QGP Instabilities in discretized Hard-Loop Approximation

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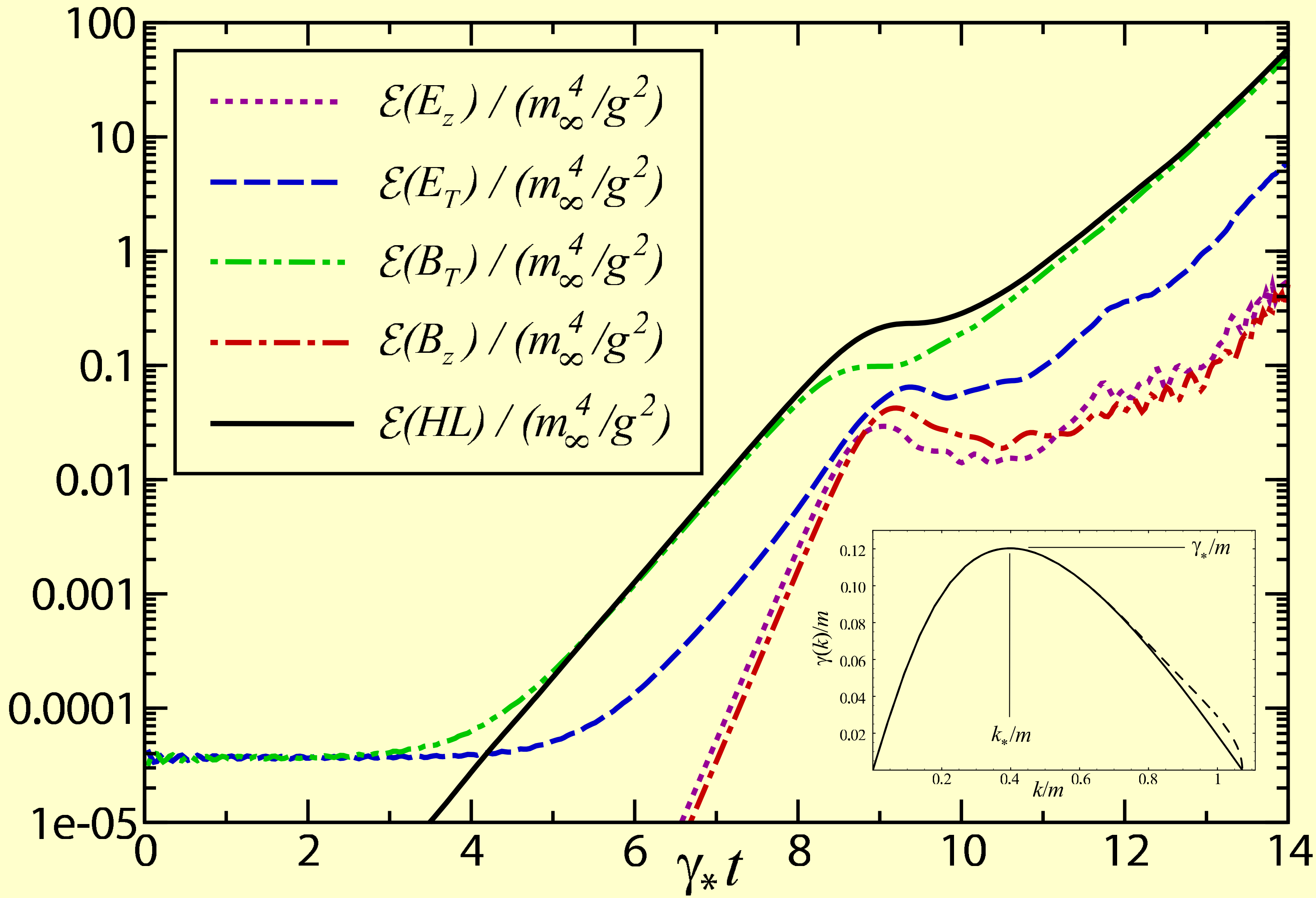
Motivation

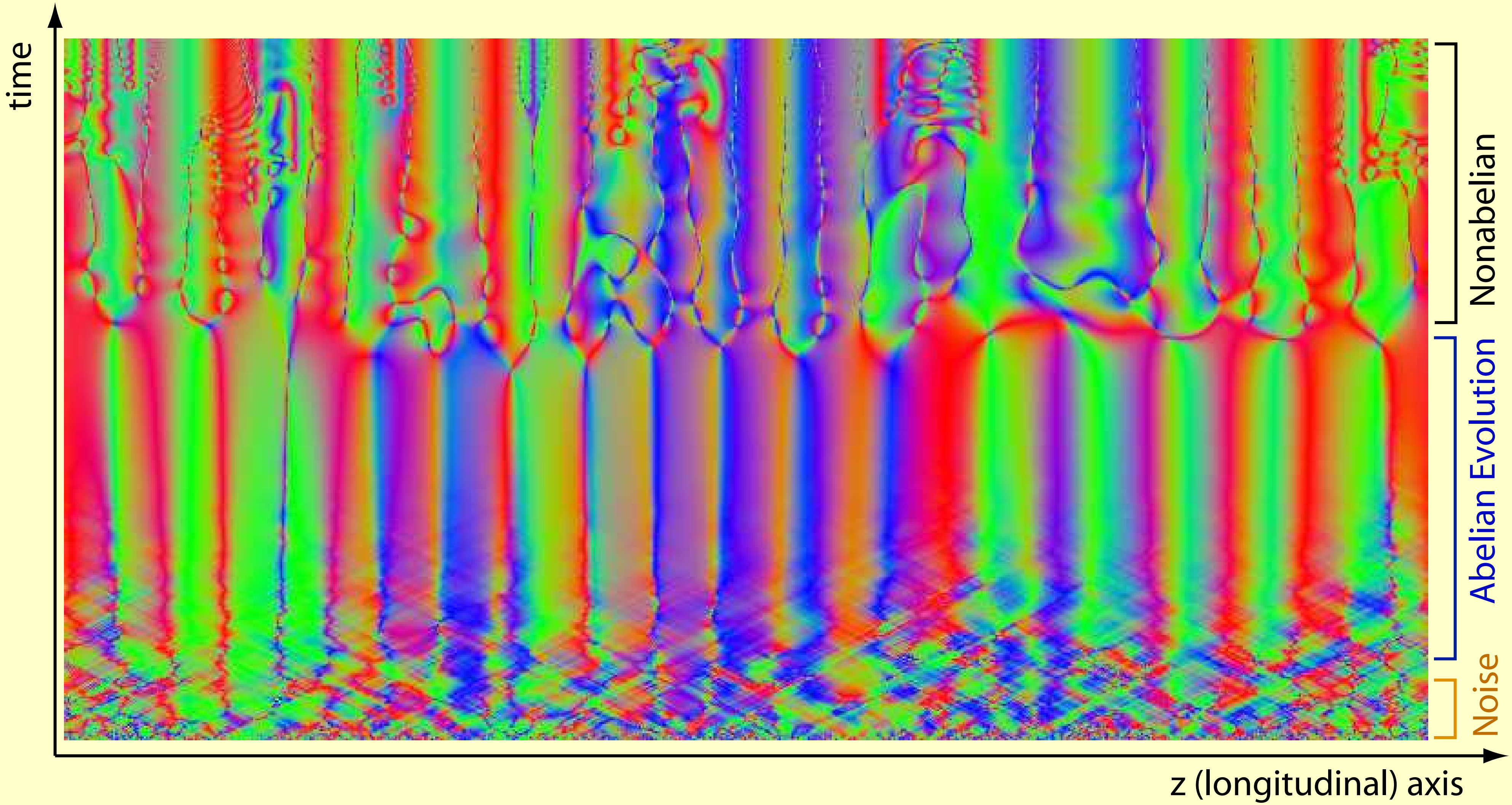
- Success of hydro models seems to imply fast isotropization (and possibly thermalization) of matter created at RHIC.
- Previous ”naive” perturbative estimates don’t seem to explain this.
- One possibility is that perturbation theory should be thrown out the window and replaced by a new (as of yet unspecified) calculational framework.
- However, the perturbative estimates to date have overlooked an important aspect of the physics, namely that in anisotropic plasmas the **collective modes** (aka mean field dynamics) are fundamentally different than in isotropic ones.
- This results in some things previously not considered in this context, namely the spontaneous generation of large soft color fields which provide additional scattering of hard particles, and a very efficient method for transferring energy from hard to soft scales.

1 space × 3 velocity Lattice results

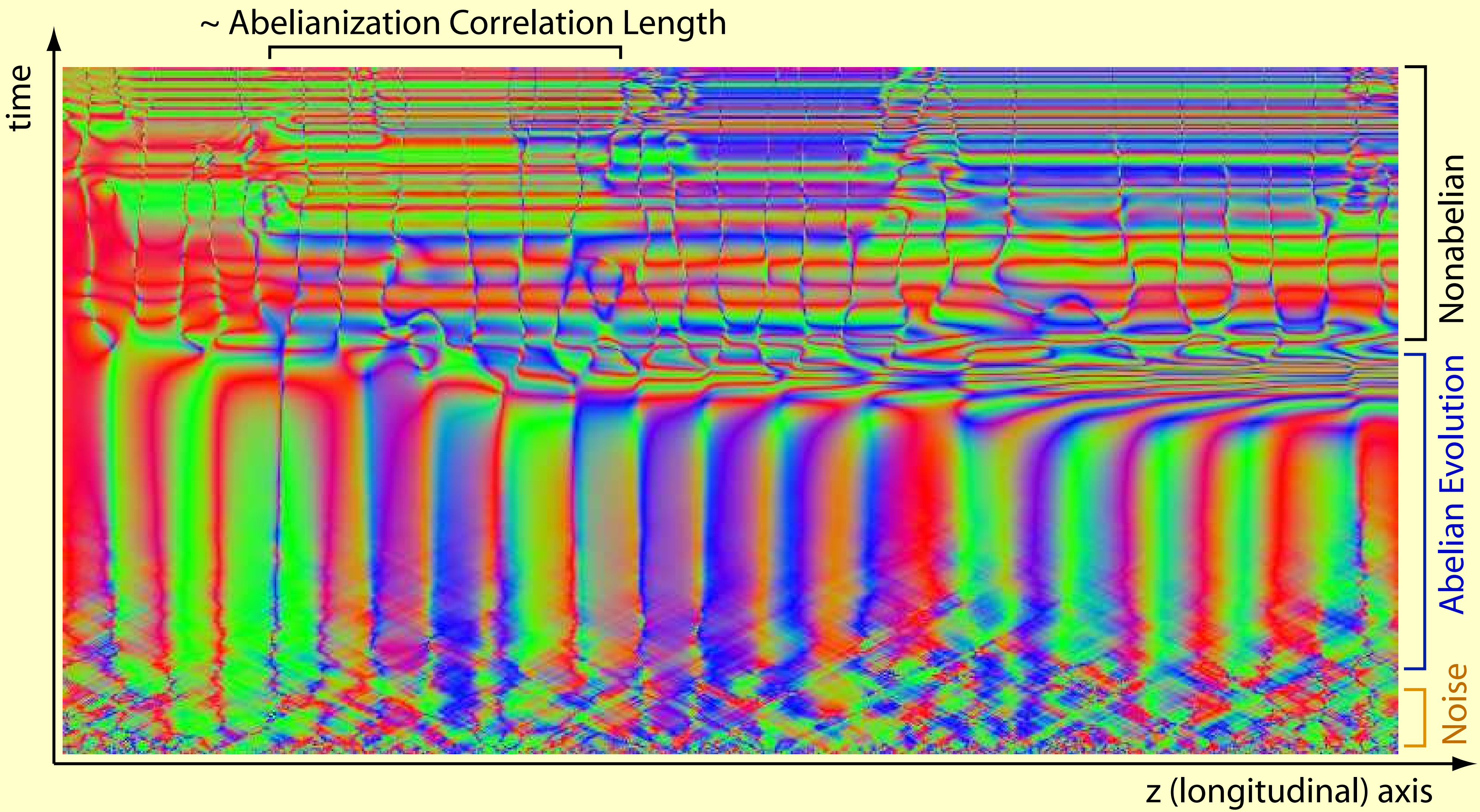


A. Rebhan, P. Romatschke, and MS, PRL 94, 102303 (2005); hep-ph/0412016.

J_x Visualization



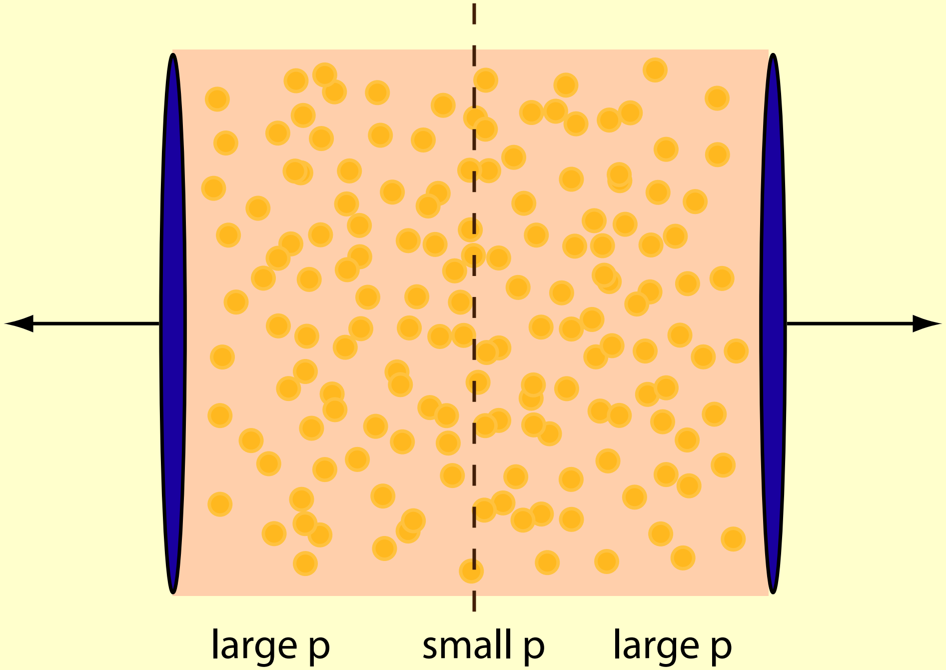
Parallel transported J_x Visualization

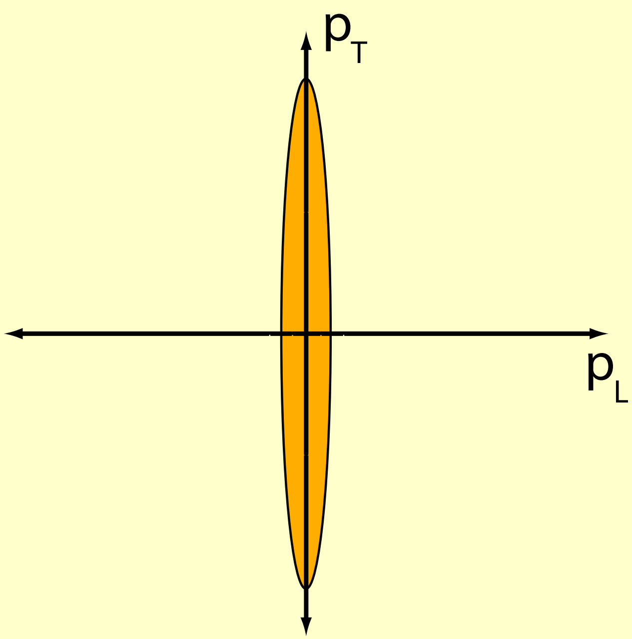


Why anisotropic distribution functions?

Because of the natural expansion of the system the gluon distribution functions created during relativistic heavy ion collisions are *generically* locally anisotropic in momentum space.

$$
\begin{aligned}
<p_T> \quad &\sim \quad Q_s \qquad \text{(nuclear saturation scale)} \\
<p_L> \quad &\sim \quad 1/\tau
\end{aligned}
$$





Collective Modes of an Isotropic QGP

The isotropic hard-thermal-loop (HTL) gluon propagator is given by

$$\Delta^{ij} \;=\; (k^2-\omega^2+\Pi_T)^{-1}(\delta_{ij}-k^ik^j/k^2)-\frac{k^2}{\omega^2}(k^2-\Pi_L)^{-1}k^ik^j/k^2$$

with

$$\Pi_T(\omega,k) \;=\; \frac{m_D^2}{2}\frac{\omega^2}{k^2}\left[1-\frac{\omega^2-k^2}{2\omega k}\log\frac{\omega+k}{\omega-k}\right],$$

$$\Pi_L(\omega,k) \;=\; m_D^2\left[\frac{\omega}{2k}\log\frac{\omega+k}{\omega-k}-1\right],$$

and $m_D \propto gT$.

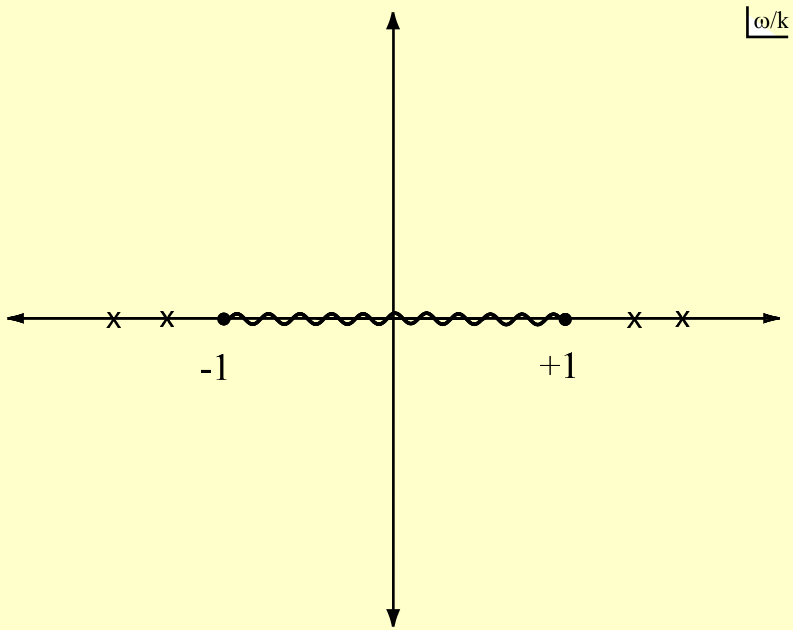
$$\lim_{\omega\to 0}\Pi_L(\omega,k) \;=\; m_D^2 \quad \text{electric screening}$$

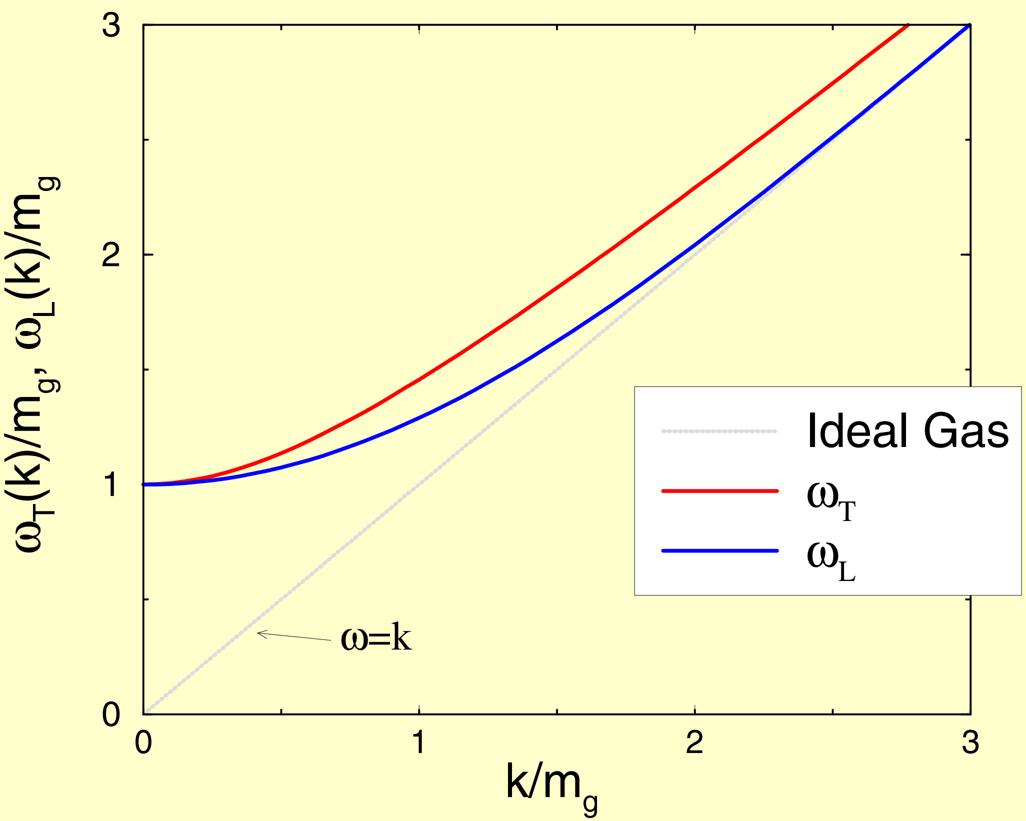
$$\lim_{\omega\to 0}\Pi_T(\omega,k) \;=\; 0 \quad \textbf{no}\ \text{magnetic screening}$$

Collective Modes of an Isotropic QGP

In the isotropic case the only poles are at real timelike ($\omega > k$) momentum. In order to determine the dispersion relations for these excitations we can then explicitly look for the poles in the propagator.

$$
\begin{aligned}
0 &= k^2 - \omega_T^2 + \Pi_T(\omega_T, k) \\
0 &= k^2 - \Pi_L(\omega_L, k)
\end{aligned}
$$





Anisotropic Gluon Polarization Tensor

In order to determine the HL gluon self-energy in a homogeneous anisotropic system we can use either three-dimensional kinetic theory or diagrammatic techniques.[4,5] The result is

$$\Pi^{ij}(K) = -g^2 \int \frac{d^3p}{(2\pi)^3} v^i \partial^l f(\mathbf{p}) \left(\delta_{jl} - \frac{v_j k_l}{K \cdot V + i\epsilon} \right) .$$

S. Mrówczyński first pointed out that within anisotropic QCD plasmas there are unstable modes which are the equivalent of QED Weibel type instabilities.[6] Recently there was renewed interest in this phenomena.[7,8]

[4] H. Elze and U. Heinz, 89; J. Blaizot and E. Iancu, 94.

[5] S. Mrówczyński, 93; S. Mrówczyński and M. Thoma, 00.

[6] E. Weibel, 59.

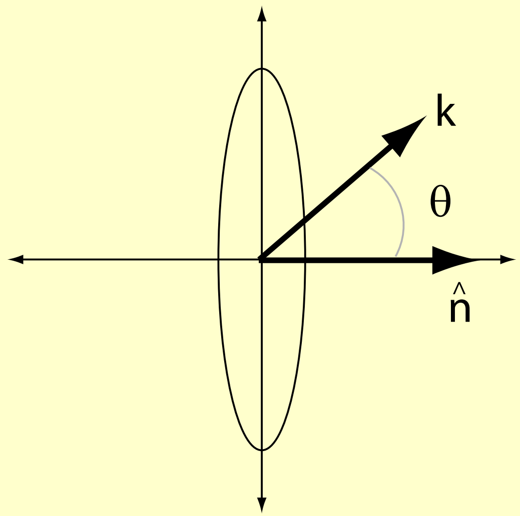
[7] P. Romatschke and MS, April 03.

[8] P. Arnold, J. Lenaghan, and G. Moore, July 03.

The nature of the anisotropy

We assume that the anisotropic distribution function can be obtained from an arbitrary isotropic distribution function by a change of its argument. *(Romatschke and MS, 03)*

$$f(p^2) \to \sqrt{1+\xi}\, f(p^2 + \xi(p\cdot n)^2)\ .$$



The polarization tensor can then be written as

$$\Pi^{ij}(K) = m_D^2 \sqrt{1+\xi} \int \frac{d\Omega}{4\pi} v^i \frac{v^l + \xi(v\cdot n)n^l}{(1+\xi(v\cdot n)^2)^2} \left(\delta^{jl} - \frac{v^j k^l}{K\cdot V + i\epsilon} \right)\ ,$$

where m_D is the *isotropic* Debye mass

$$m_D^2 = -\frac{g^2}{2\pi^2} \int_0^\infty dp\, p^2 \frac{df(p^2)}{dp}\ .$$

Tensor basis

We can construct a symmetric 3d tensor basis with the following four tensors

$$A^{ij} = \delta^{ij} - k^i k^j / k^2 \qquad B^{ij} = k^i k^j / k^2 \ ,$$
$$C^{ij} = \tilde{n}^i \tilde{n}^j / \tilde{n}^2 \qquad D^{ij} = k^i \tilde{n}^j + \tilde{n}^i k^j \ ,$$

where $\tilde{n} \equiv n^i A^{ij}$. We can then decompose the propagator and gluon polarization tensor in this tensor basis.

$$\Pi^{ij} = \alpha A^{ij} + \beta B^{ij} + \gamma C^{ij} + \delta D^{ij} \ ,$$

where

$$\begin{aligned} k^i \Pi^{ij} k^j &= k^2 \beta \ , \\ \tilde{n}^i \Pi^{ij} k^j &= \tilde{n}^2 k^2 \delta \ , \\ \tilde{n}^i \Pi^{ij} \tilde{n}^j &= \tilde{n}^2 (\alpha + \gamma) \ , \\ \mathrm{Tr}\Pi^{ij} &= 2\alpha + \beta + \gamma \ . \end{aligned}$$

Anisotropic Propagator and Static Limit

This allows us to express the propagator in terms of three functions

$$\begin{aligned}\Delta_A^{-1}(K) &= k^2 - \omega^2 + \alpha \, , \\ \Delta_\pm^{-1}(K) &= \omega^2 - \Omega_\pm^2 \, ,\end{aligned}$$

where

$$2\Omega_\pm^2 = \bar{\Omega}^2 \pm \sqrt{\bar{\Omega}^4 - 4((\alpha + \gamma + k^2)\beta - k^2 \tilde{n}^2 \delta^2)} \, ,$$

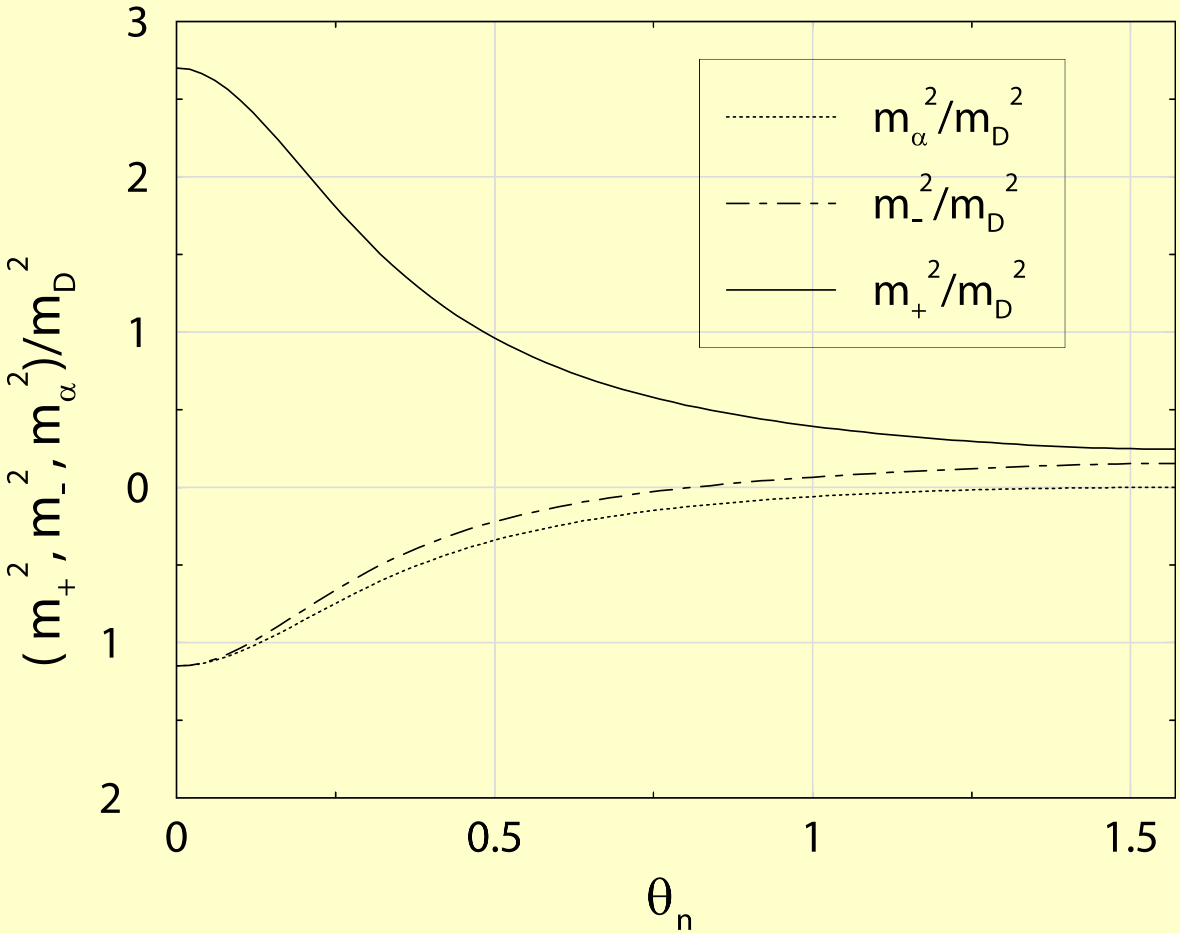
and $\bar{\Omega}^2 = \alpha + \beta + \gamma + k^2$.

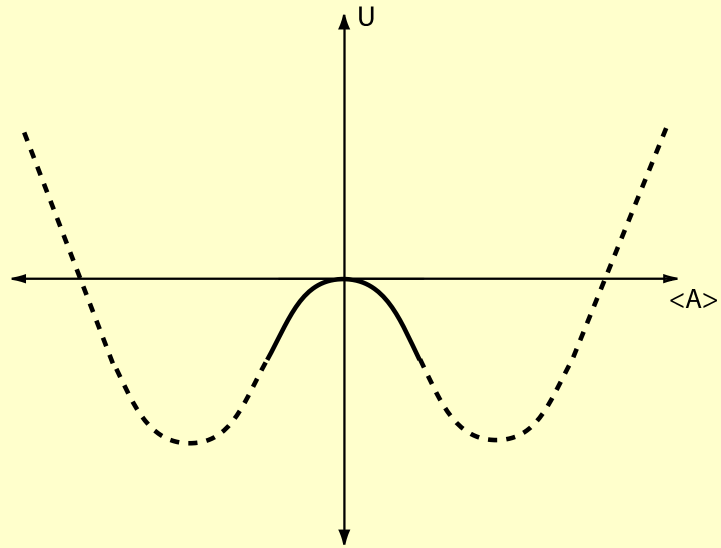
Taking the static limit we can define three mass scales: $m_\pm$ and m_α.

In the isotropic limit, $\xi \to 0$, $m_\alpha^2 = m_-^2 = 0$ and $m_+^2 = m_D^2$ and for finite ξ it is possible to evaluate these masses analytically.[9]

P. Romatschke and MS, 03.

New Mass Scales – $\xi > 0$



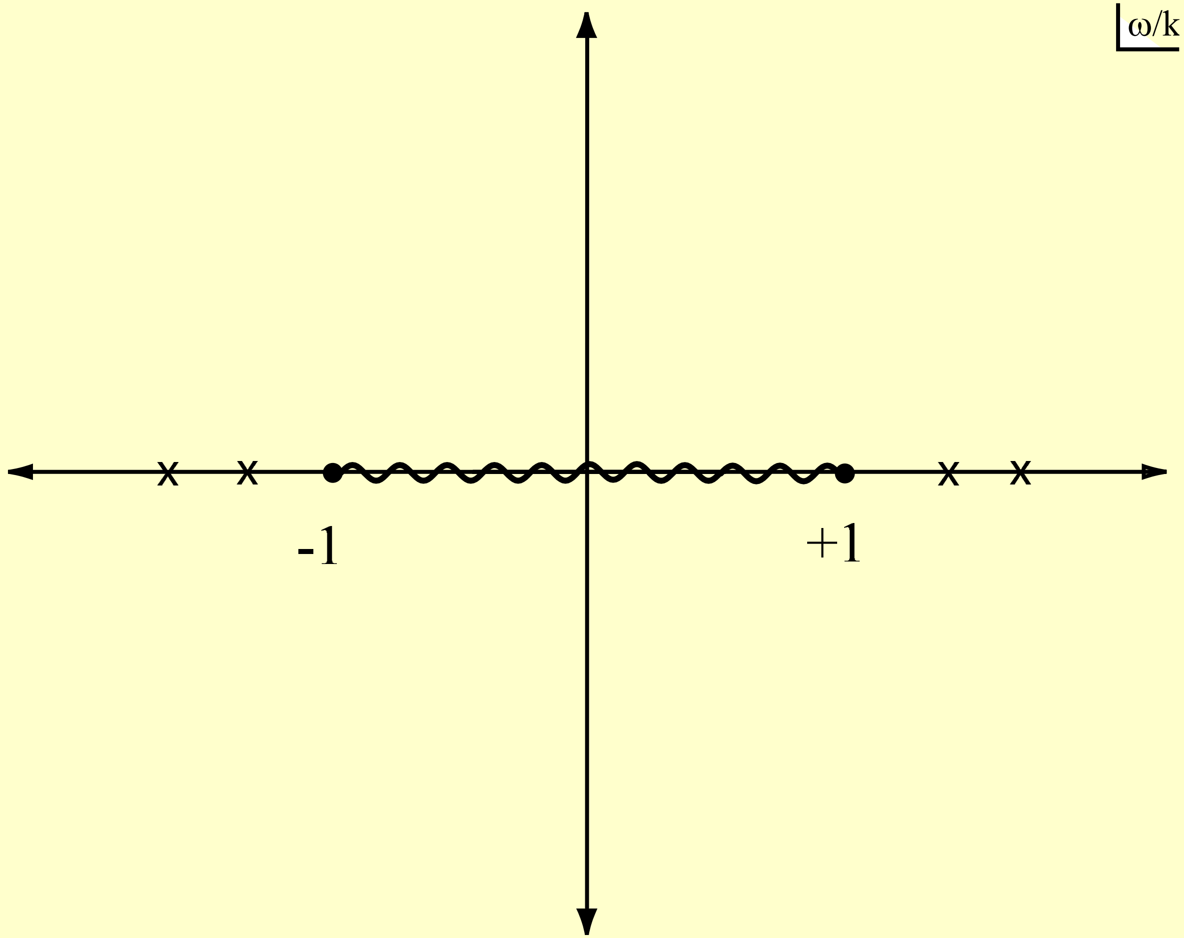


Sketch of the effective potential of an unstable mode.

Angular dependence of m_α^2, m_+^2, and m_-^2 at fixed $\xi = 10$.

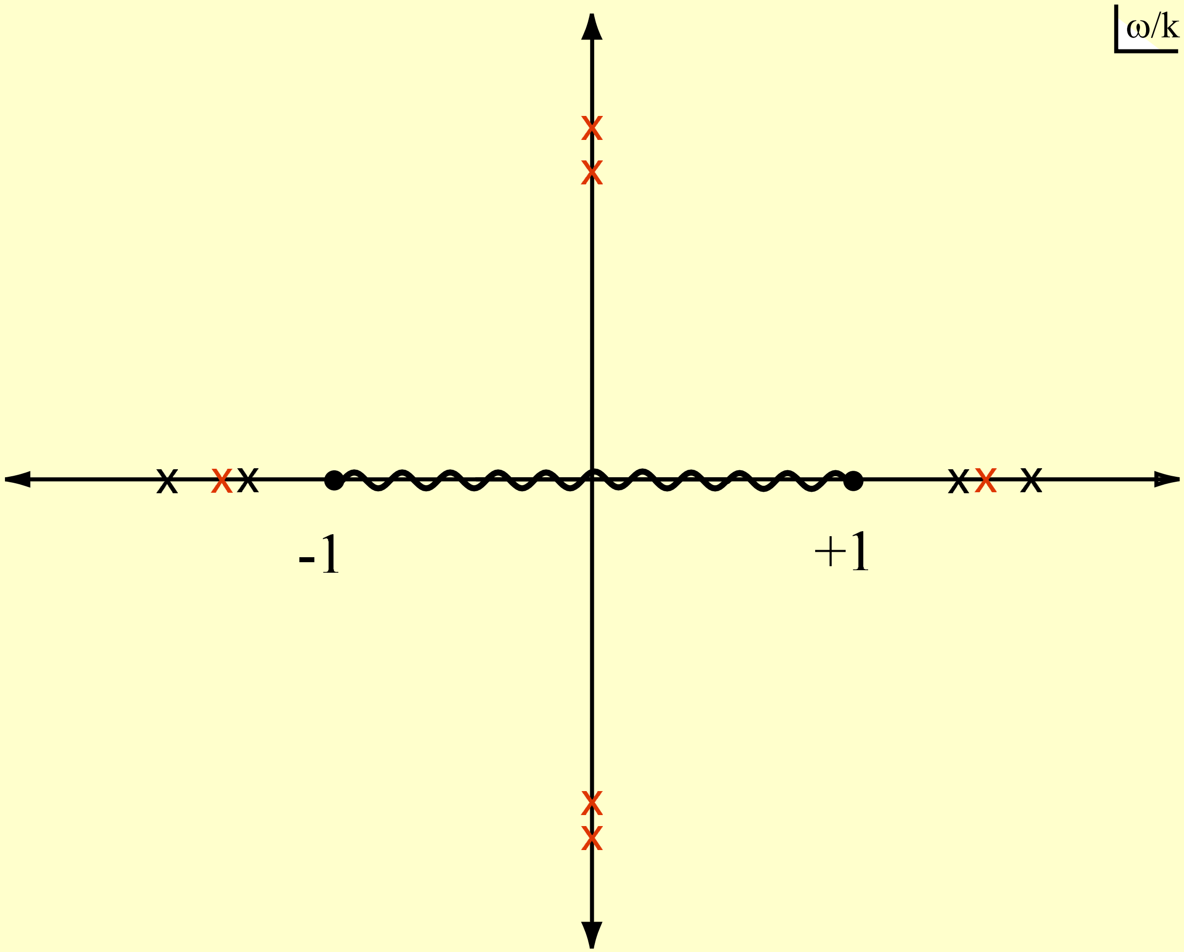
P. Romatschke and MS, 03.

Isotropic Collective Modes



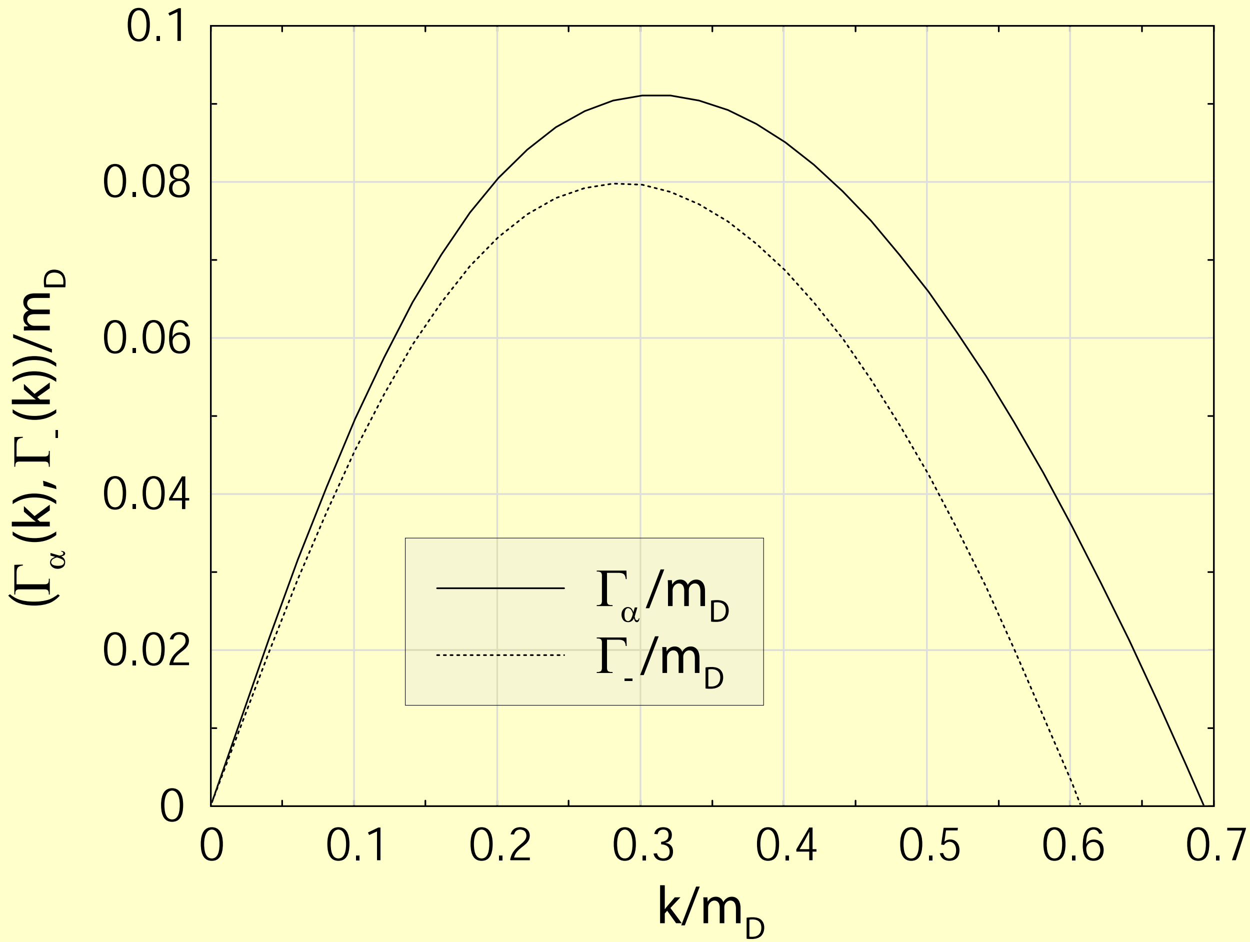
Isotropic poles ($\xi = 0$).

Anisotropic Collective Modes



Anisotropic poles at positive ξ.

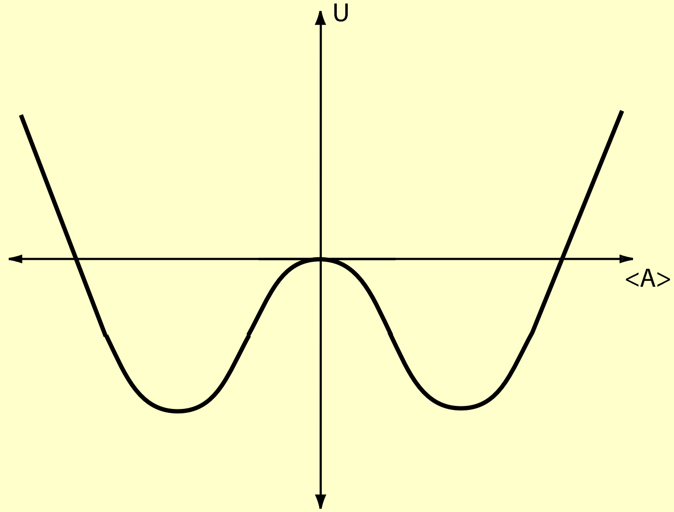
Unstable Modes – $\xi > 0$



$\Gamma_\alpha(k)$ and $\Gamma_-(k)$ as a function of k for $\xi = 10$ and $\theta_n = \pi/8$.

P. Romatschke and MS, 03.

Anisotropic HL Effective Action



Using the requirement of gauge invariance it is possible to determine all n-point functions in the same way as in the isotropic case.[10]

$$S_{\rm HL} = -\frac{g^2}{2}\int_x \int_{\mathbf{p}} \left\{ f(\mathbf{p}) F^a_{\mu\nu}(x) \left(\frac{p^\nu p^\rho}{(p\cdot D)^2} \right)_{ab} F_\rho{}^{b\,\mu}(x) + i\frac{C_F}{2}\tilde{f}(\mathbf{p})\bar{\Psi}(x)\frac{p\cdot\gamma}{p\cdot D}\Psi(x) \right\} .$$

For example, from this we can obtain the anisotropic 3-gluon vertex

$$\Gamma^{\mu\nu\lambda}(k,q,r) = \frac{g^2}{2}\int_{\mathbf{p}} \frac{\partial f(\mathbf{p})}{\partial p^\beta}\, \hat{p}^\mu \hat{p}^\nu \hat{p}^\lambda \left(\frac{r^\beta}{\hat{p}\cdot q\;\hat{p}\cdot r} - \frac{k^\beta}{\hat{p}\cdot k\;\hat{p}\cdot q} \right) .$$

S. Mrówczyński, A. Rebhan, and MS, 04.

Real-Time Lattice Simulation

In order to answer this we must numerically solve the equations of motion resulting from the HL effective action.

$$j^\mu[A] = -g^2 \int_{\mathbf{p}} \frac{1}{2|\mathbf{p}|}\, p^\mu \,\frac{\partial f(\mathbf{p})}{\partial p^\beta} W^\beta(x;\mathbf{v})$$

with

$$[v \cdot D(A)] W_\beta(x;\mathbf{v}) = F_{\beta\gamma}(A) v^\gamma$$

and $v^\mu = p^\mu/|\mathbf{p}| = (1, \mathbf{v})$.

This has to be solved with

$$D_\mu(A) F^{\mu\nu} = j^\nu$$

where $\nu = 0$ is the Gauss law constraint.

A. Rebhan, P. Romatschke, and MS, PRL 94, 102303 (2005); hep-ph/0412016.

$\vec{\mathbf{v}}$-discretized equations of motion

Assuming cylindrically symmetric anisotropies, we can parameterize them by $f(\mathbf{p}) = f(|\mathbf{p}|, p^z)$ allowing us express the current in terms of two auxiliary W fields

$$j^\mu = -\frac{1}{2} g^2 \int_{\mathbf{p}} v^\mu \left[f_1(|\mathbf{p}|, p^z) W^0(x,v) + f_2(|\mathbf{p}|, p^z) W^z(x,v) \right]$$

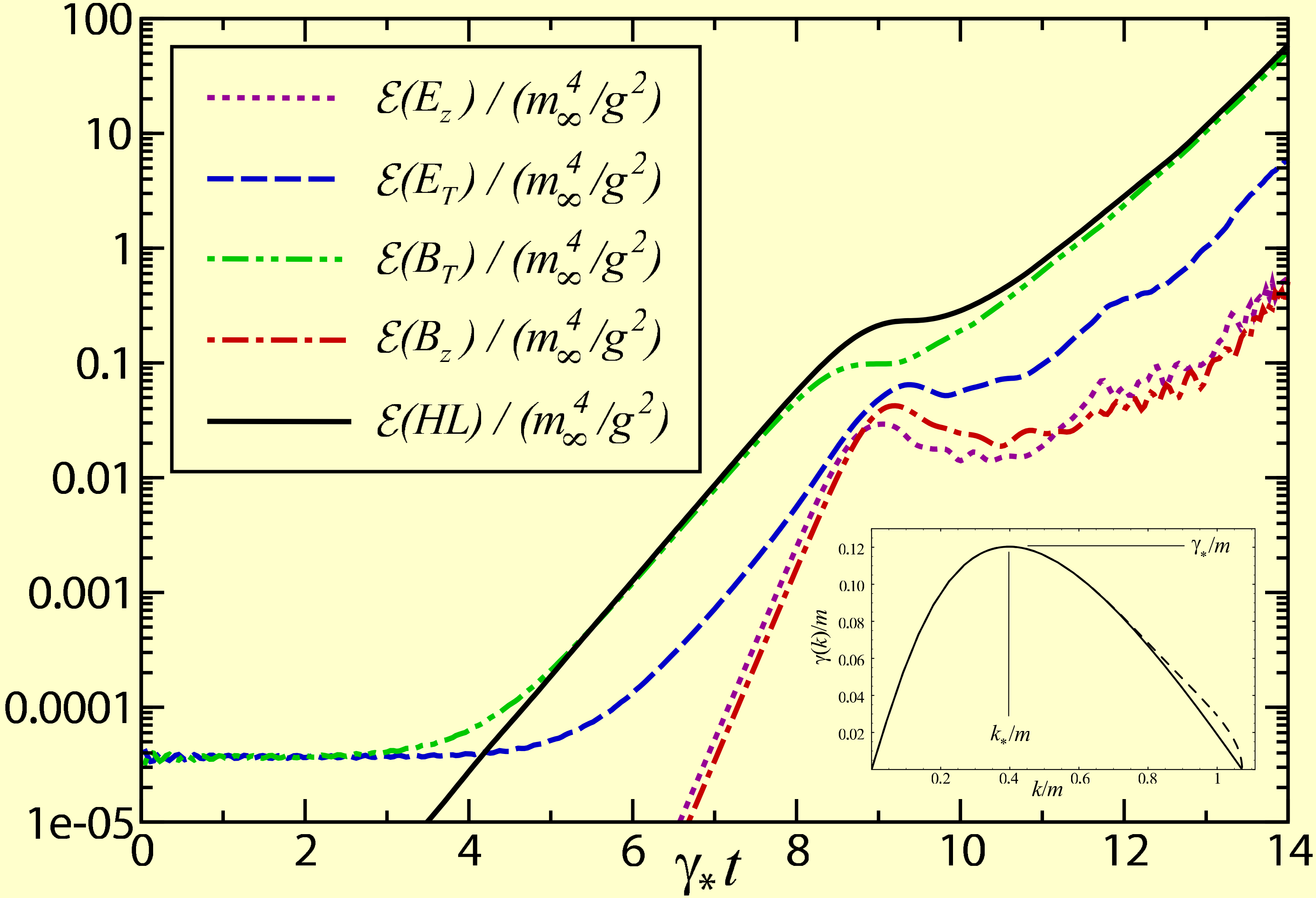
A closed set of gauge-covariant equations is obtained when the angular integral over $\hat{\mathbf{p}}$ is discretized. The full HL dynamics is then approximated by the following set of equations

$$[v \cdot D(A)] \mathcal{W}_{\mathbf{v}} = (a_{\mathbf{v}} F^{0\mu} + b_{\mathbf{v}} F^{z\mu}) v_\mu$$

$$D_\mu(A) F^{\mu\nu} = j^\nu = \frac{1}{\mathcal{N}} \sum_{\mathbf{v}} v^\nu \mathcal{W}_{\mathbf{v}},$$

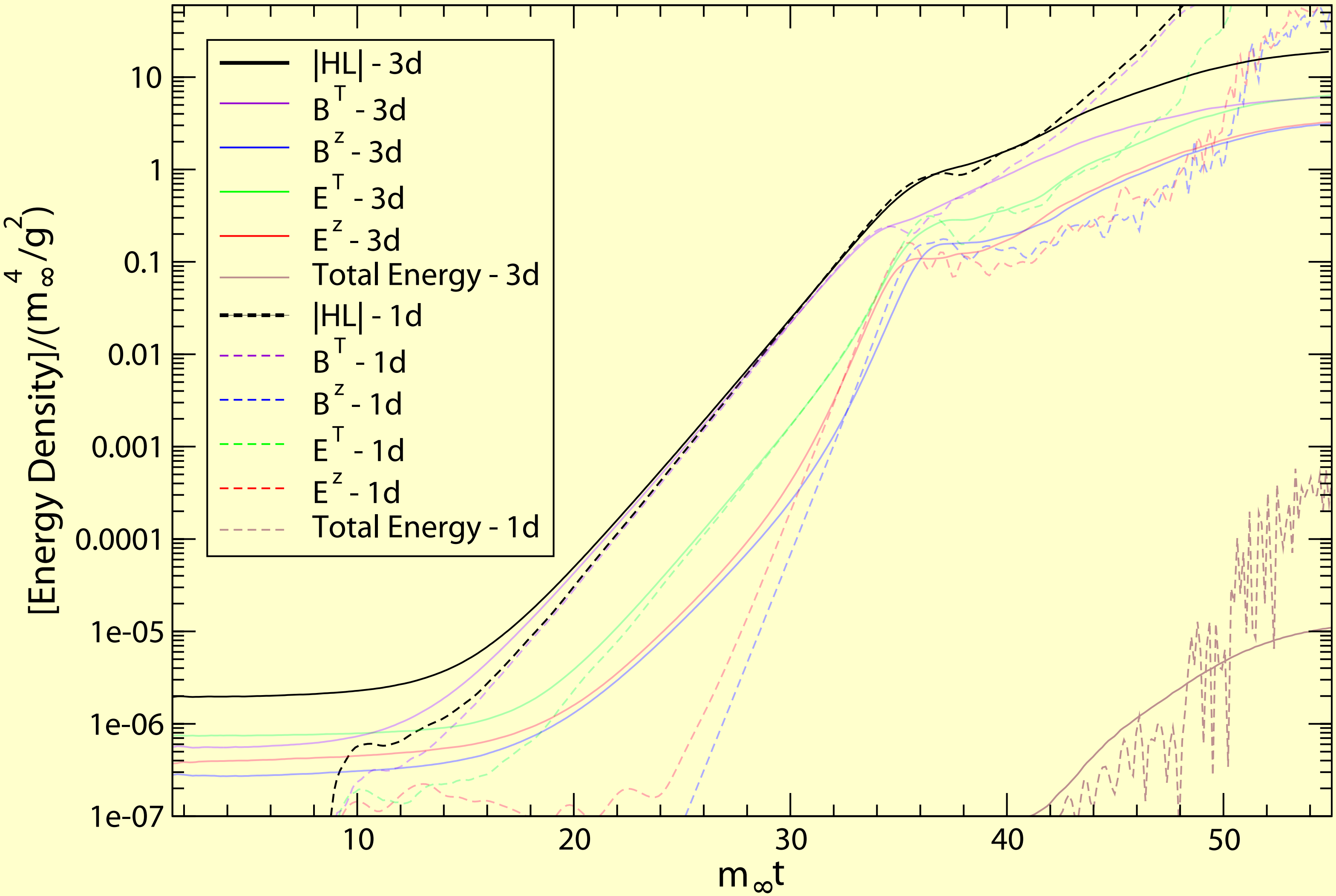
which can be systematically improved by increasing $\mathcal{N}$.

1s × 3v Lattice results



A. Rebhan, P. Romatschke, and MS, PRL 94, 102303 (2005); hep-ph/0412016.

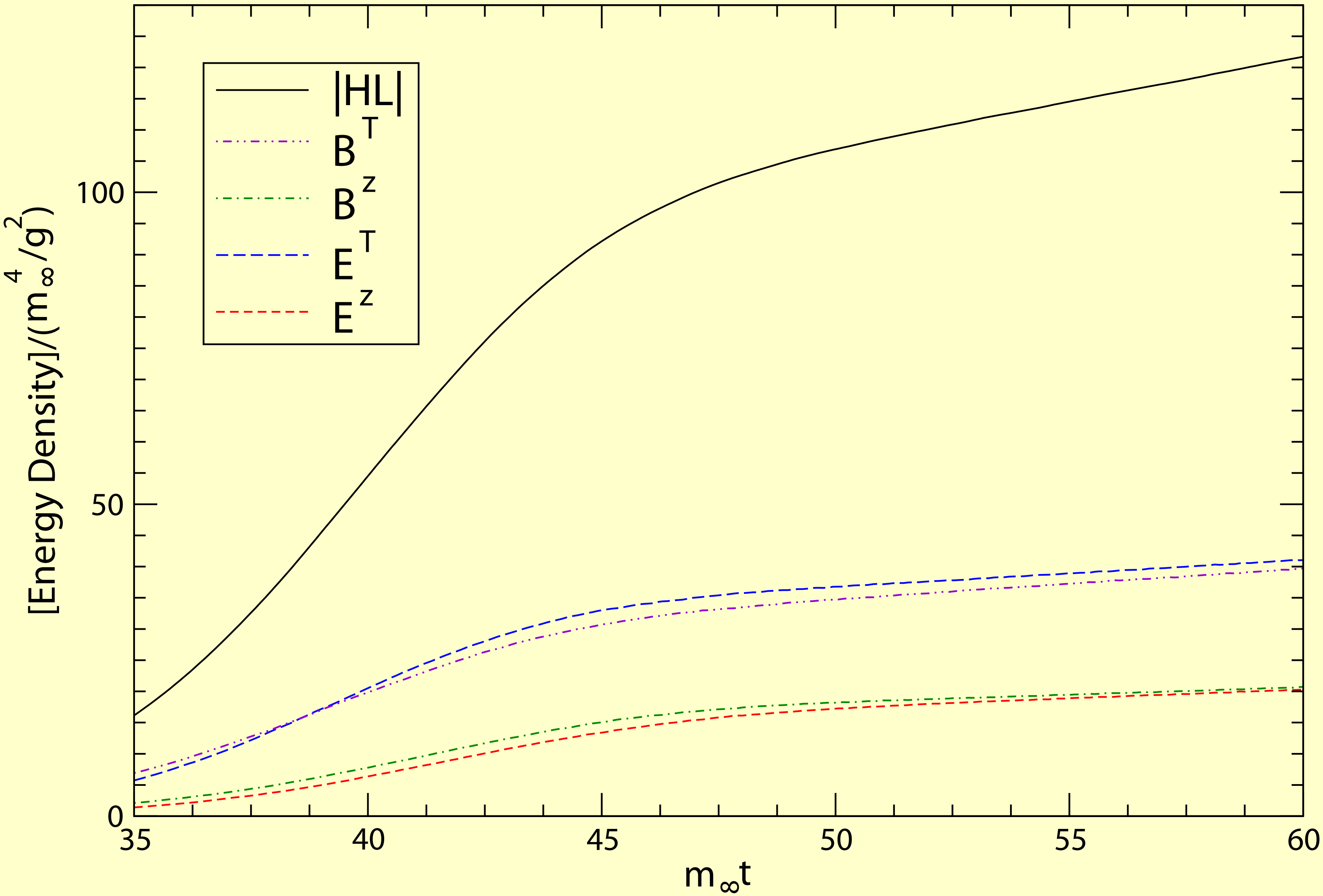
3s × 3v Lattice results



A. Rebhan, P. Romatschke, and MS, hep-ph/0505261.

See also: P. Arnold, G. Moore, L. Yaffe, hep-ph/0505212.

3s × 3v Lattice results contd.



A. Rebhan, P. Romatschke, and MS, hep-ph/0505261.

See also: P. Arnold, G. Moore, L. Yaffe, hep-ph/0505212.

Conclusions

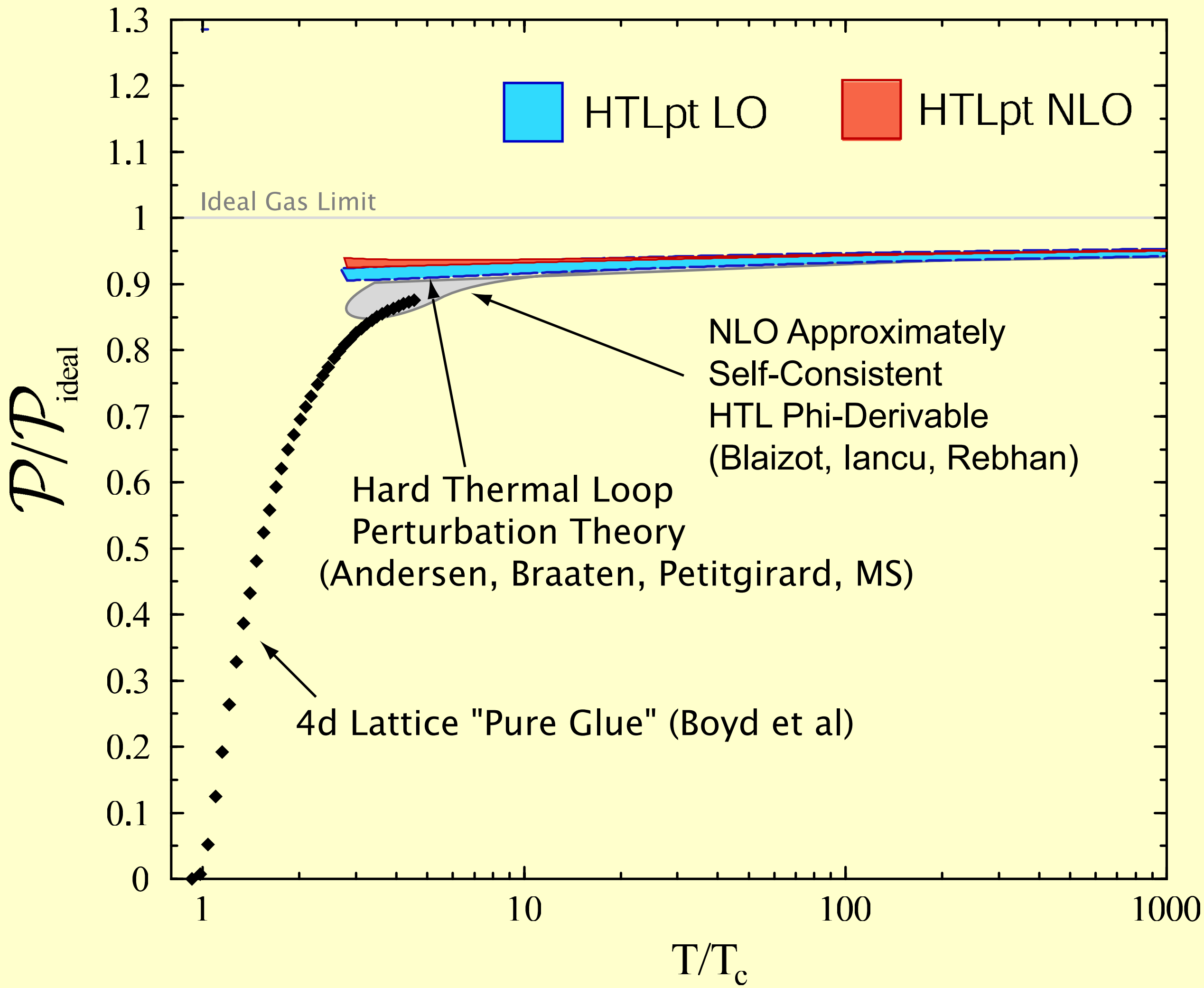
- Anisotropic plasmas are qualitatively different than isotropic ones. An entirely new phenomena associated with unstable modes appears.
- In 1s$\times$3v soft unstable modes grow non-perturbatively large until their energy is on the order of the energy in the hard particles (and the HL approximation breaks down at long times).
- 3s$\times$3v lattice simulations show, however, that soft unstable modes grow only to the “nonabelian” scale and then become power-law.
- Method less efficient due to the reduced field amplitudes but it means that the system stays within the bounds of the HL effective theory and therefore we can study the generated soft field and its affects on the hard particles in a systematic way!
- In the next talk by A. Dmitru we will hear about impressive attempts to include the backreaction of the hard particles using 1s $\times$ 3v nonabelian ”color particle-in-cell” (CPIC) simulations.

To do list

- Study effect of linear growth phase:
  - What is the size of the field correlations as a function of time?
  - Does this behavior change in going from SU(2) to SU(3)?
  - What is the effect on hard particles flying in this background?
  - Is it similar to the mechanism of ”collisionless magnetic reconnection” in Abelian plasmas?
  - Can the properties of this phase be determined by a linearization around the 1+1 solutions?
- Include expansion dynamically
- Inhomogeneous systems
- NLO hard loops ... unsolved in isotropic case
- 1s $\times$ 3v CPIC simulations including elastic collisions
- 3s $\times$ 3v CPIC simulations including elastic and inelastic collisions

The Big Question

- How relevant is this heavy-ion collisions?



More Questions . . .

- Are there distinctive signatures of plasma instabilities and/or their ”late-time” consequences?
  - v2 fluctuations?
  - enhanced energy loss/broadening??
  - rapidity dependence of photons/dielectrons???
  - photon bremsstrahlung due to isotropization of hard particles????
- Are large background fields enough to provide the enhanced cross sections deemed necessary for the creation of the ’sQGP’?
- Does this mean that parton cascade codes need to be rewritten to include realistic mean-field dynamics?