Quark-antiquark production from classical fields and chemical equilibration

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August 2005

Abstract

We compute by numerical integration of the Dirac equation the number of quark-antiquark pairs produced in the classical color fields of colliding ultrarelativistic nuclei. The backreaction of the created pairs on the color fields is not taken into account. While the number of $q\bar{q}$ pairs is parametrically suppressed in the coupling constant, we find that in this classical field model it could even be compatible with the thermal ratio to the number of gluons. After isotropisation one could thus have quark-gluon plasma in chemical equilibrium.

Outline

- Background
- Results
- Conclusions

Motivation

- Heavy quark production: doable perturbatively, but do the strong color fields change the result?

- Chemical equilibration, light quark production? ▶ Essential for understanding how and if the CGC turns into (thermalized) QGP.

Related calculations

- Analytical calculation in MV model: lowest order and pA: Gelis, Venugopalan, Fujii[1,2]

- $\mathbf{k}_T$–factorized calculation with "CGC" distributions: Kharzeev, Tuchin[3,4]

- Corresponding calculation in QED can be done analytically to all orders: Baltz, Gelis, McLerran, Peshier[5,6]

- Analytical calculation in a more general setting by Dietrich[7]

[1] F. Gelis and R. Venugopalan, Phys. Rev. **D69**, 014019 (2004), [hep-ph/0310090].
[2] H. Fujii, F. Gelis and R. Venugopalan, hep-ph/0504047.
[3] D. Kharzeev and K. Tuchin, Nucl. Phys. **A735**, 248 (2004), [hep-ph/0310358].
[4] K. Tuchin, Phys. Lett. **B593**, 66 (2004), [hep-ph/0401022].
[5] A. J. Baltz and L. D. McLerran, Phys. Rev. **C58**, 1679 (1998), [nucl-th/9804042].
[6] A. J. Baltz, F. Gelis, L. D. McLerran and A. Peshier, Nucl. Phys. **A695**, 395 (2001), [nucl-th/0101024].
[7] D. D. Dietrich, Phys. Rev. **D70**, 105009 (2004), [hep-th/0402026].

Background field from MV, KMW model

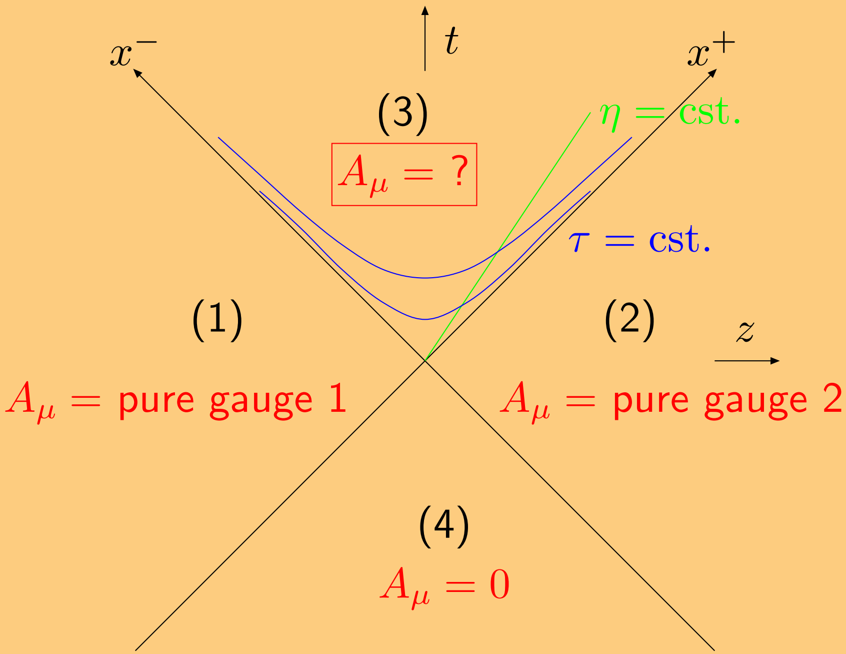
The MV[8] model, collision of two ions studied analytically by KMW[9] and numerical formulation by Krasnitz & Venugopalan[10]

$$[D_\mu, F^{\mu\nu}] = J^\nu,$$

$$J^\mu = \delta^{\mu +}\rho_{(1)}(\mathbf{x}_T)\delta(x^-) + \delta^{\mu -}\rho_{(2)}(\mathbf{x}_T)\delta(x^+),$$

$$\langle \rho^a(\mathbf{x}_T)\rho^b(\mathbf{y}_T)\rangle = g^2\mu^2\delta^{ab}\delta^2(\mathbf{x}_T - \mathbf{y}_T).$$

$$g^2\mu \sim Q_\mathrm{s} \quad \text{(very roughly)}$$



[8] L. D. McLerran and R. Venugopalan, Phys. Rev. **D49**, 2233 (1994), [hep-ph/9309289].
[9] A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. **D52**, 3809 (1995), [hep-ph/9505320].
[10] A. Krasnitz and R. Venugopalan, Nucl. Phys. **B557**, 237 (1999), [hep-ph/9809433].

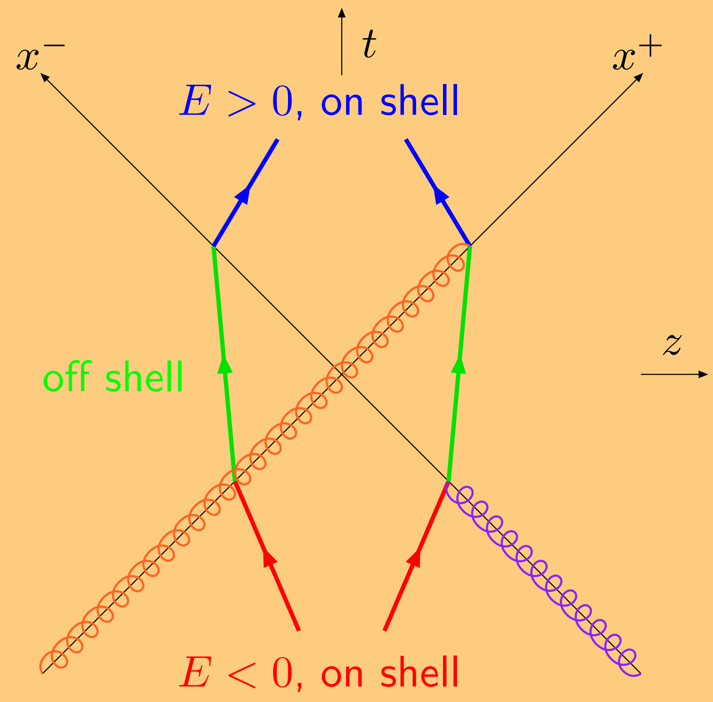
Dirac equation in background field

Method explained in [11] and numerics tested in a 1+1-dimensional toy model. Solve Dirac equation in backgroud field.

- Initial condition: negative energy plain wave
- Integrate D.E. forward in time using coordinates

$$\boxed{\tau, z, \mathbf{x}_T}$$

- Two separate branches of the solution; (amplitude linear superposition of two terms; think of u, t-channels in Abelian case).
- Projection to positive energy states gives number of *quark pairs* produced.

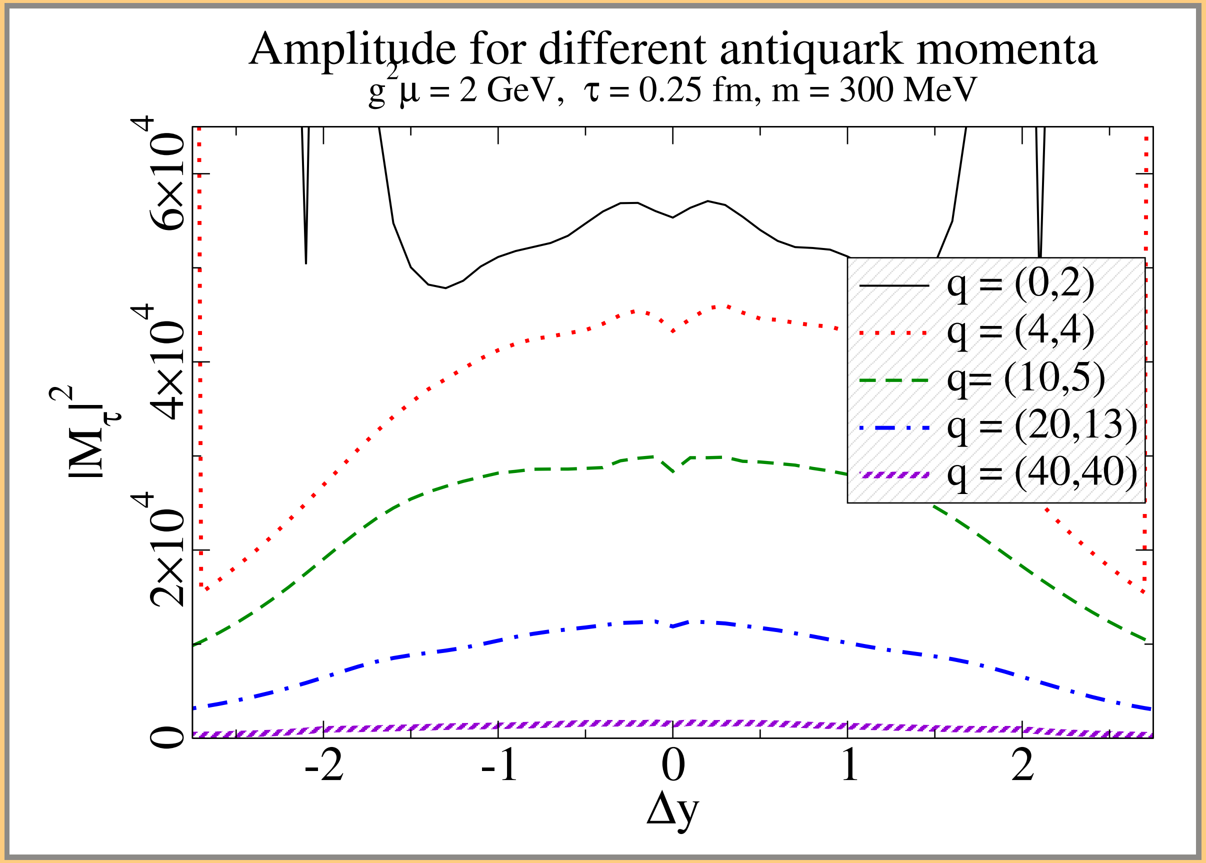


Parameters: $g^2\mu$ (bg field), R_A (system size), and m (quark mass).

[11] F. Gelis, K. Kajantie and T. Lappi, Phys. Rev. **C71**, 024904 (2005), [hep-ph/0409058].

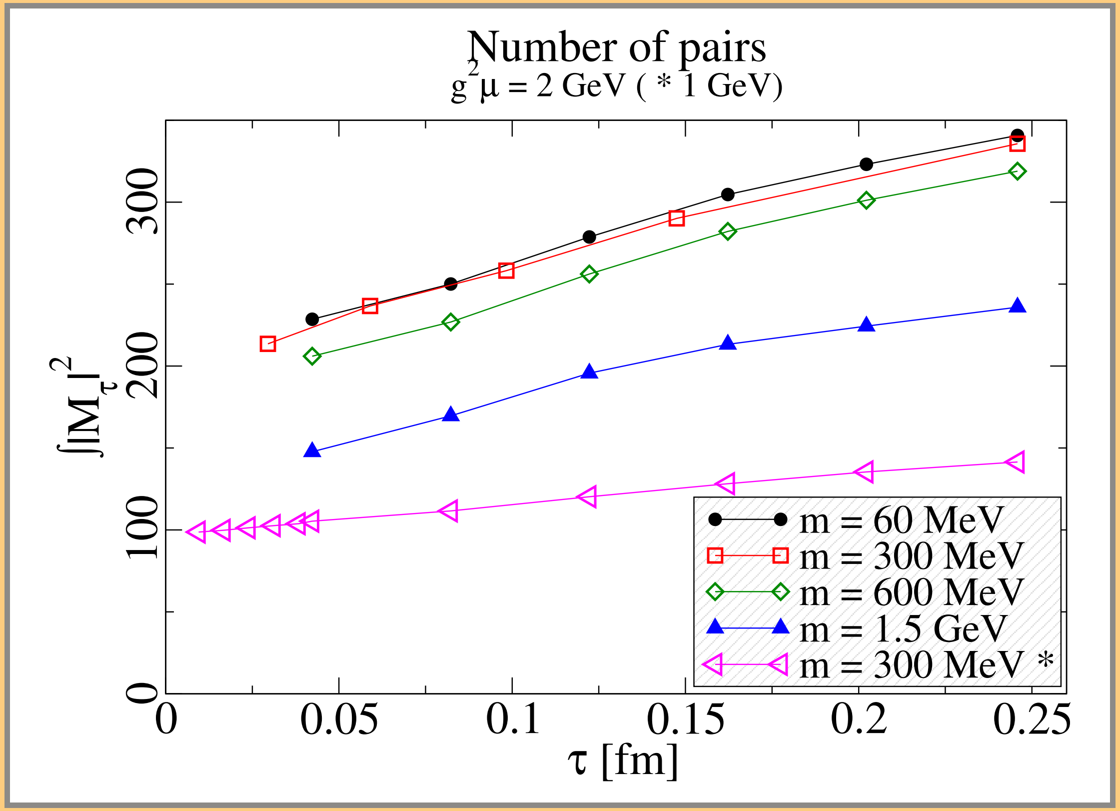
Amplitude for different antiquark momenta

For each antiquark momentum $\mathbf{q}_T$, the projection gives an amplitude M:



Finite $\mathrm{d}z$ ▶ UV cutoff in $p_z \sim \sinh y_p$. The numerical calculation breaks down for large y_p, small $m, \mathbf{q}_T$.

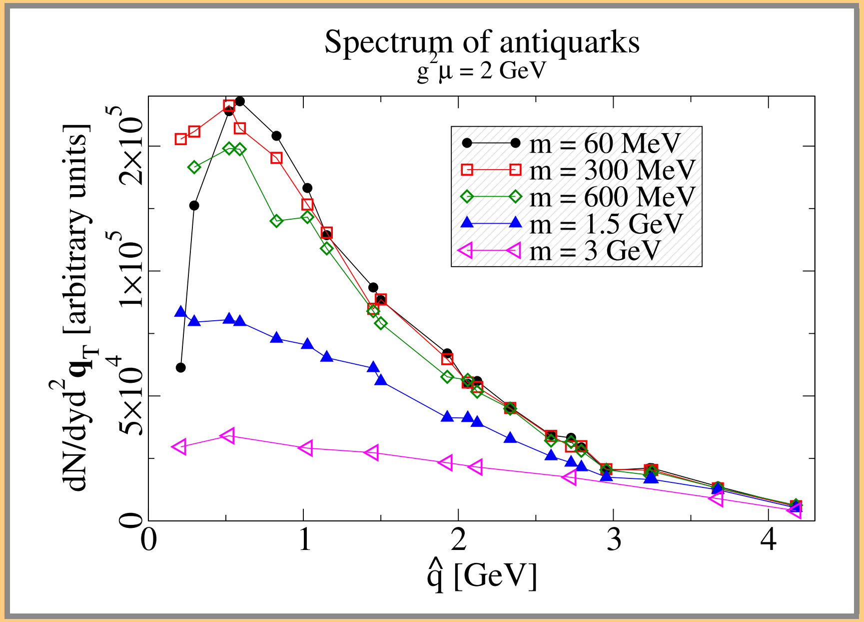
Number of pairs, time dependence

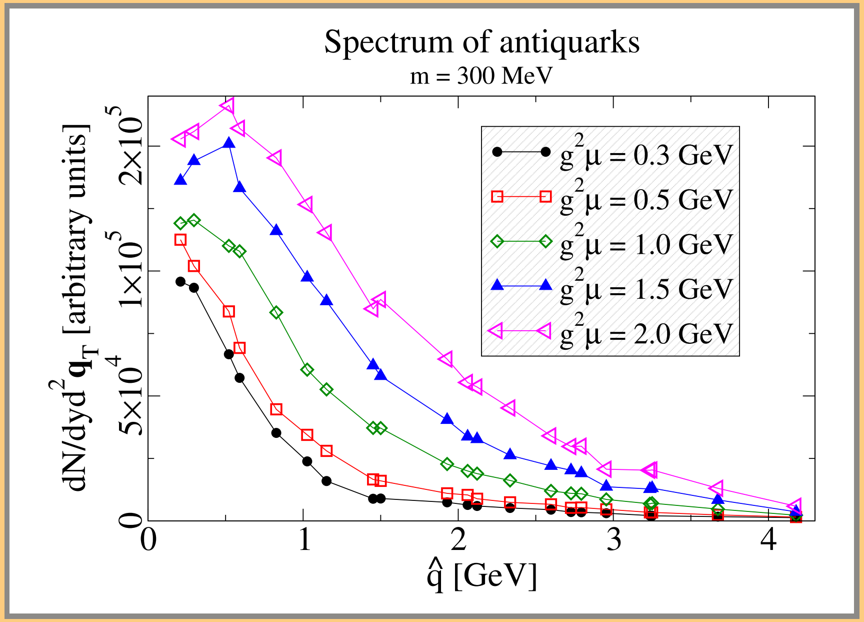


Most of the pairs are produced at $\tau = 0$, then the number increases in the background field.

Note: This is for one flavor of mass m and one unit of rapidity y.

(Anti)quark spectrum

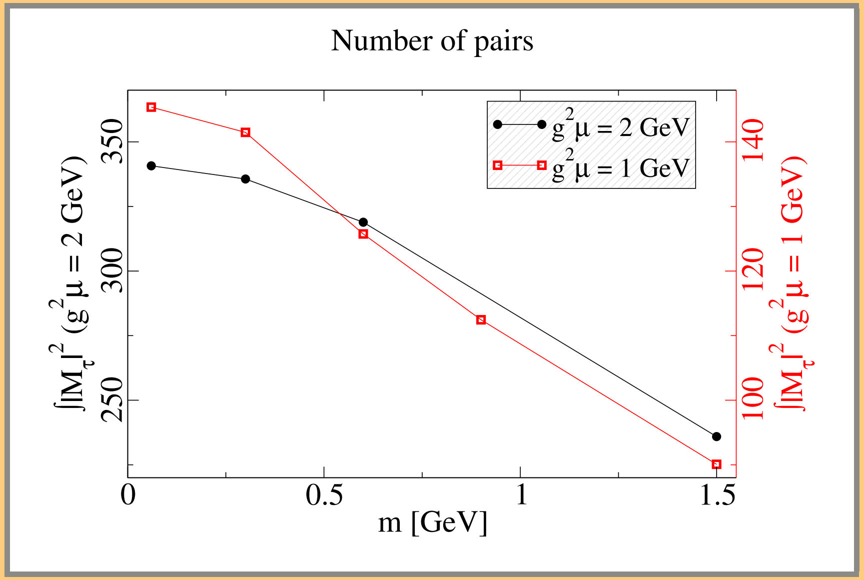




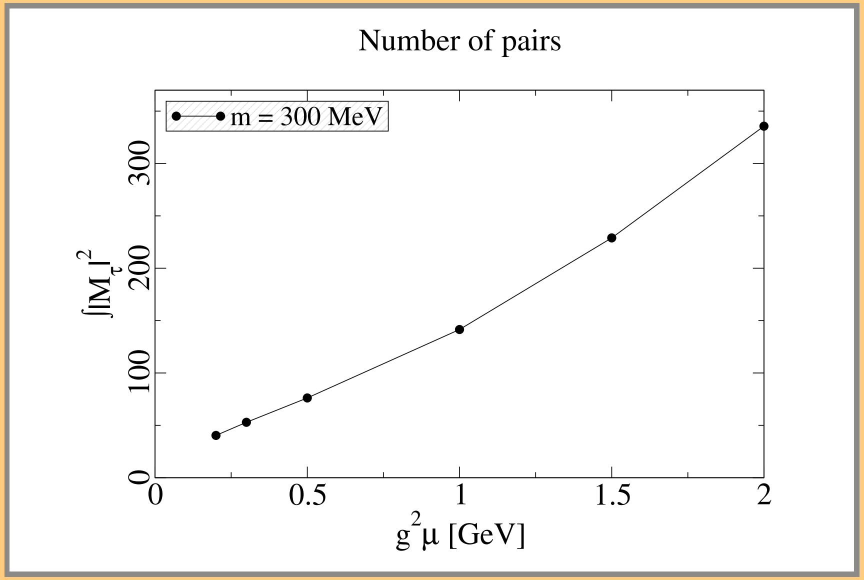
The spectrum gets harder and the number decreases with increasing quark mass, but not as strongly as one would expect (see next slide).

Both the normalization and the momentum scales increase with $g^2\mu$. Also this dependence is perhaps weaker than expected on dimensional grounds (see next slide).

Number of pairs, dependence on mass and $g^2\mu$



Dependence on mass



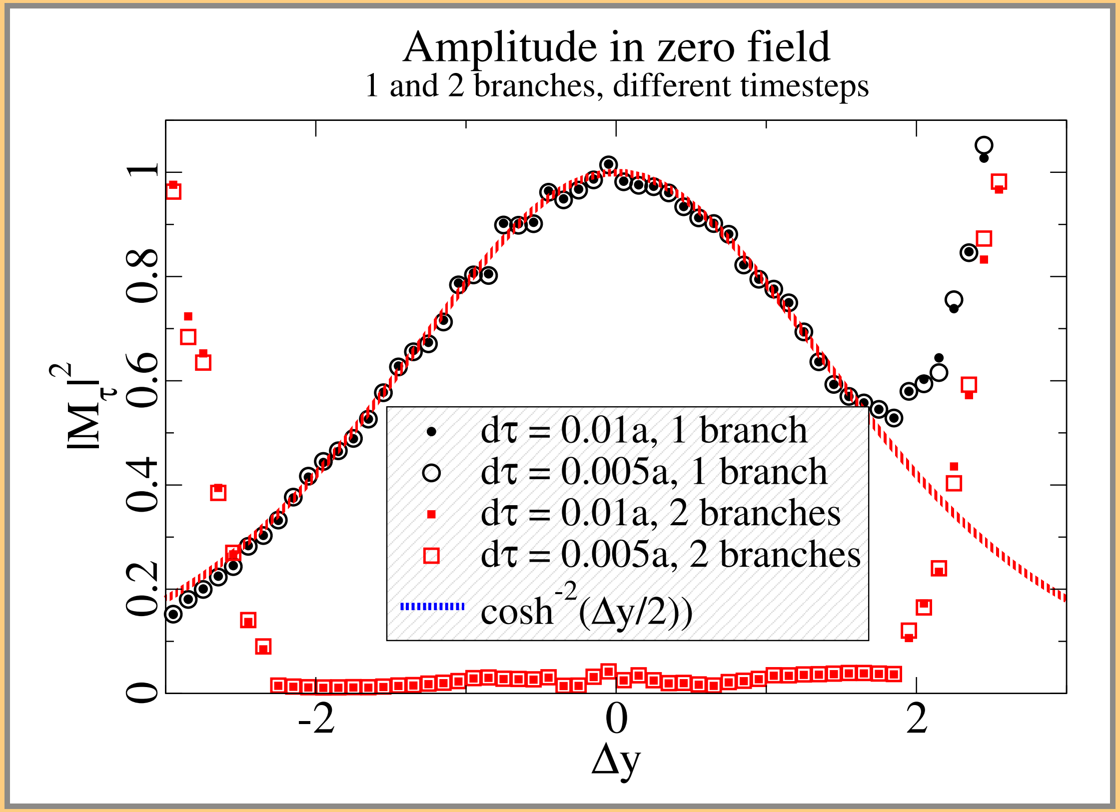
Dependence on $g^2\mu$

Numerical calculation

- Background field generated by separate (old) code and stored on disk
- Dirac equation discretization:
  - Transverse lattice treated in standard way
  - Longitudinal (z) direction discretized *implicitly* to handle the curved coordinate system.
- Memory requirement: 400×180^2–lattice with $4 \times (N_\mathrm{c} = 3)$ complex components in spinor: $1.2\ \mathrm{GB}$ memory in single precision.
- Most computations performed on `ametisti` (66×2–processor $1.8\ \mathrm{GHz}$ AMD Opteron linux cluster at University of Helsinki), over 10^{17} flop used so far.

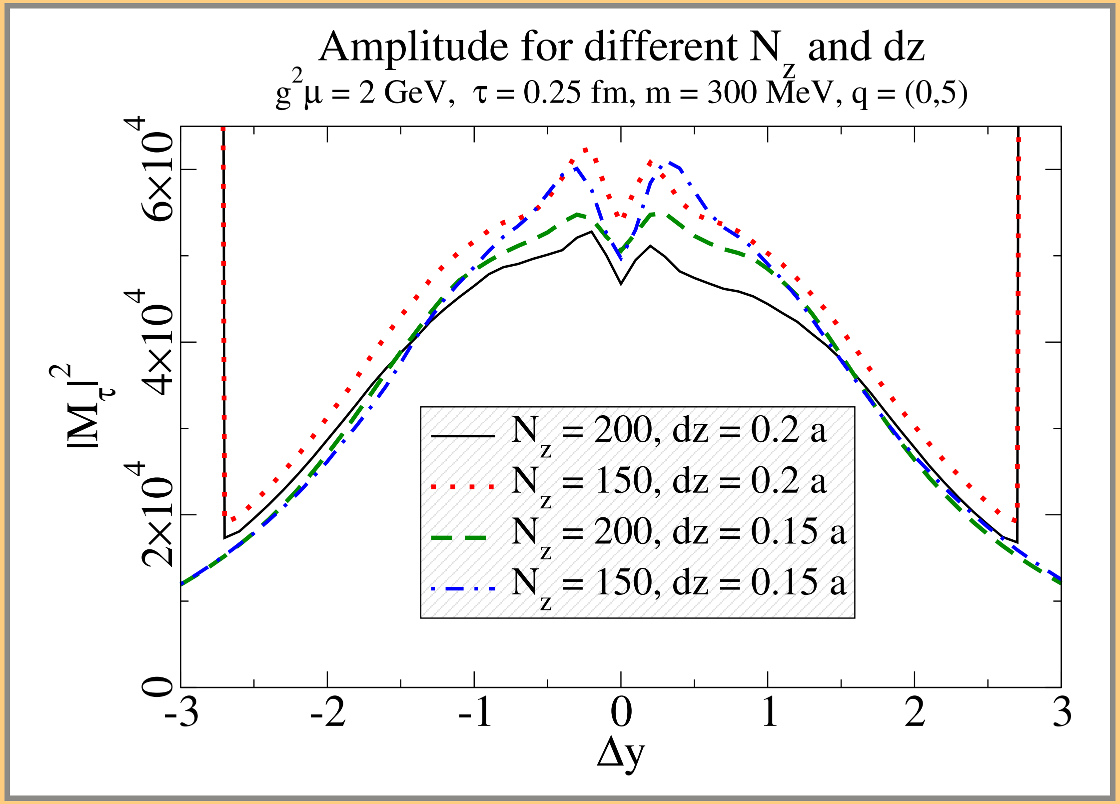
Testing the numerics: zero external field

For zero field: $|M|^2$ from one branch is $\frac{1}{\cosh^2 \Delta y/2}$ and the branches cancel each other.



Theoretically understood curves are reproduced; give some idea of the numerical inaccuracy.

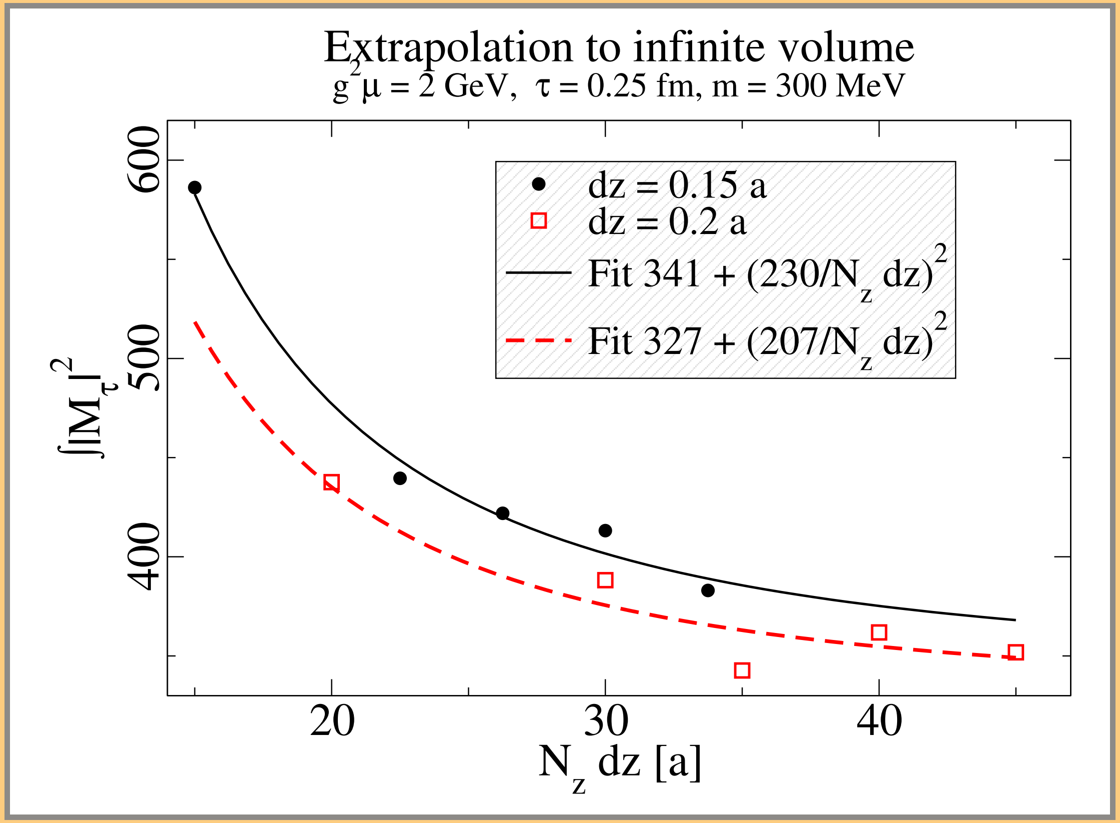
Testing the numerics: amplitude for different $\mathrm{d}z$ and N_z



The differences between these curves are purely numerical effects.

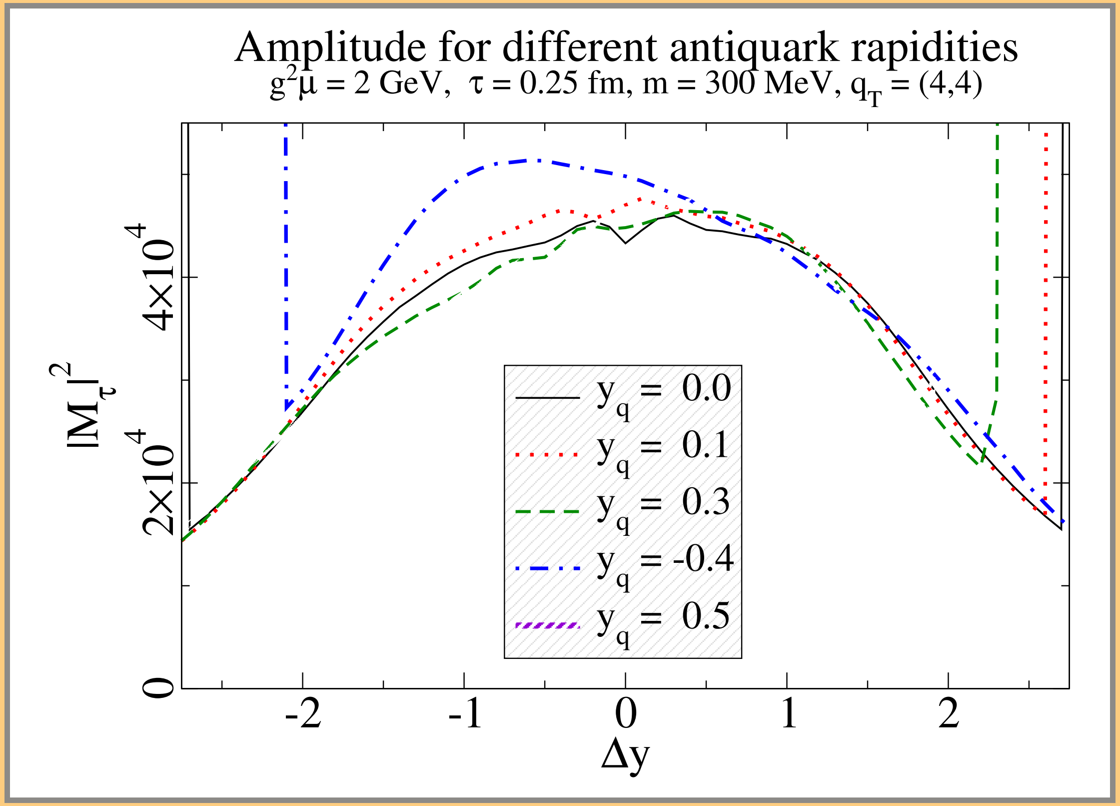
Having a smaller $\mathrm{d}z$ enables going to larger rapidities ($\sim$ larger p_z).

Testing the numerics: extrapolation to infinite volume



The dependence on $\mathrm{d}z$ and N_z is weak ▶ extrapolating to the limit $\mathrm{d}z \to 0$, $N_z\,\mathrm{d}z \to \infty$ is possible, but requires a lot of data. So far use mostly $\mathrm{d}z = 0.2a$, $N_z = 200$.

Testing the numerics: boost invariance

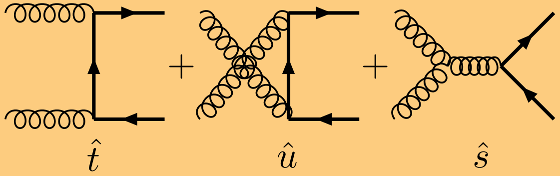


Background field boost independent ▶ The amplitude should be a function of $\Delta y = y_p - y_q$ only, independent of y_q. This is a nontrivial test of the numerics in τ, z–coordinates.

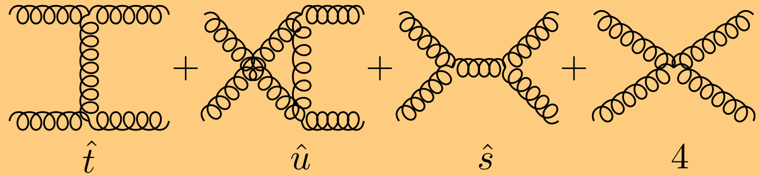
Discussion: does this make any sense?

Conventional wisdom: initial state gluonic. Our result: number of quark pairs large. Is it reasonable to compare quark and gluon numbers?

- Collinear pQCD calculation (à la EKRT[12]) ▶ $2 \to 2$, IR cutoff.



vs.



  suppressed by $\sim 210 = \overbrace{7}^{\text{color}} \times \overbrace{30}^{\text{diags}}$ ▶ at RHIC $gq \to gq,\ g\bar{q} \to g\bar{q}$ dominate over this contribution.

- This calculation ▶ gluons produced in $2 \to 1$, quarks in $2 \to 2$ ▶ Quarks suppressed by a power of the coupling, kinematics completely different.

[12] K. J. Eskola, K. Kajantie, P. V. Ruuskanen and K. Tuominen, Nucl. Phys. **B570**, 379 (2000), [hep-ph/9909456].

Phenomenology

I.e. what does this result mean if one does take it seriously?

Assuming that the subsequent evolution of the system conserves entropy $\sim$ multiplicity we should have ~ 1000 particles (gluons, quarks, or antiquarks) in the initial state.

- If these are all gluons, we need $g^2\mu \sim 2\mathrm{GeV}$[13].
- If also quarks are amply present, we could have $g^2\mu \sim 1.3\mathrm{GeV}$, ~ 400 gluons, $\gtrsim 100 N_\mathrm{f}$ quarks and $\gtrsim 100 N_\mathrm{f}$ antiquarks, close to the thermal ratio $N_g/N_q = 64/21 N_\mathrm{f}$.

[13] T. Lappi, Phys. Rev. **C67**, 054903 (2003), [hep-ph/0303076].

Conclusions

Quark pair production from classical background field of McLerran-Venugopalan model studied by solving the 3+1–dimensional Dirac equation numerically in this classical background field.

- Number of quarks produced large (▶ chemical equilibration)
- Mass dependence surprisingly weak, no conclusions on heavy quarks yet.
- Numerical computation still continuing.