Numerical studies of QGP instabilities and implications

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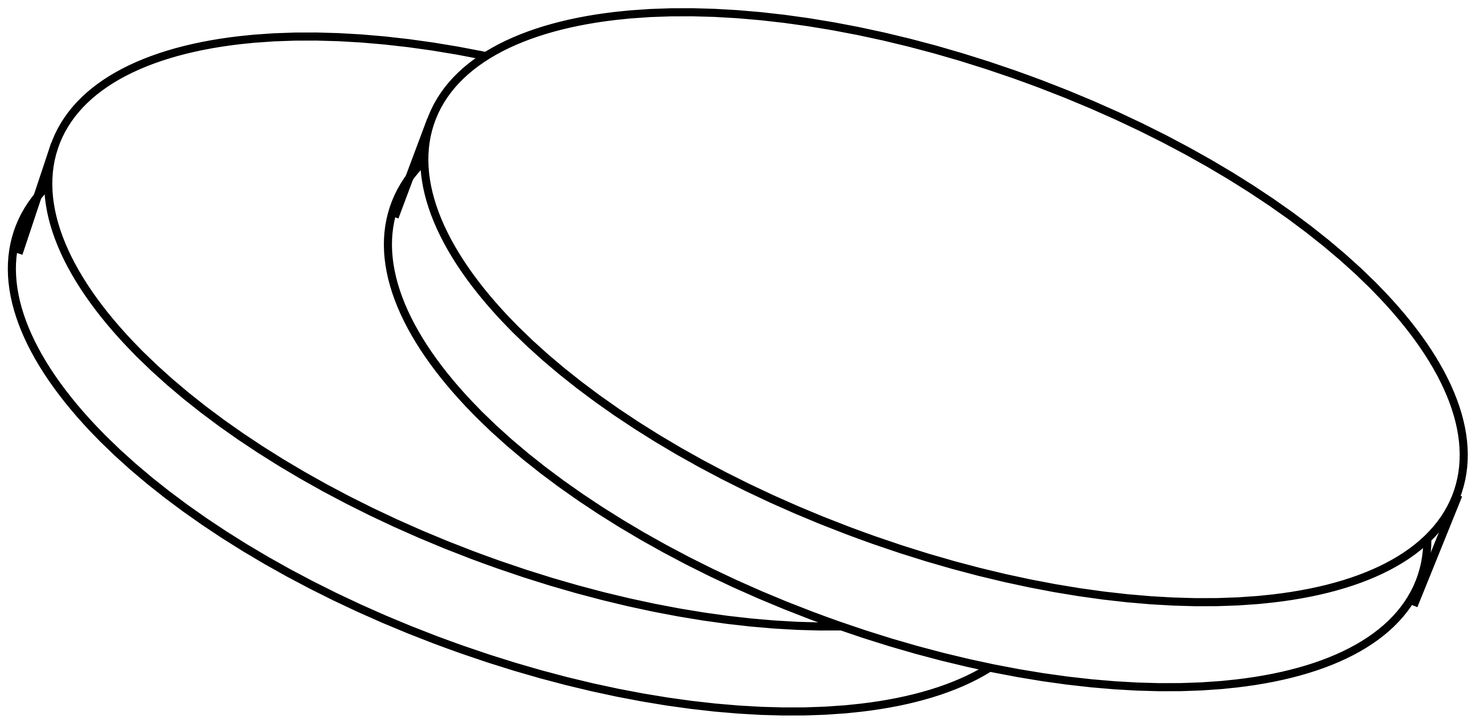
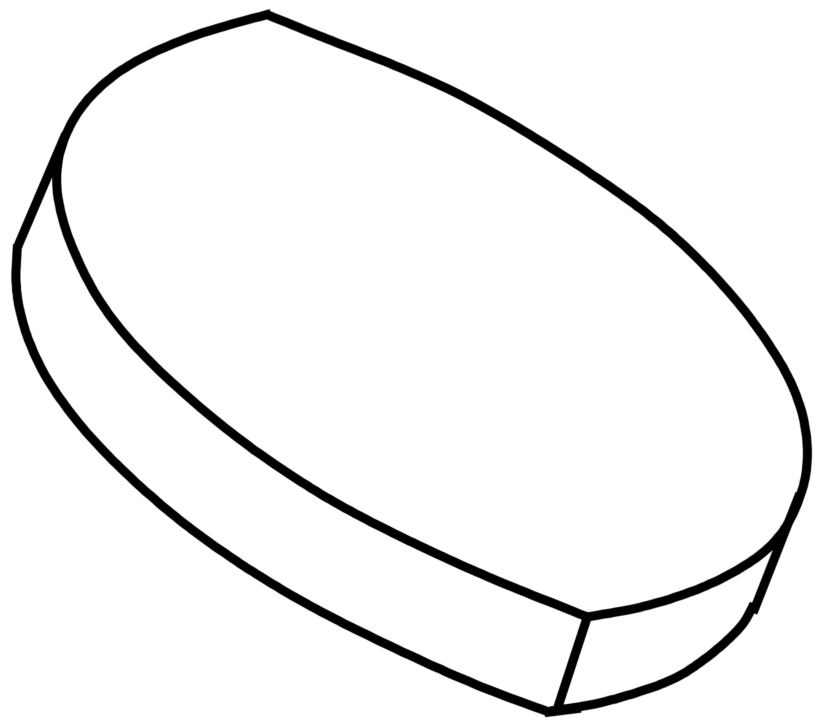
- The Setting
- Is Weak Coupling Ruled Out?
- Weibel Instability
- Nonabelian lattice study

Just before collision



Lorentz contracted nuclei, striking with nonzero impact parameter

Region where Collision Occurs:



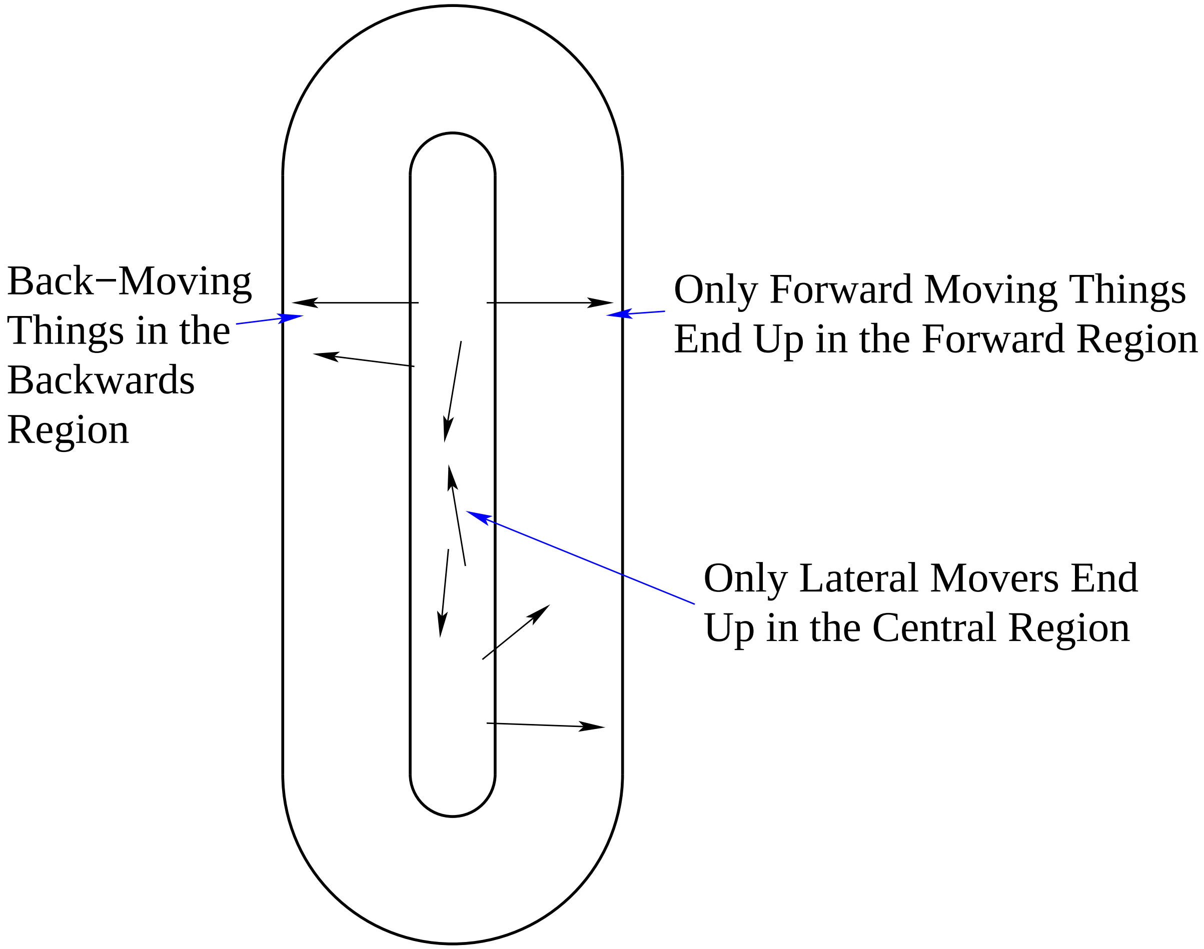
Irregular "flat almond" shaped overlap region:

Longer than it is wide, Wider than it is thick

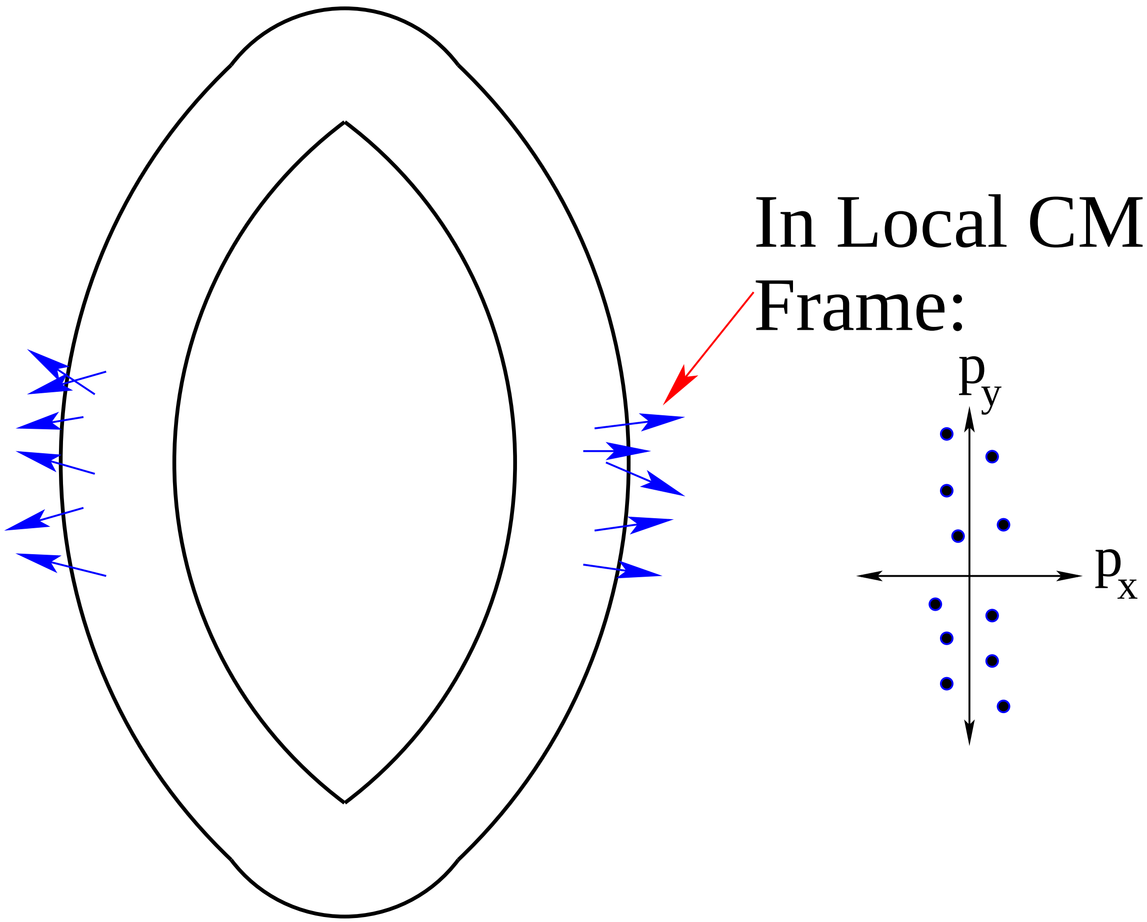
~ 2000 partons, p distribution azimuthally isotropic

Momentum Selection

Side-on view of the flat almond as it expands



Same is true in transverse plane



Free expansion: squeezed momentum distribution

If particles re-scatter: CM p distribution becomes more isotropic.

Regions on sides: increases p_x^2 at expense of p_y^2.

Geometry: more "sides" than "top and bottom."

Net increase in p_x^2 at expense of p_y^2: eliptic flow.

(Enhanced $p_\|^2$ at expense of $p_\perp^2$ should be even larger.)

Data: effect nearly maximal (always near locally isotropic distribution)

Is This in Conflict with a Weakly-Coupled Description?

Before we give up on weak coupling hypothesis, we should understand what its predictions actually are.

Assume $\alpha_{\rm s} \ll 1$ expansion has some validity

Longitudinal expansion: $f(p \geq Q_s) < 1/g^2$ from quite early times.

Conditions for kinetic or Boltzmann-Vlasov description:

$p \geq Q_s$: Classical Particles

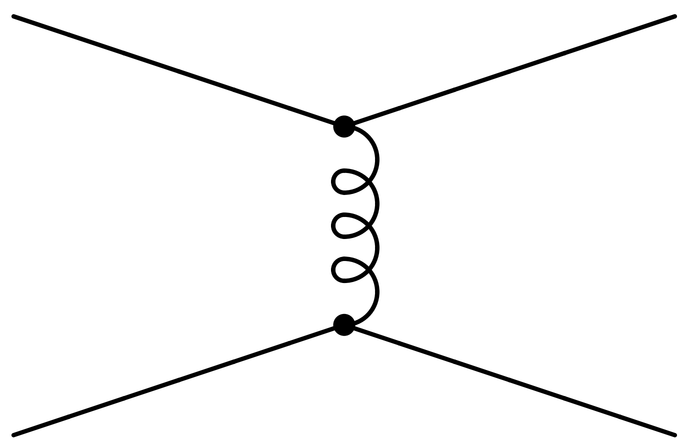
$p \ll Q_s$: Classical Fields

How does equilibration occur?

Old picture: scatterings

most particles have "typical" momenta $\sim p_{\rm typical}$ (think T)

Dominant interaction: $2 \leftrightarrow 2$ scattering



Equilibration rate: $\Gamma \sim g^4 n/p^2$ (think $g^4 T$)

Log divergent, cut off by plasma screening

Plasma screening

Gauge fields of wavenumber k bend particles, $F = gv \times B$

$$\delta p \sim g\frac{B}{k} \sim gA$$

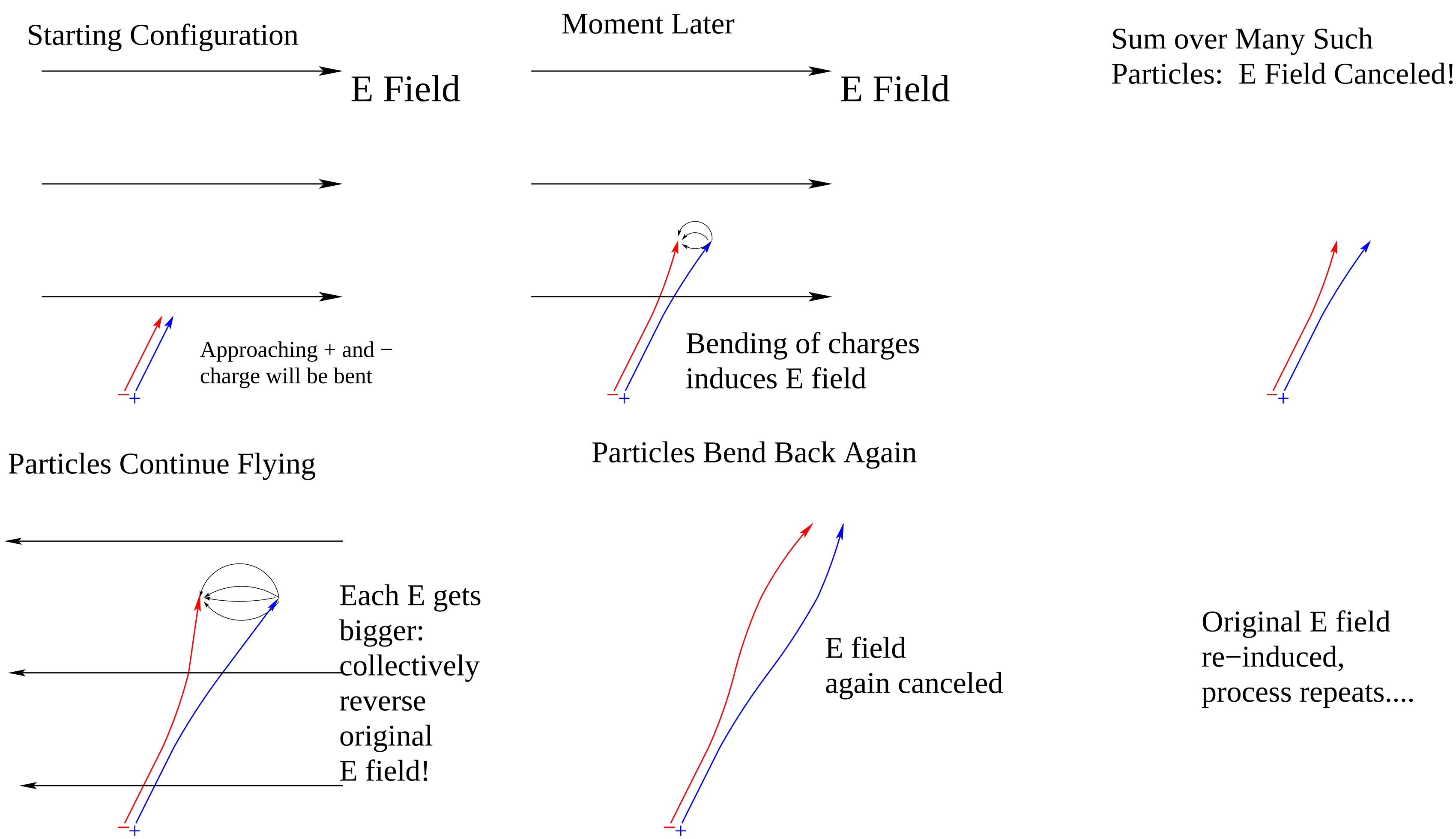
inducing a current $j \sim n\delta p/p$ which is important if

$$\nabla \times B \sim gj \qquad \text{or} \qquad k^2 A \sim gj \sim \frac{g^2 n}{p} A$$

These effects are therefore important at and below

$$k \sim g\sqrt{\frac{n}{p}} \qquad \text{(think } gT\text{)}$$

Example: plasma oscillations



More on Plasma Screening

Collective: small disturbance to many particles.

evolve on quite short time scale!

Equilibrium: All modes **Stable**.

Anisotropic: **Always** unstable wave vectors!

Exponential growth in some IR gauge fields.

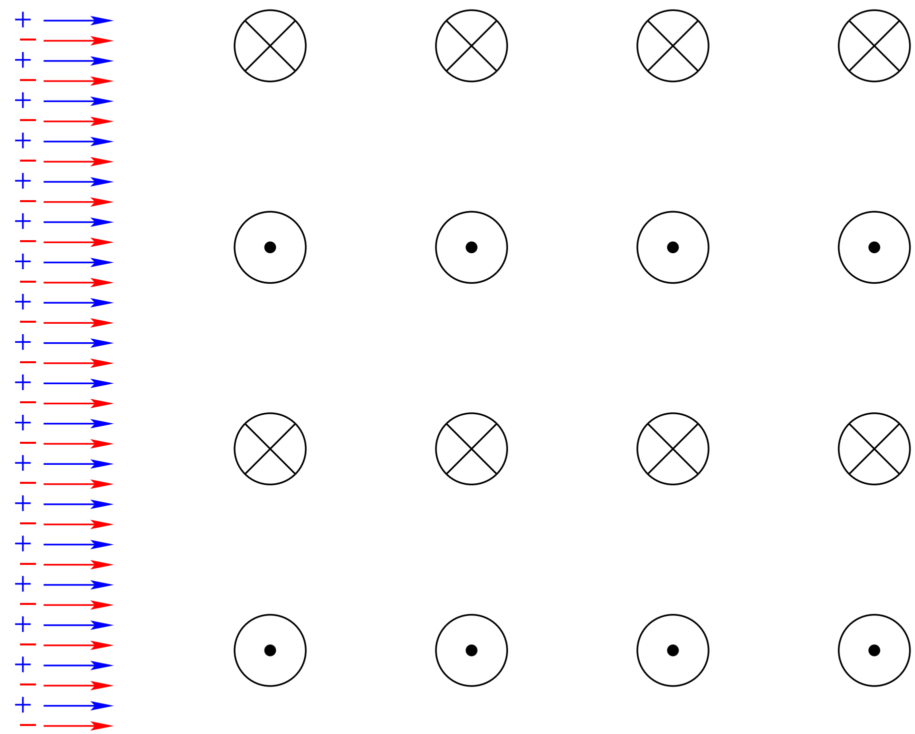
Physical origin of instability

Suppose charges, $p \parallel \hat{x}$

B field, $\hat{k} = \hat{z}$

A in x direction

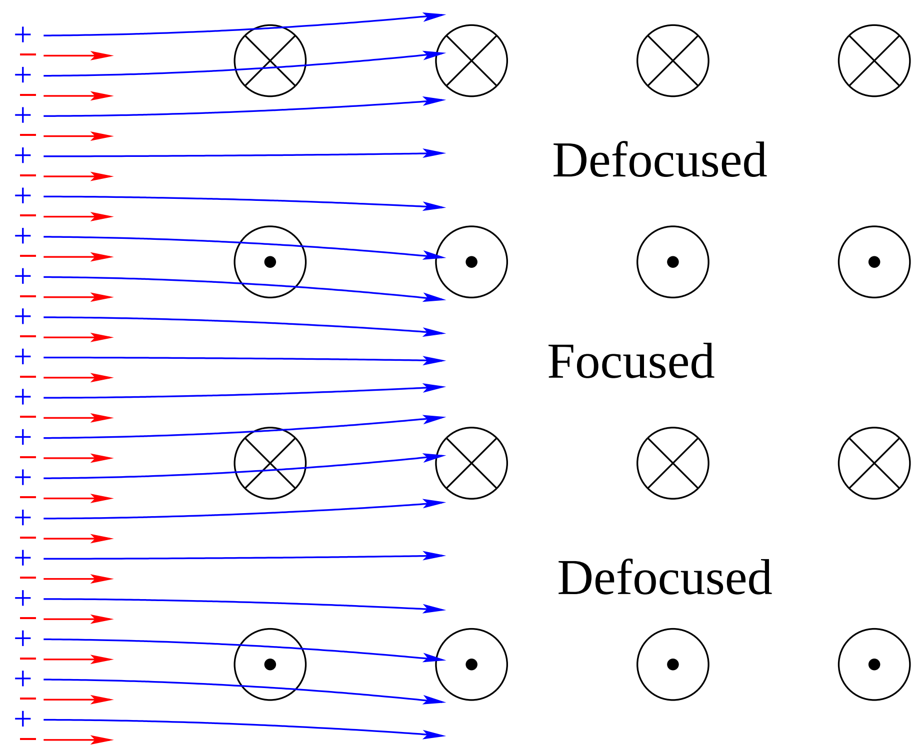
How will p be bent?



$F = gv \times B$. Consider $+$ charges first:

Physical origin of instability

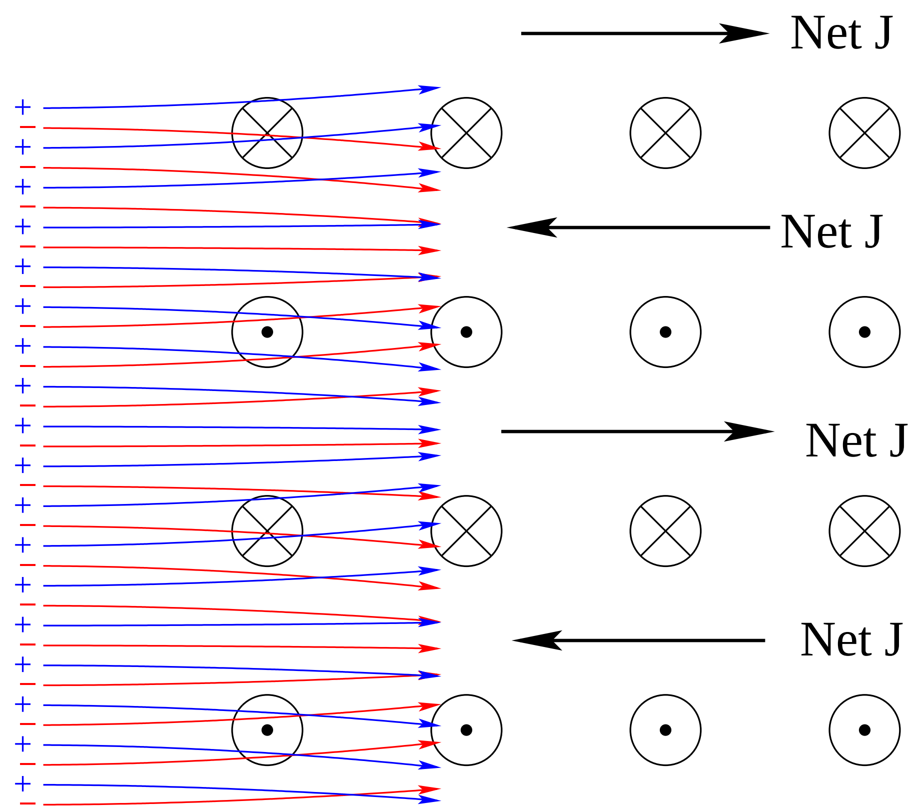
Positive charges deflected as shown: Alternately focused and de-focused



Charges coming from opposite side (not shown) are focused where these are defocused.

Physical origin of instability

− charges defocused where + are focused → net J is induced, grows with time



J **Strengthens** B! Weibel instability.

Studies which include only scattering (parton cascade. . .):
equilibrium probably not reached. Seem to conflict with data.

But instability is fastest process.

Longitudinal expansion–quickly becomes highly anisotropic

Growth rate of instabilities always faster than expansion rate

Must re-consider equilibration and isotropization including plasma instabilities.

Goal (Someday)

Understand full implications for thermalization in heavy ion collisions. Do these instabilities isotropize the system?

Goal (So far)

Understand how instability works when unstable modes' mutual nonabelian interactions become important. Work in weak coupling *limit*.

Information we will need to achieve the big goal.

Approach: Kinetic Theory

Variables:

- High p particle excitations

  Dominate energy density. Anisotropic. Well defined p, x. Each colored, but collection nearly color neutral.

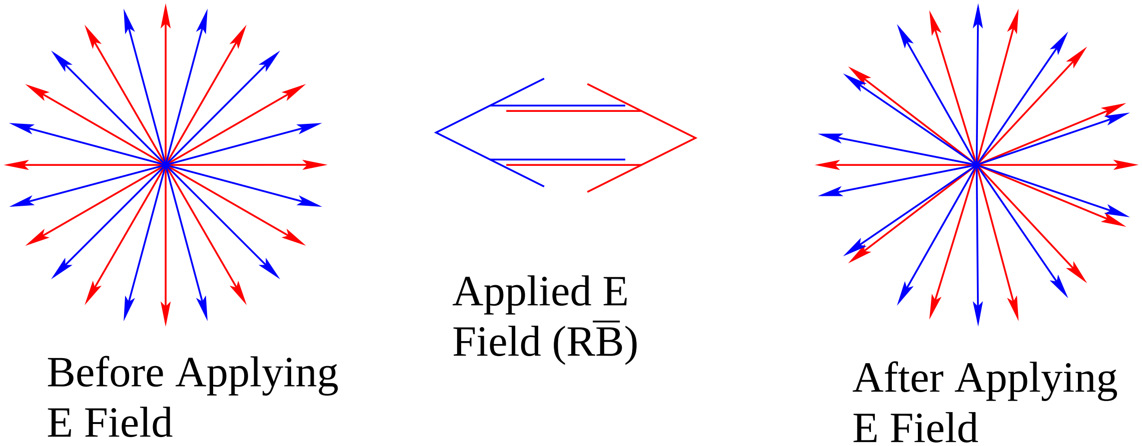
  Describe in terms of $f(x, p)$ a (color matrix) statistical function.

- Low p classical field excitations

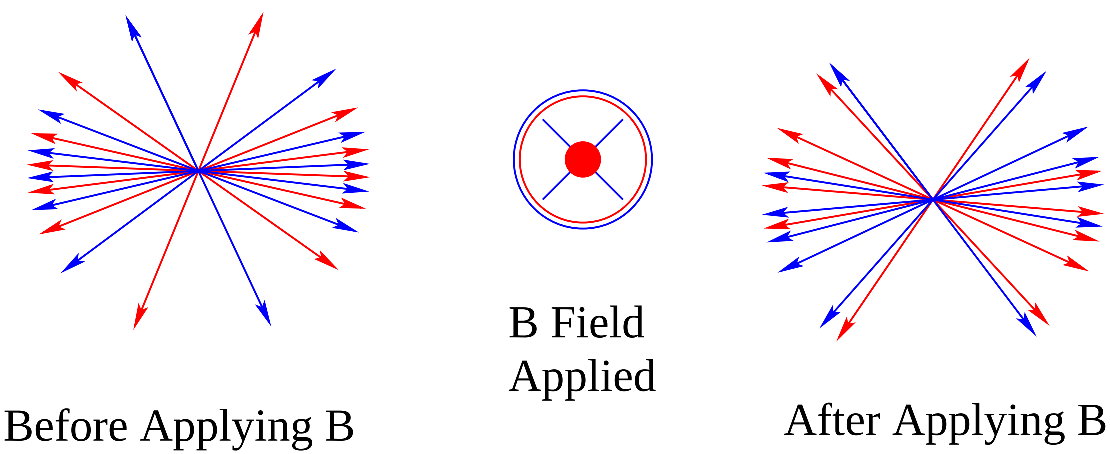
  Obey classical Maxwell/Yang-Mills equations. Particles act as color charges for them.

  Describe in terms of $A^a_\mu(x)$ a classical color gauge field.

Equilibrium: E fields induce color currents.



Anisotropic: B fields also induce color currents.



Always color octet. Y_{lm}: $l = 1$ (isotropic), $l > 1$ (aniso).

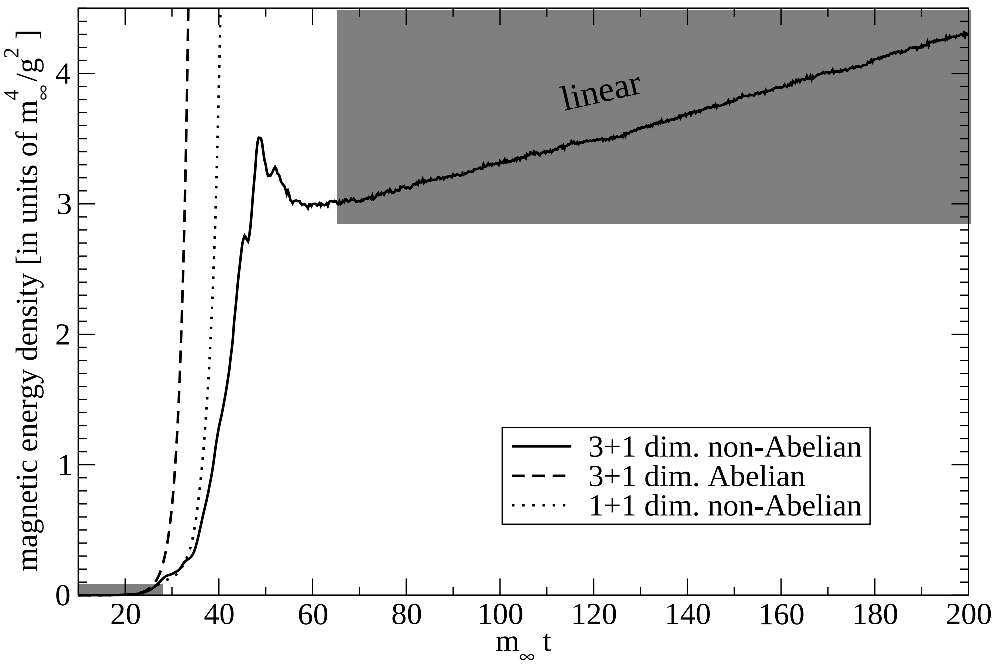
Define $W^a(x, \vec{v})$: mean color of particles going in $\vec{v}$ direction at point x.

$$\begin{aligned} D_t W^a(x,\vec{v}) &= -\vec{v}\cdot\vec{D}W^a(x,\vec{v}) + m_\infty^2 \text{ Source} \\ \text{Source} &= 2\Omega(\mathbf{v})\vec{v}\cdot\vec{E} - \vec{E}\cdot\frac{\partial}{\partial\vec{v}}\Omega(\mathbf{v}) - F_{ij}v_i\frac{\partial\Omega(\mathbf{v})}{\partial v_j} \\ D_\mu F^{\nu\mu} &= J^\nu = \int_{\mathbf{v}} v^\nu \, W(\mathbf{v}) \end{aligned}$$

$\Omega(\mathbf{v}) =$ angle dependence of anisotropic p distribution of (color neutral) bulk of particles.

Put on lattice: must also discretize $\vec{v}$ space. Spherical harmonics, with $l_{\max}$ cutoff. SU(2) for simplicity.

If gauge fields start with small fluctuations:



first: exponential growth. Fields get nonperturbatively large. Switches to linear growth in energy.

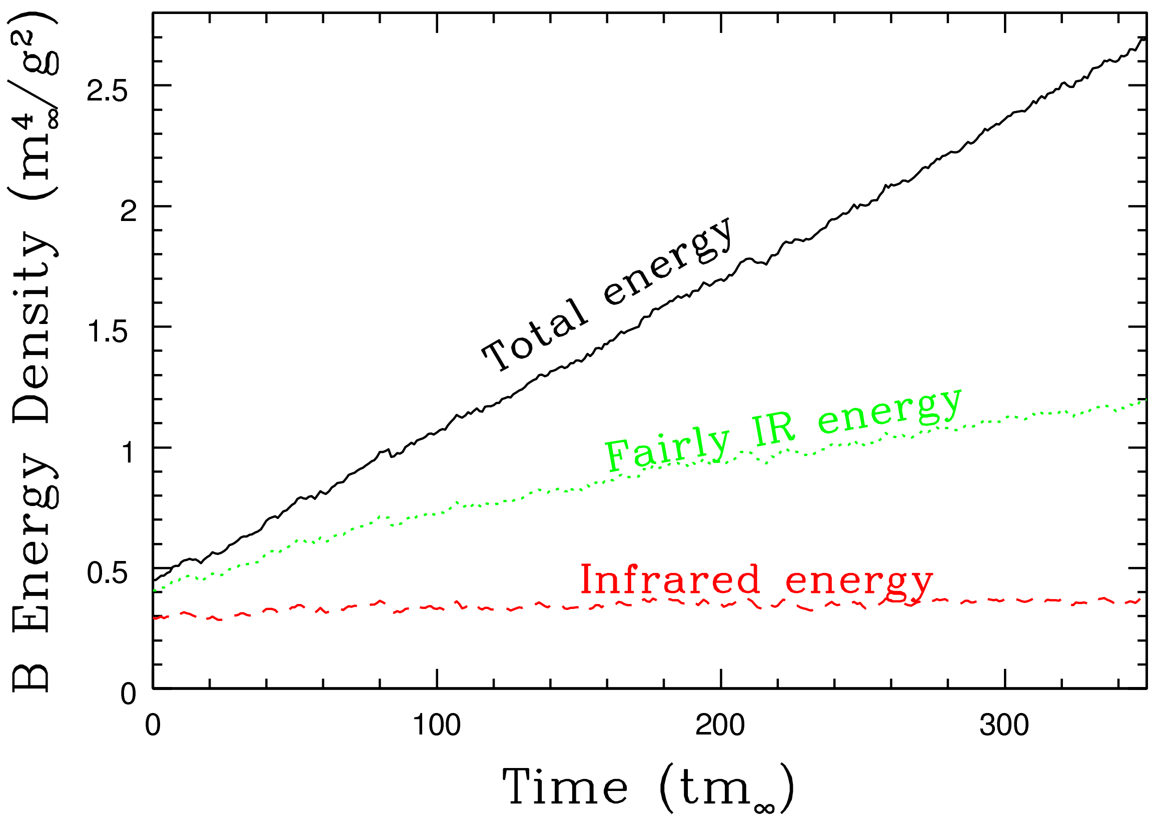
Time scales: exp. time shorter than system age.

all the time spent in the linear growth part.

Concentrate on nonperturbative linear part

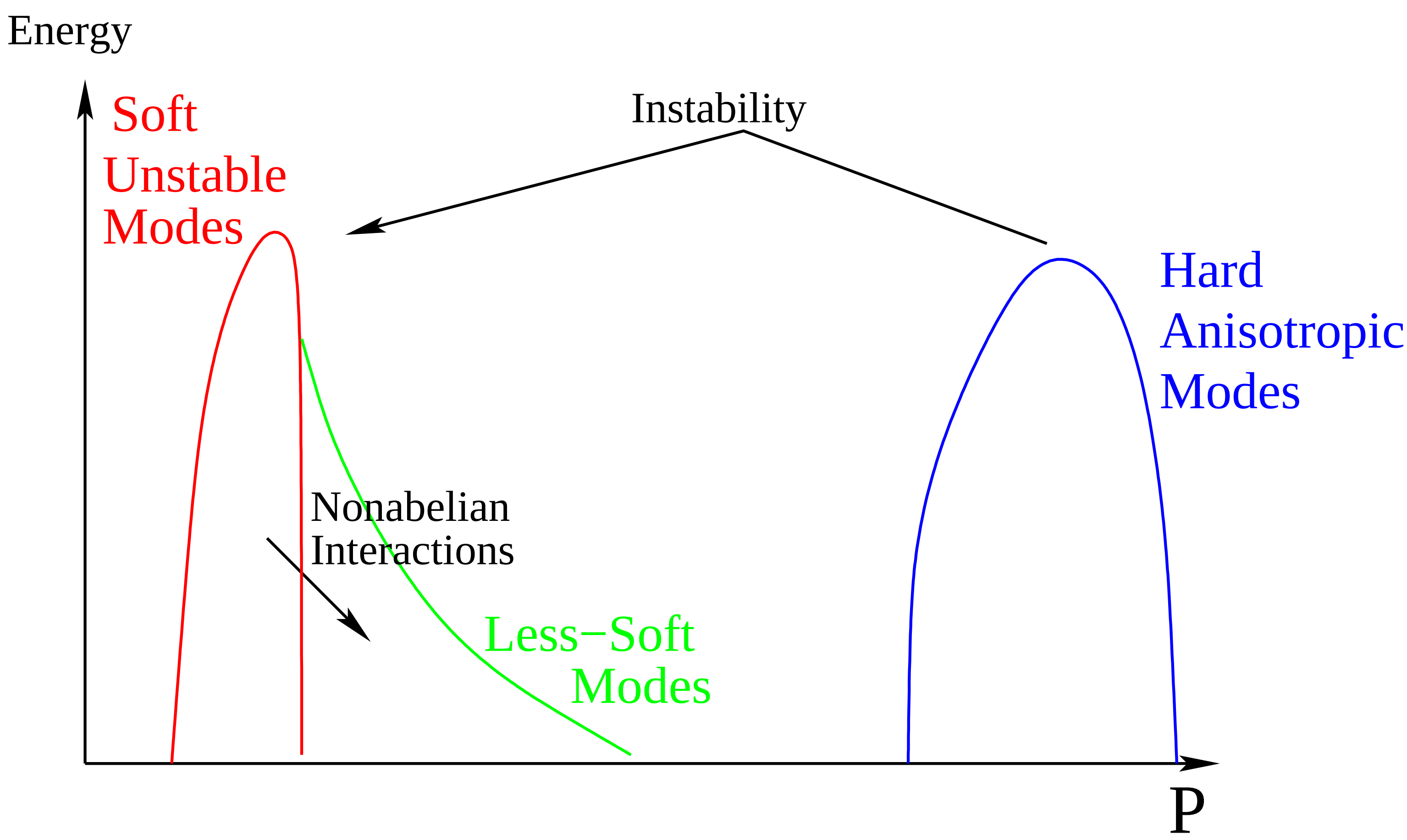
Energy in gauge fields grows linearly with time

Field smearing lets us see how much is very IR energy.

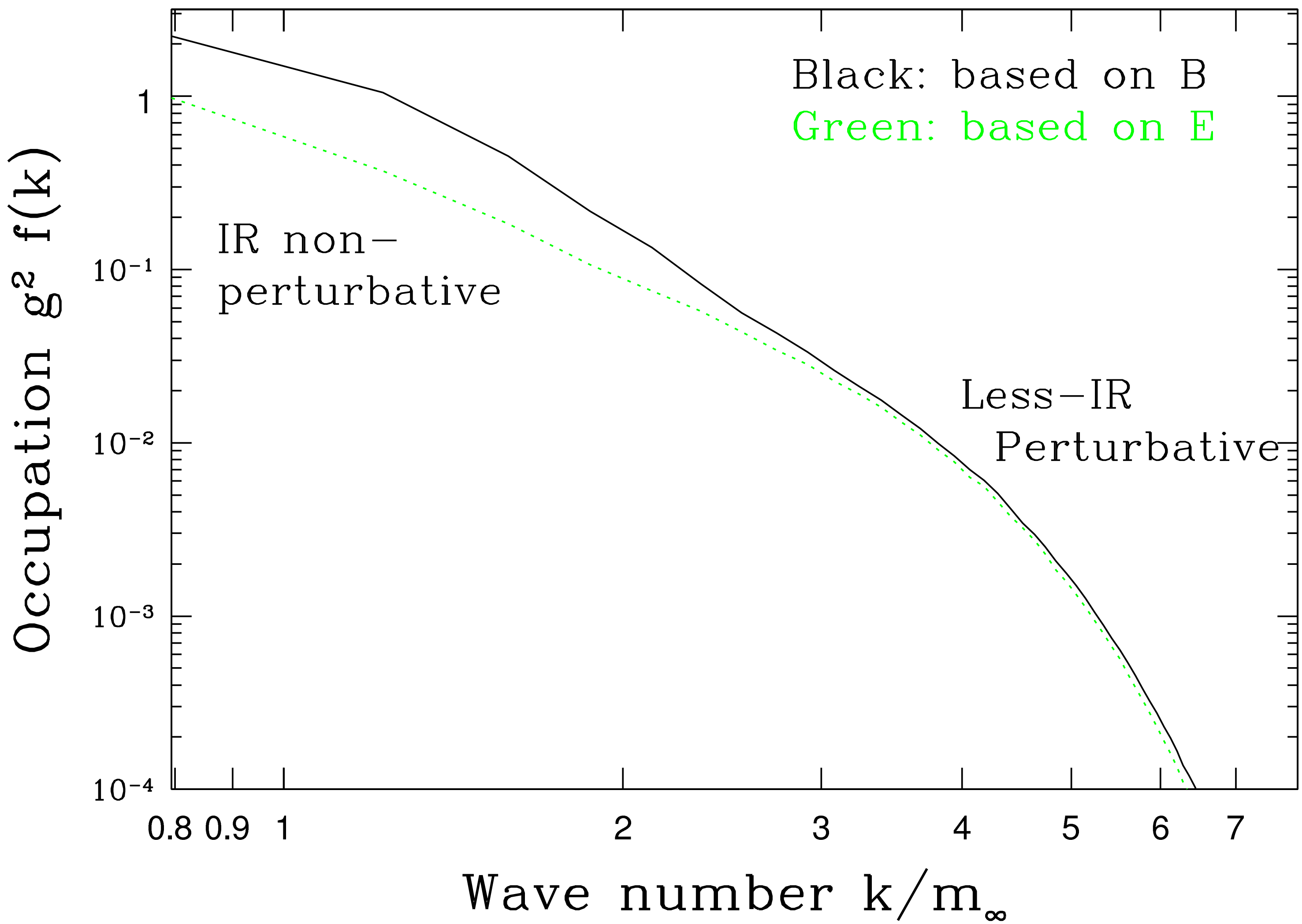


Very soft fields constant. Medium-soft grow slower.

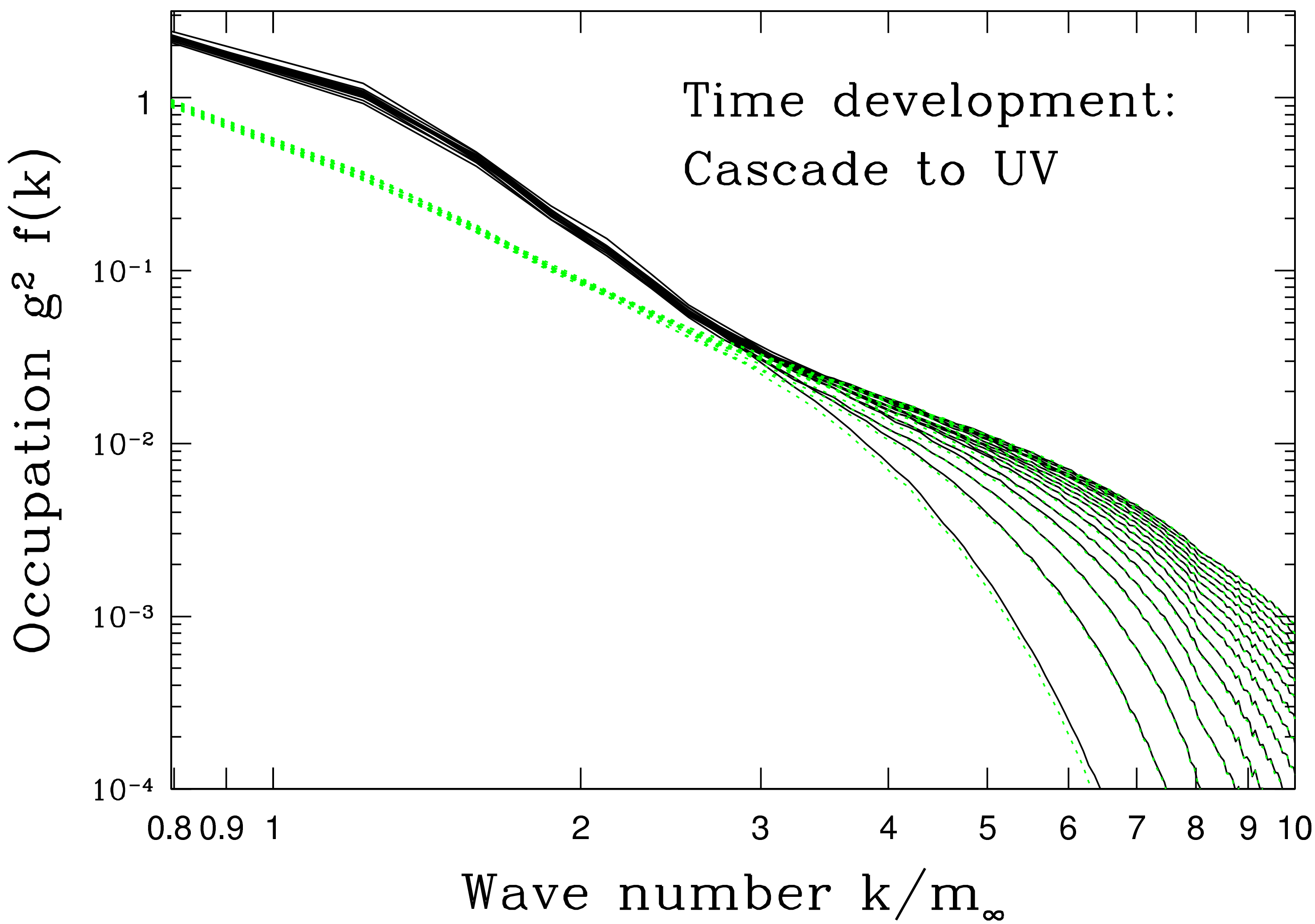
Instability pumps soft modes. Nonabelian interaction cascades energy into less-soft modes.



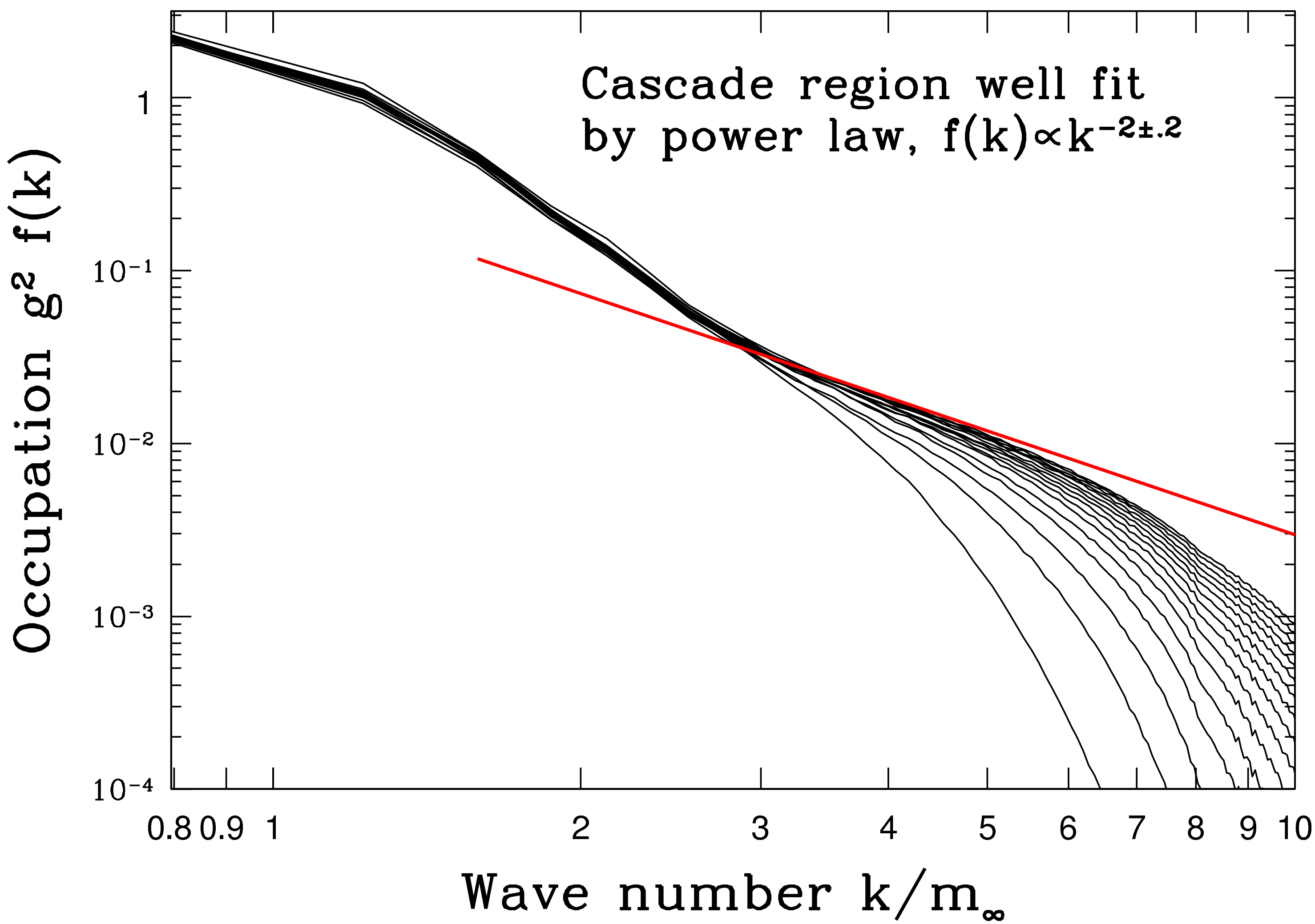
Coulomb gauge power spectrum: Initially



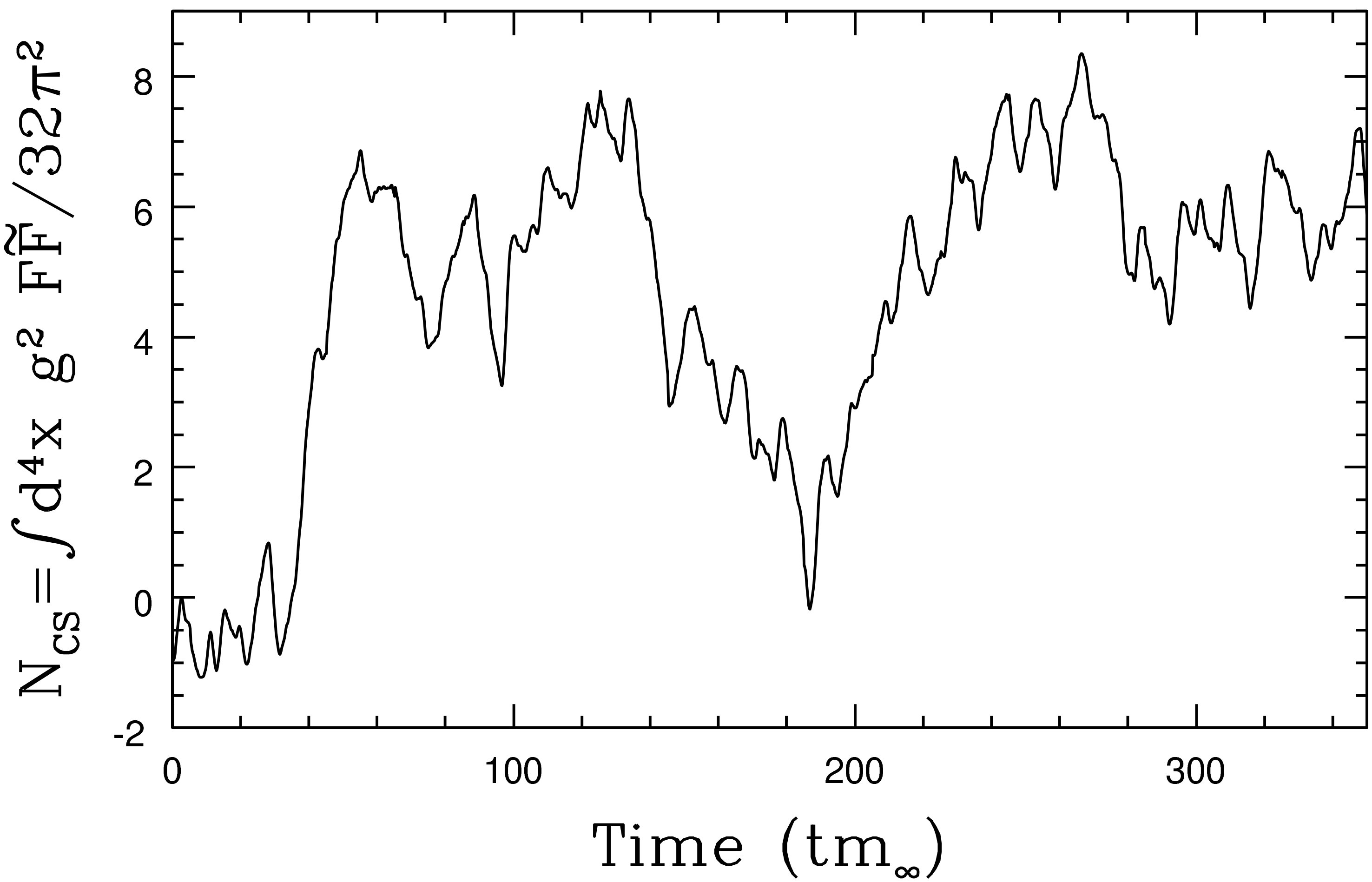
Time development of Coulomb gauge spectrum



Power-law behavior with moving cutoff



Soft nonabelian fields are large and randomly fluctuating, as seen in chaotic evolution of Chern-Simons number:



What We Have Learned about Instability

- Anisotropic momentum distribution pumps energy into IR color fields
- IR color fields become nonperturbatively strong
- Nonabelian interactions cause cascade of energy into less-IR color fields
- Spectral index of cascade is $f(k) \propto k^{-2}$

Implications

Spectral index means screening and scattering dominated by IR nonabelian fields.

$$\text{Screening} \propto \int \frac{d^3k}{k} f(k) \sim \int \frac{d^3k}{k^3}$$

$$\text{Scattering} \propto \int d^3k f(1+f) \sim \int \frac{d^3k}{k^4}$$

Parametrically more scattering than without instability.

Systems with anisotropic flow should show parametrically more scattering than spherically expanding systems.

Conclusions

IF you believe in a weak coupling treatment:

MUST include plasma instabilities

Relative to naive weak coupling treatment,

- Scattering rate is much higher
- $\Rightarrow$ Faster isotropization, eliptic flow
- $\Rightarrow$ More collinear radiation
- $\Rightarrow$ More jet energy loss

How big are these effects? Not yet known.