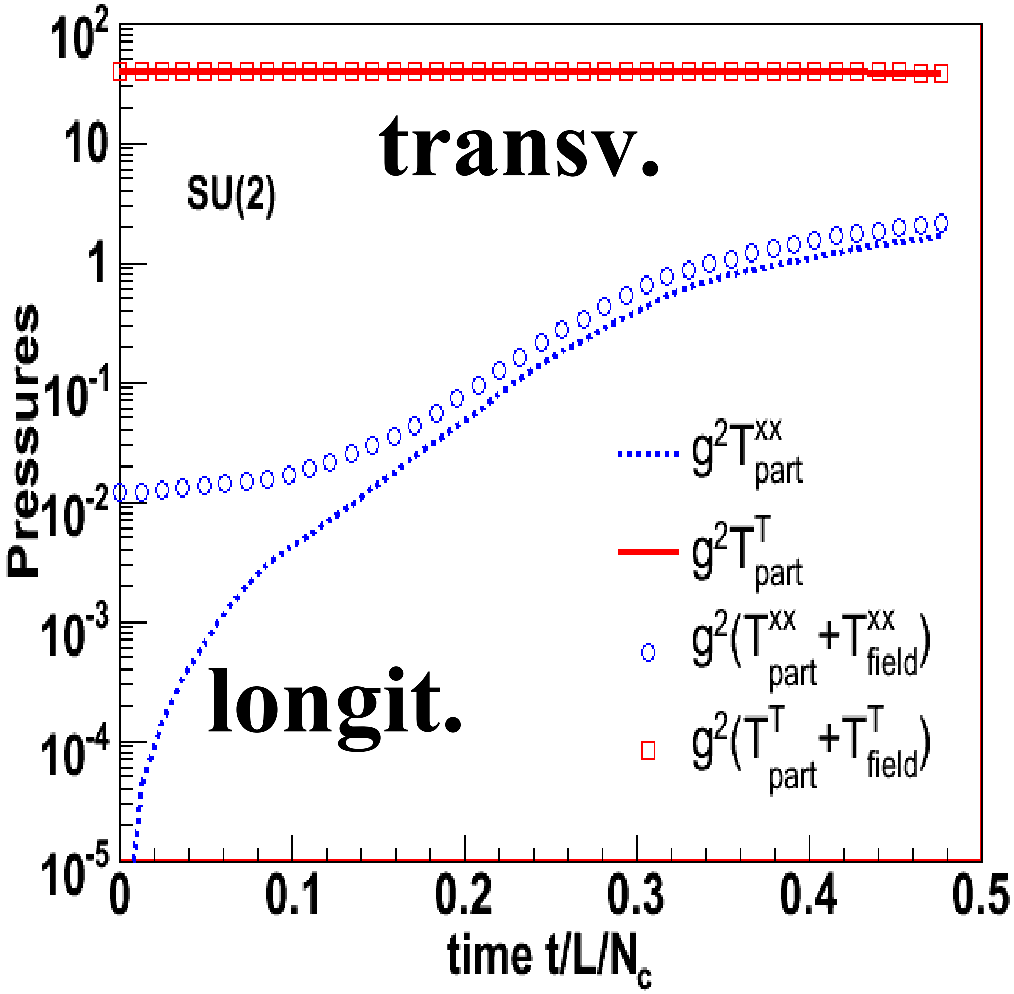
$$f(\boldsymbol{p}) \sim \delta(p_x)\, e^{-\sqrt{p_x^2 + p_y^2}/p_{hard}}$$

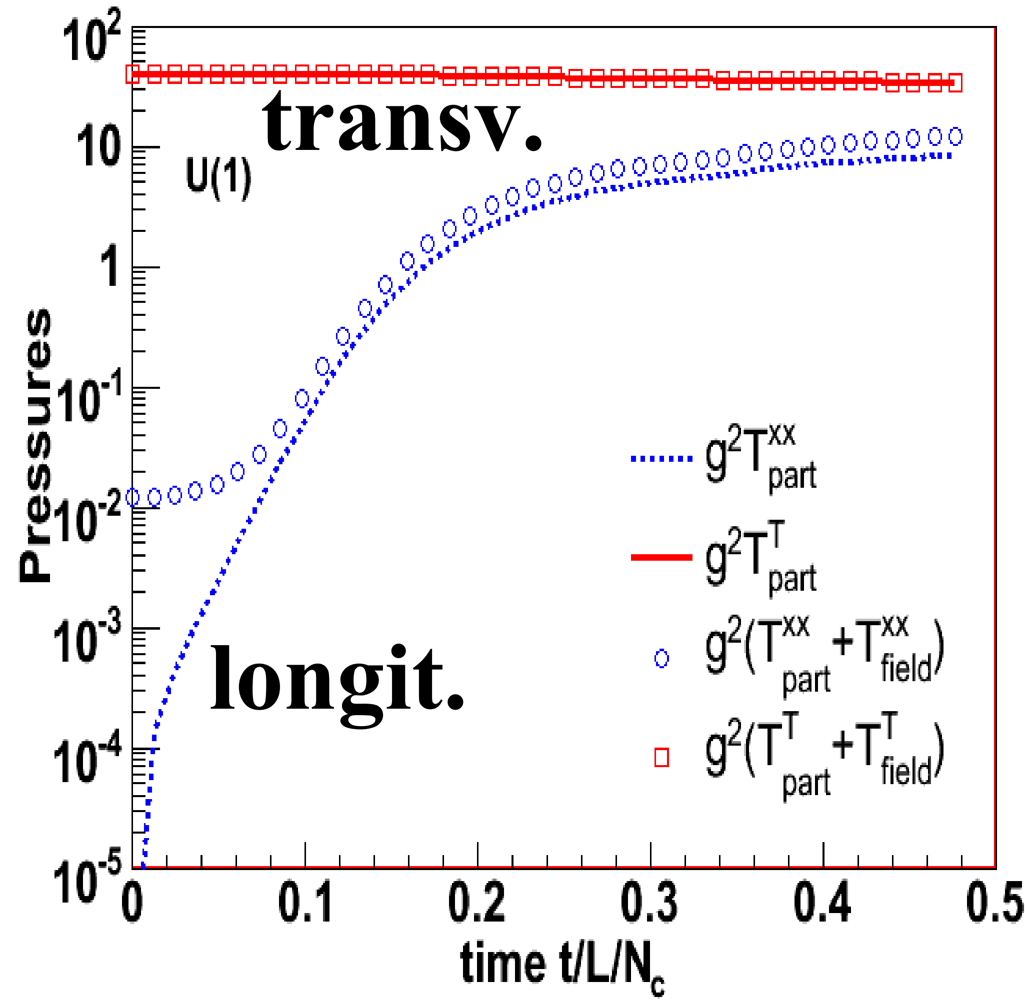
Weak initial fields

- Confirms existence of instabilities.
- Particles ARE affected
- Saturation well before fields reach particle scale.



(Partial) Isotropization



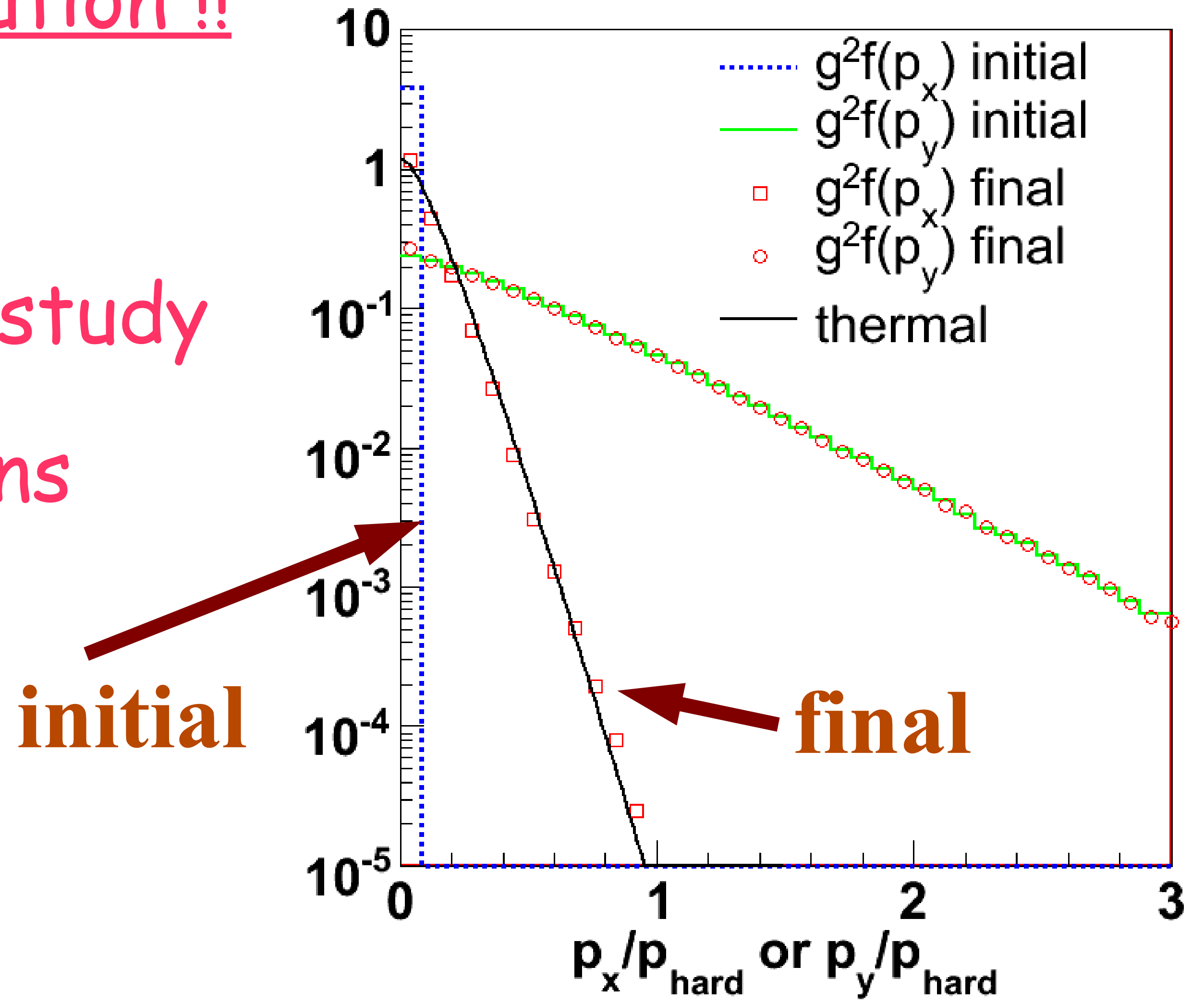


exponential growth of long. pressure

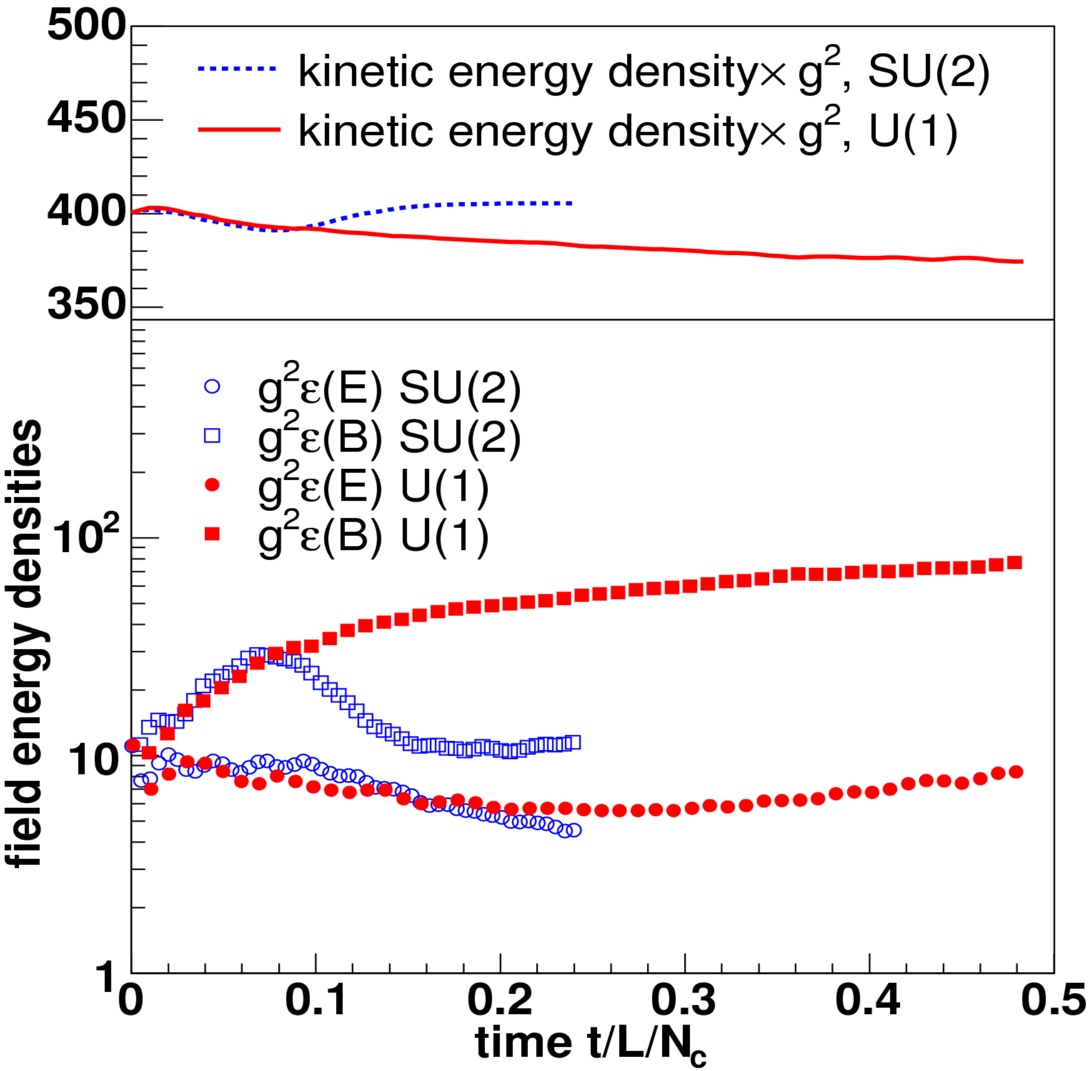
$$\textit{field part}: T_{0x} = T_{transverse} = 0, \quad T_{xx} = T_{00}$$

Momentum distributions of the hard particles

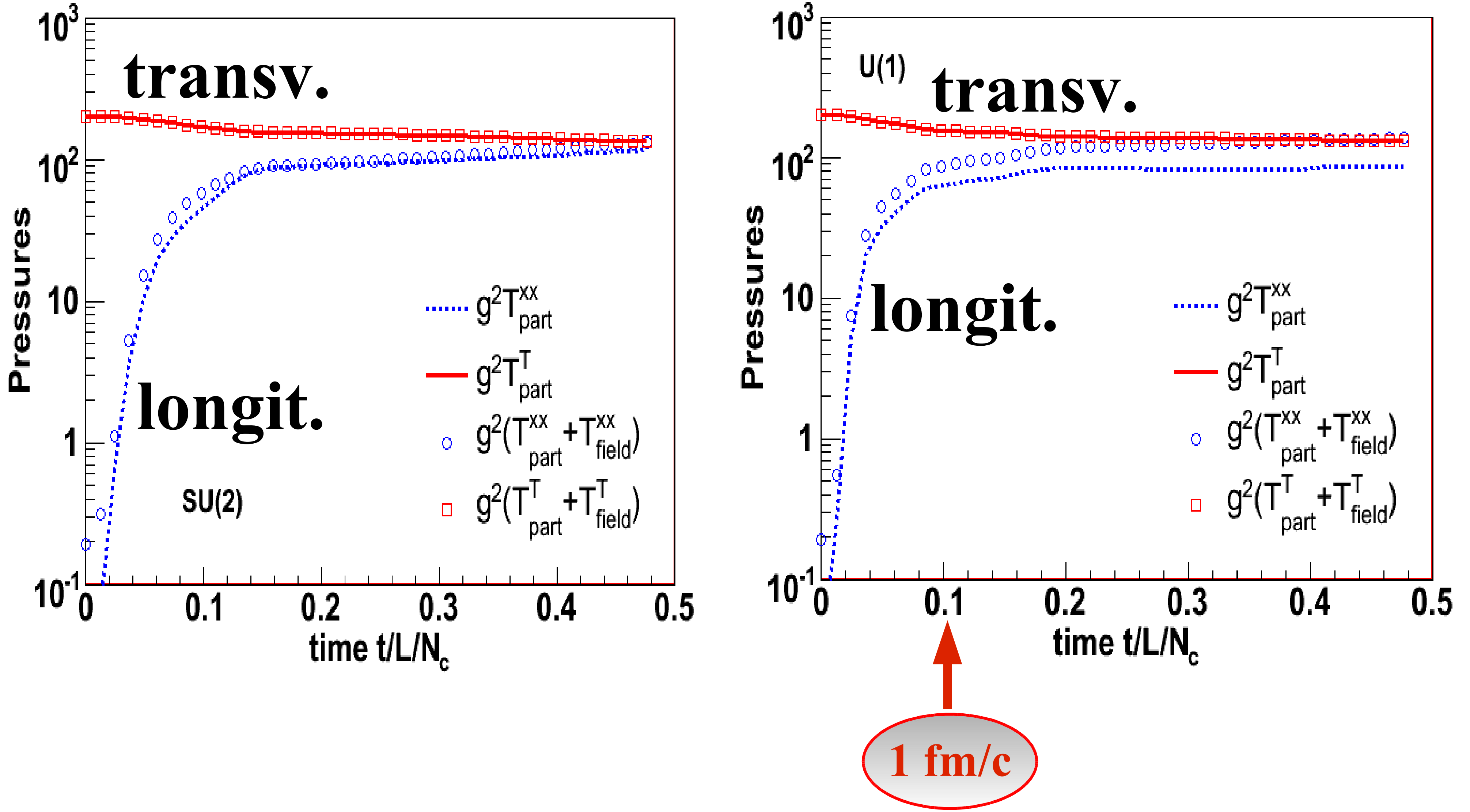
- Exponential distribution !!
- Longitudinal Cooler
- Full 3d required to study transverse directions



2) Strong initial fields



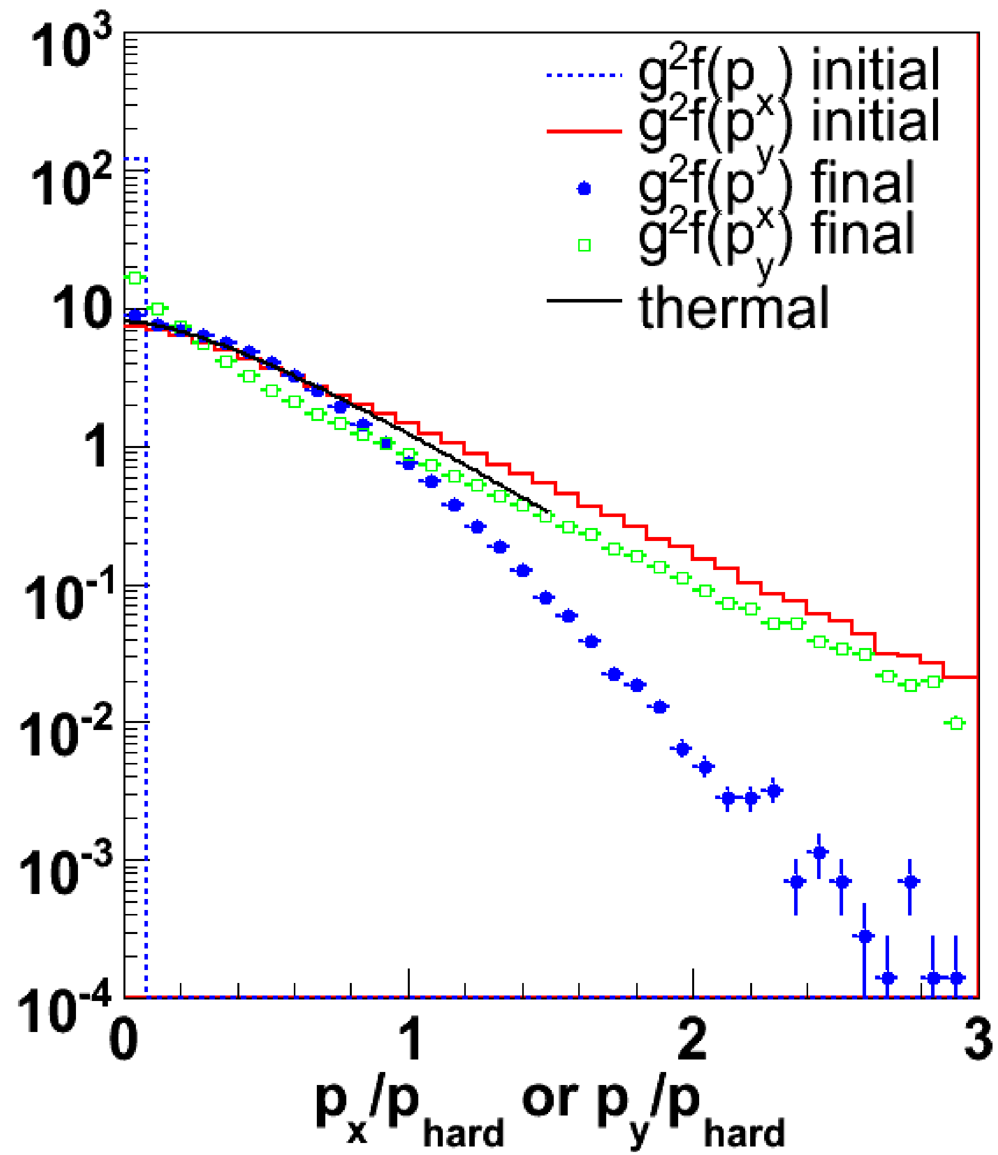
- No real instabilities seen...
- Soft fields irrelevant?



- Rapid (and complete!) isotropization by deflection in strong field.
- Shuts off instability.
- Even so, time scale at least as small as scattering.
- Soft fields need to be taken into account !

Momentum distributions of the hard particles

- Nearly thermal up to ~ p_{hard}
- ONE Temp !
- Stay tuned for long. Bj expansion, 3d, collisions, ...



Isotropization:
small-angle scattering vs deflection by field

- Boltzmann eq: $p_\mu \partial^\mu f \ = \ C[f]$

  Particle entropy $s = \int_p \{(1+f)\log(1+f) - f\log f\}$

  increases

- Collisionless Vlasov: entropy conserved (Liouville theorem)
  - moderate logarithmic amount of entropy can be produced by averaging over random initial field configurations

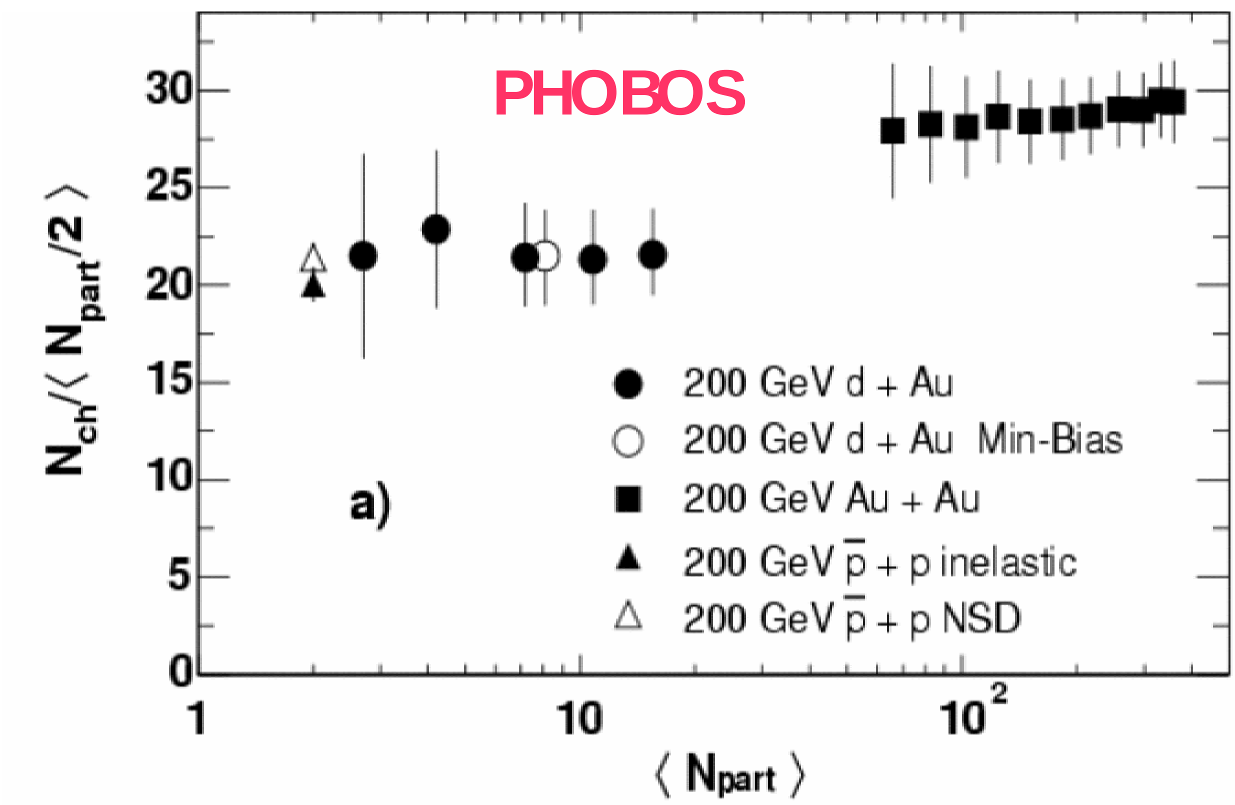
$$(1+f)\log(1+f) - f\log f \ \sim \ \log f$$

cf. discussion by Mueller & Son, PLB 582 ('04)

- Compare entropy with and w/o equilibration:

  central vs peripheral AA;   pA vs AA

- 0^{th} order test: multiplicity per participant



- How to measure entropy in HIC:

  e.g. Müller & Rajagopal '05,   Pal & Pratt PLB'04

Isotropic $T^{\mu\nu} \Rightarrow$ Ideal Flow ?

$$u_\mu \partial_\nu T^{\mu\nu} = 0 \quad , \quad T^{\mu\nu} = (e+p) u^\mu u^\nu - p g^{\mu\nu}$$

$$\Rightarrow \; u\cdot\partial e + (e+p)\,\partial\cdot u = 0$$

- Now use 1st law: $\quad de = T\,ds + \mu\,dn$
  and *Gibbs-Duhem* $\Rightarrow$ $\quad e + p = T\,s + \mu\,n$

$$\Rightarrow \; T\; u\cdot\partial s + T\; s\;\partial\cdot u \; + \underbrace{\mu\; u\cdot\partial n + \mu\; n\;\partial\cdot u}_{\mu\,\partial_\nu (n u^\nu) = 0,\ \text{current conserv.}} = 0$$

$$\Rightarrow \; \partial_\nu (s\, u^\nu) = 0, \quad \textbf{\textit{entropy conservation}}$$

BUT: Gibbs-Duhem does not apply for non-extensive statistics (Tsallis) **!**

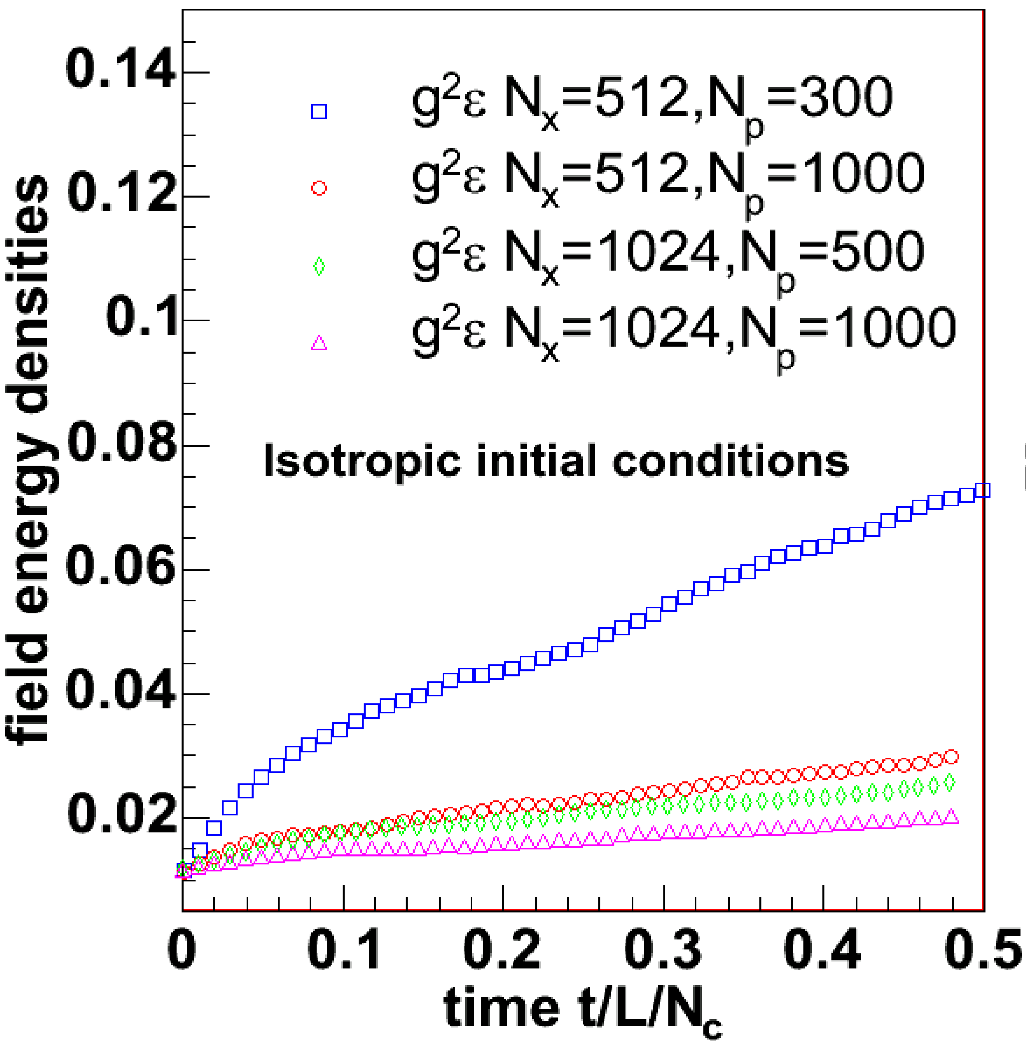
Pointed out to me by Takeshi Kodama

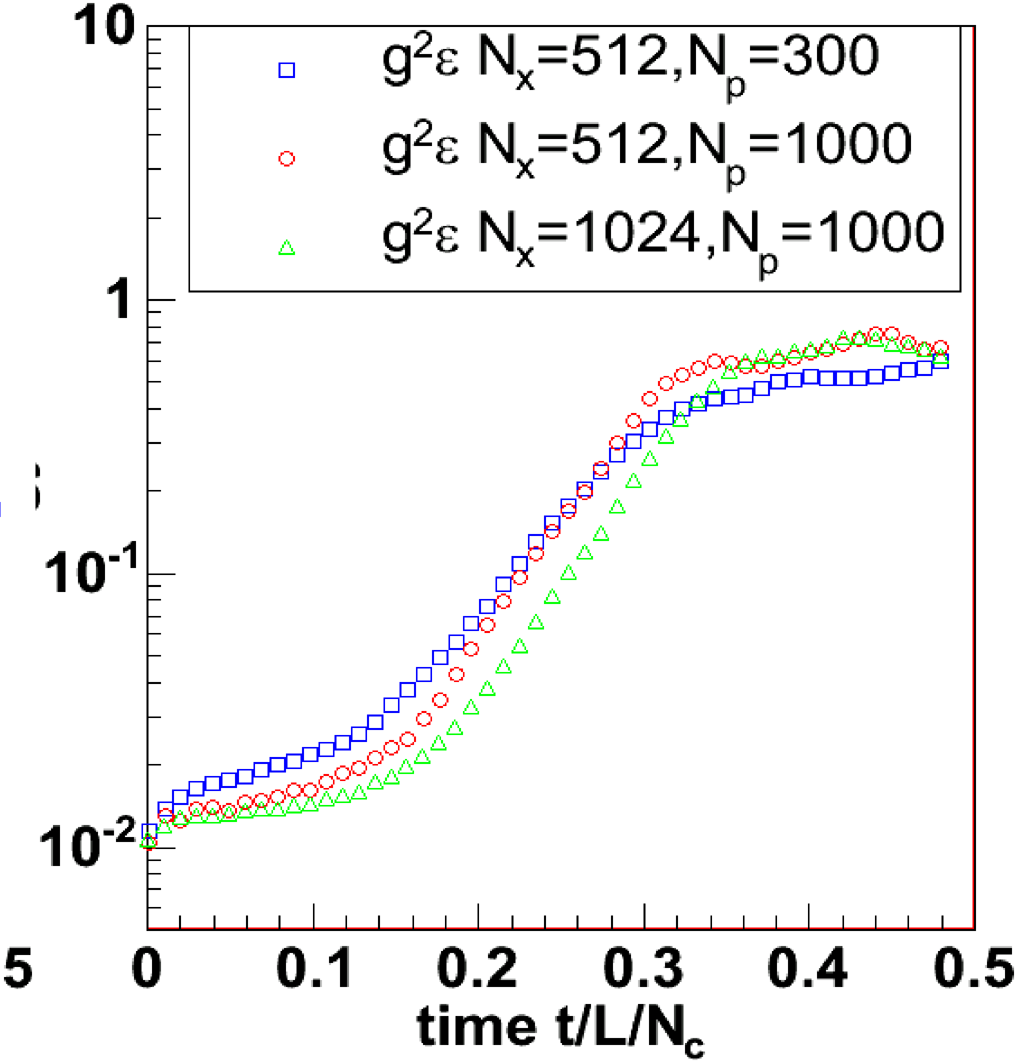
Summary and Outlook

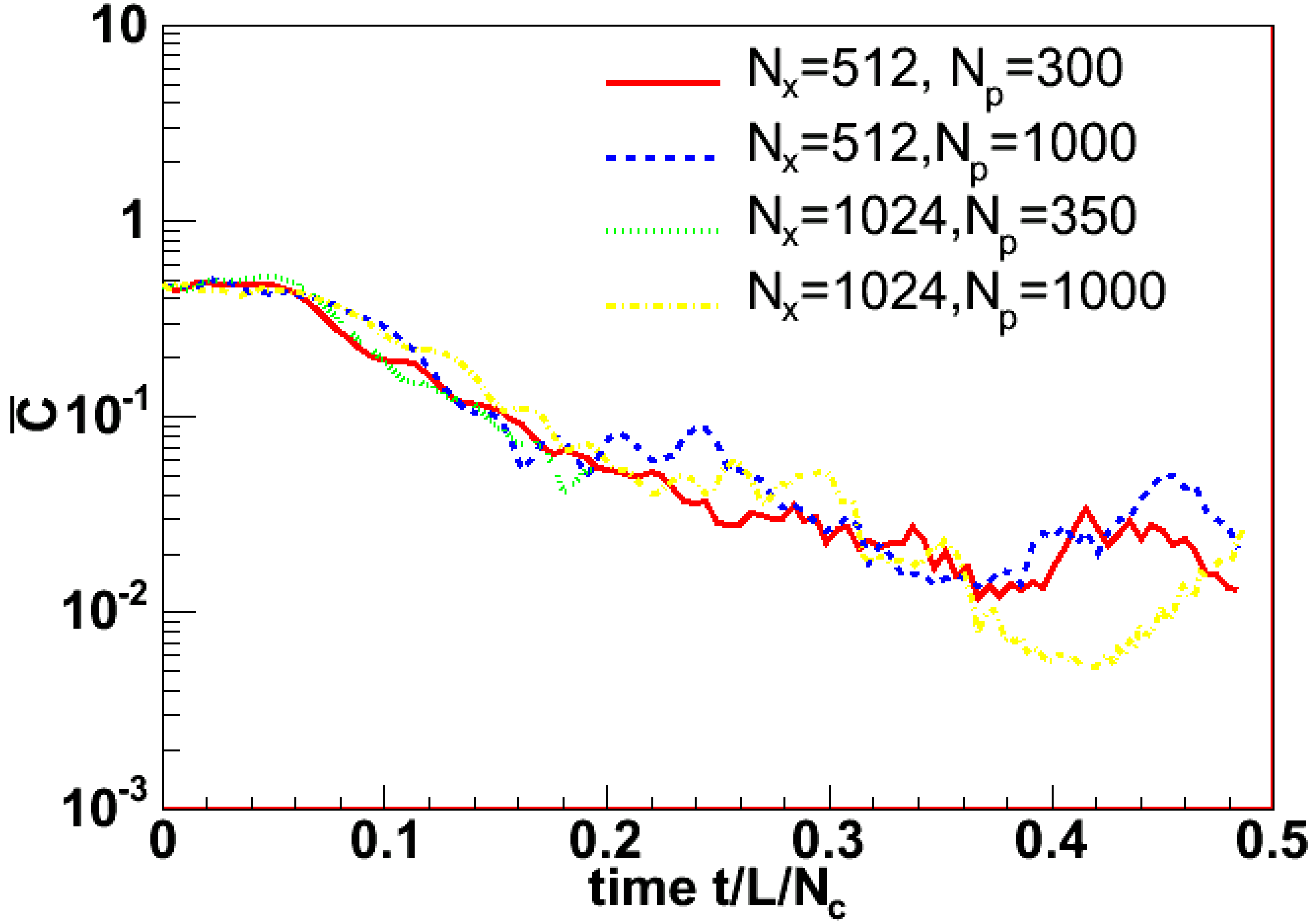
- Particle-field (Vlasov-YM) simulations in QCD
- So far only 1+1 D
- Collective effects due to color field play important role for the pre-equilibrium dynamics

  - Include longitudinal Bj expansion
  - Full 3D simulation
  - Collision term

lattice size and test particle dependence







10
1
10^-1
10^-2
10^-3
C
0
0.1
0.2
0.3
0.4
0.5
time t/L/Nc
Nx=512, Np=300
Nx=512,Np=1000
Nx=1024,Np=350
Nx=1024,Np=1000