Isotropization vs thermalization

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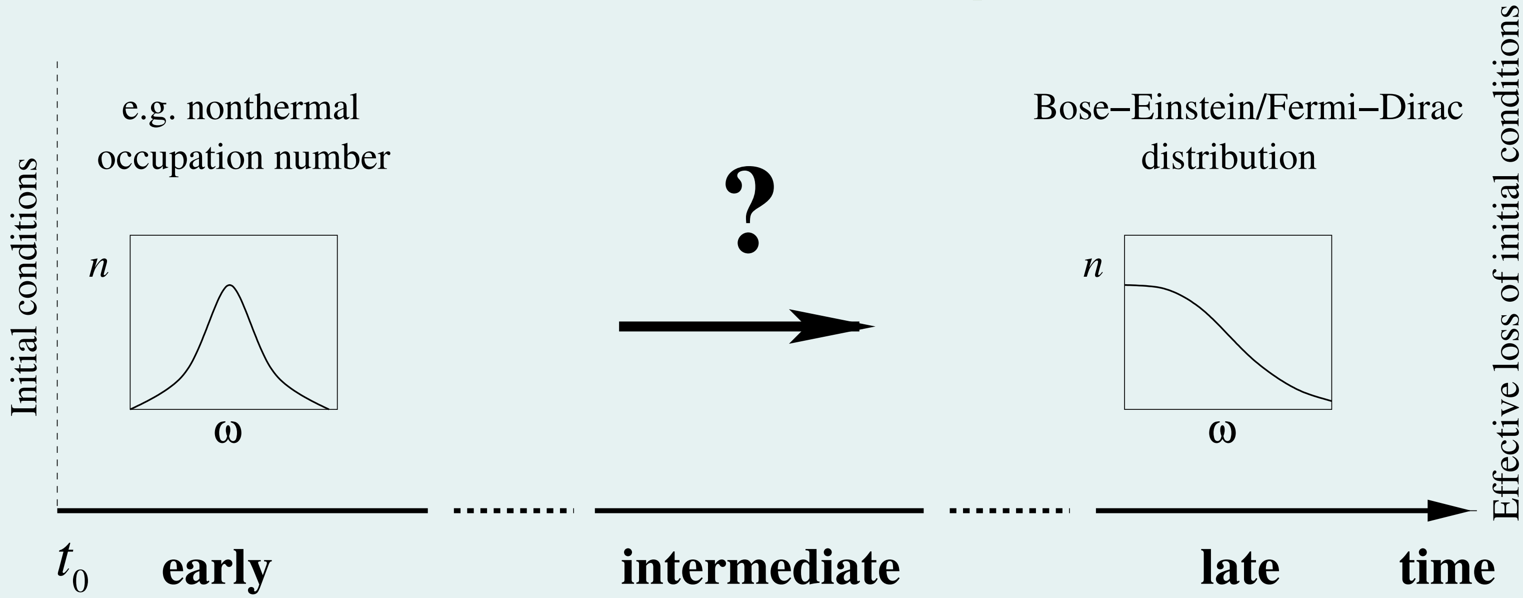
Outline

2PI equations of motion for a scalar theory
Weakly coupled theory
Anisotropic initial condition
What determines the isotropization rate?
On what time scale does chemical equilibrium reached?
Does the Boltzmann equation give a proper estimate?

Solve an initial value problem



Initial conditions: *Gaussian fluctuations*

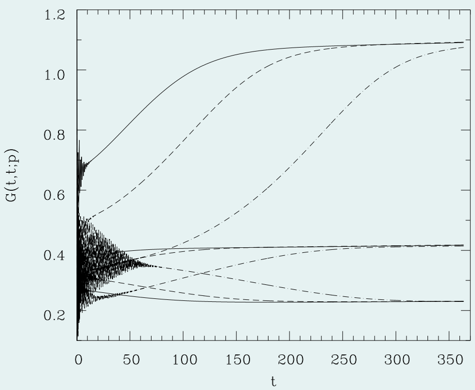
From the 2PI equations of motion:

effective loss of initial information

`Cox, Berges 2000`

Comparison to classical evolution:

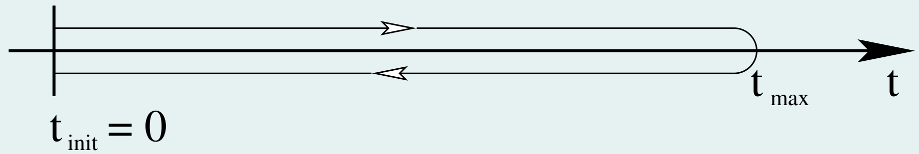
`Aarts, Berges 2002`



The 2PI effective action

Cornwall, Jackiw, Tomboulis 1974 ; Calzetta, Hu 1988
Ivanov, Knoll, Voskresensky 1988 ;

Define path integral along the *closed time path* contour



$$Z\left[J,K\right] = \int \mathcal{D}\varphi_c \exp\Big(i\mathcal{S}[\varphi] + iJ_a\varphi_a + \tfrac{i}{2}\varphi_a K_{ab}\varphi_b\Big) \ ,$$

$$W\left[J,K\right] = -i\log\left(Z\left[J,K\right]\right)$$

$$\delta W[J,K]/\delta J = \phi \quad \delta W[J,K]/\delta K = (\phi^2 - G)/2$$

$$\Gamma\left[\phi,G\right] = W\left[J,K\right] - J_a\phi_a - \tfrac{1}{2}K_{ab}\left[G_{ab} + \phi_a\phi_b\right]$$

2PI equations of motion

(a) $\frac{\delta\Gamma[\phi,G]}{\delta\phi_a(x)} = -J_a(x) - \int_{\mathcal{C}} \mathrm{d}^4y \left[K_{ab}(x,y)\,\phi_b(y)\right] \overset{!}{=} 0$

(b) $\frac{\delta\Gamma[\phi,G]}{\delta G_{ab}(x,y)} = -\frac{1}{2}K_{ab}(x,y) \overset{!}{=} 0 \quad \rightarrow \quad G_{ab}(x,y;\phi) = \langle \mathcal{T}_{\mathcal{C}}\hat{\varphi}(x)\hat{\varphi}(y)\rangle_c$

Decomposition:

$$\Gamma_b[\phi,G] = S[\phi] + \frac{i}{2}\mathrm{tr}_{\mathcal{C}}\left[\log\left[G^{-1}\right]\right] + \frac{i}{2}\mathrm{tr}_{\mathcal{C}}\left[G_0^{-1}G\right] + \Gamma_{\mathrm{int}}[\phi,G] + \mathrm{const}$$

$$\Gamma_f[\psi,D] = S[\psi] - i\mathrm{tr}_{\mathcal{C}}\left[\log\left[D^{-1}\right]\right] - i\mathrm{tr}_{\mathcal{C}}\left[D_0^{-1}D\right] + \Gamma_{\mathrm{int}}[\psi,D] + \mathrm{const}$$

With $\Sigma_f(x,y) \equiv 2i\frac{\delta\Gamma_{\mathrm{int}}[G]}{\delta G(y,x)} \qquad \Sigma_s(x,y) \equiv -i\frac{\delta\Gamma_{\mathrm{int}}[D]}{\delta D(y,x)}$

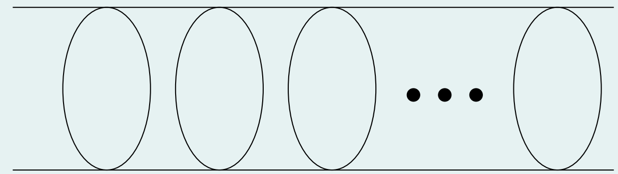
$$(\partial_x^2 + m^2)G_{ab}(x,y) = \int_{\mathcal{C}} \mathrm{d}^4z\,\Sigma_{ab}(x,z;G,D)\,G_{bc}(z,y) + \delta_{\mathcal{C}}(x,y)\delta_{ab},$$

$$(\not{\partial}_x + im_f)D_{ij}(x,y) = \int_{\mathcal{C}} \mathrm{d}^4z\,\Sigma_{ik}(x,z;G,D)D_{kj}(z,y) + \delta^4_{\mathcal{C}}(x,y)\delta_{ij}$$

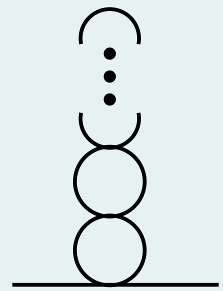
$\leftarrow$ equivalent to Kadanoff–Baym equations

2PI: Ladder ressummation

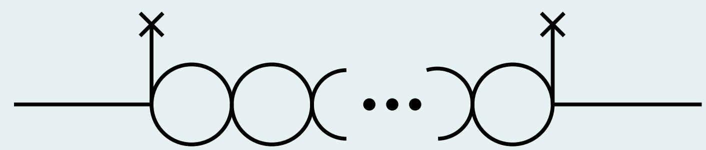
Propagator in the symmetric phase:



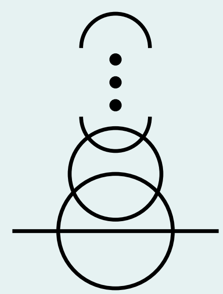
For $n > 2$
or in the non-symmetric case:
n-point defined from the
1PI effective action:
All 3 channels resummed



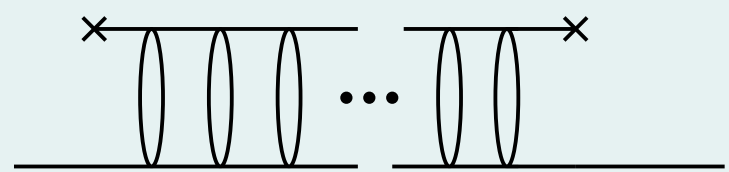
2PI, LO



2PI resummed 1PI

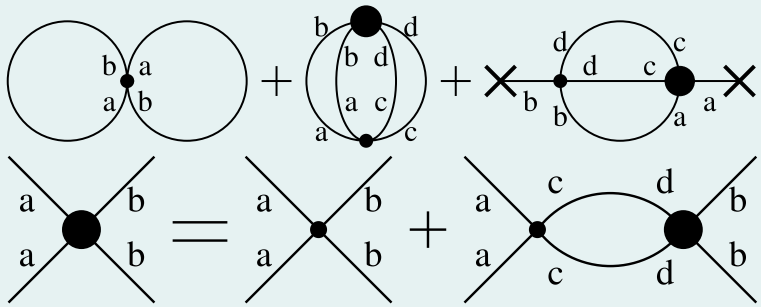


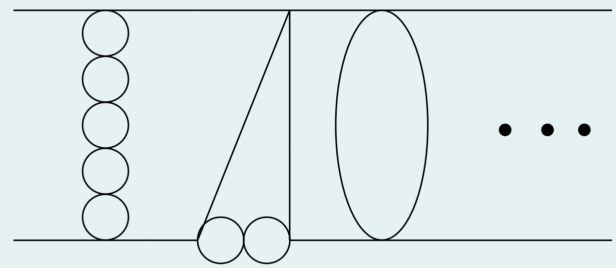
2PI, NLO



2PI resummed 1PI

2PI $1/N$ resummation





Berges 2000 ; Aarts, Ahrensmeyer, Berges, Baier, Serreau 2002

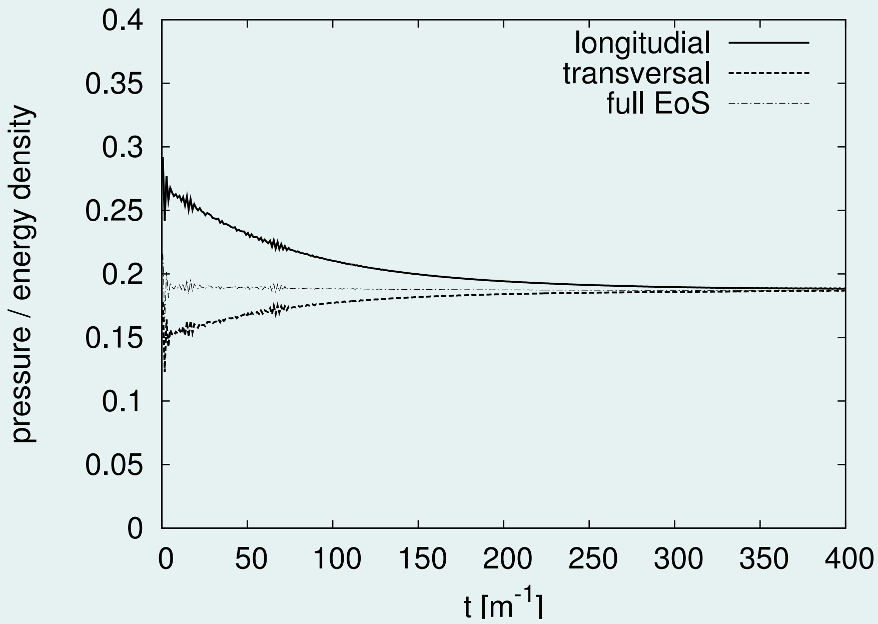
- Shear viscosity from correlators of the energy momentum tensor
  Aarts, Resco 2004
- Nontrivial (not mean field) critical exponents
  Alford, Berges, Cheyne 2004
- Parametric resonance
  Berges, Serreau 2002
- Tachyonic instability
  Arrizabalaga, Smit, Tranberg 2004

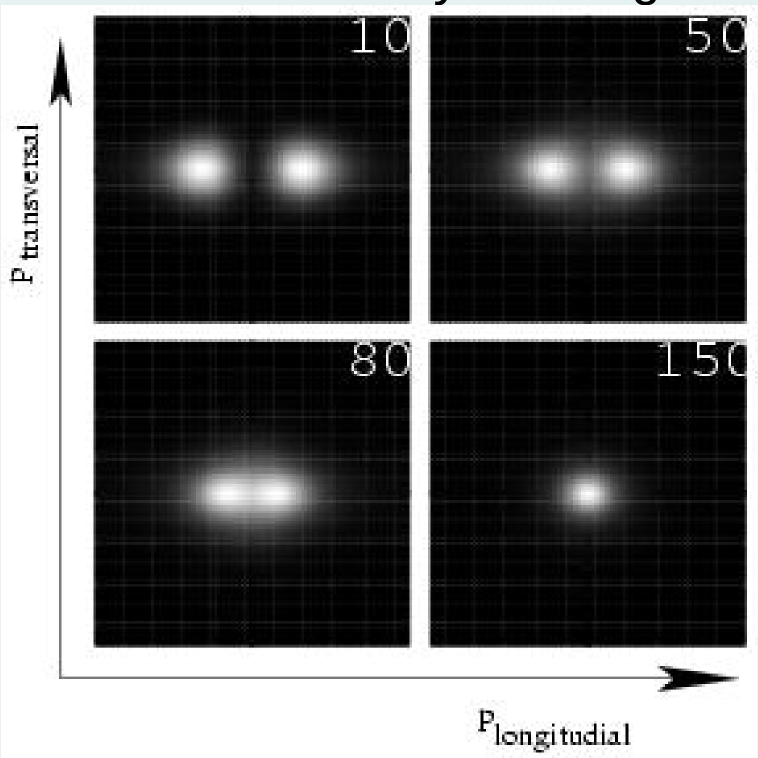
Isotropization

Berges, Borsanyi, Wetterich 2005

Ideal hydrodynamics requires
the isotropization of the ***pressure***

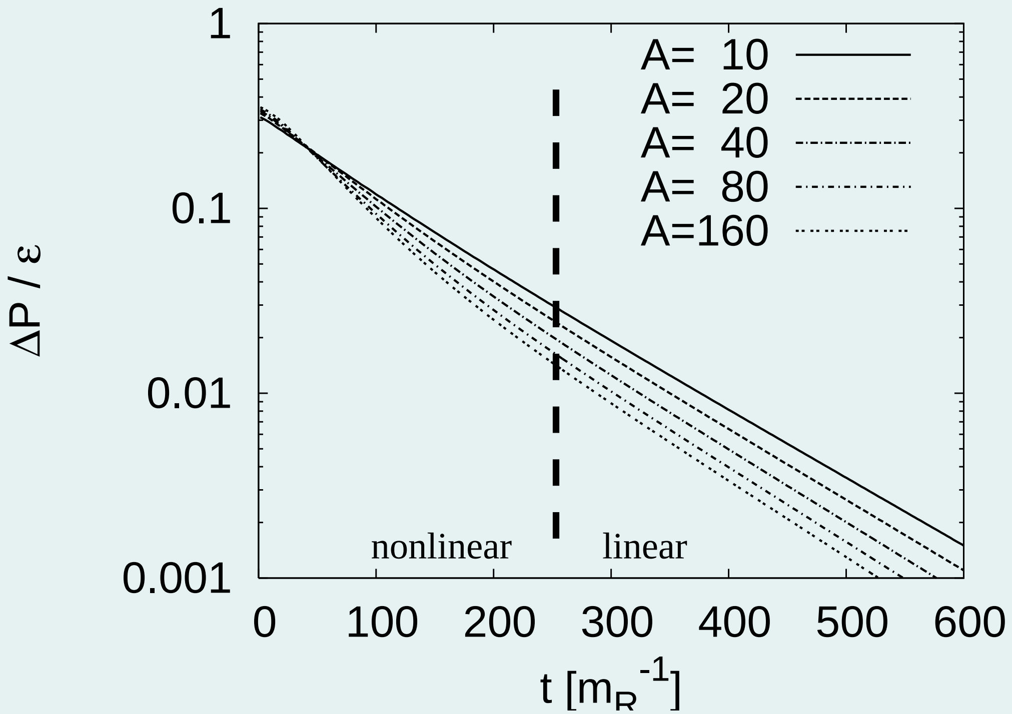
64^3 lattice,
100Gb memory integral

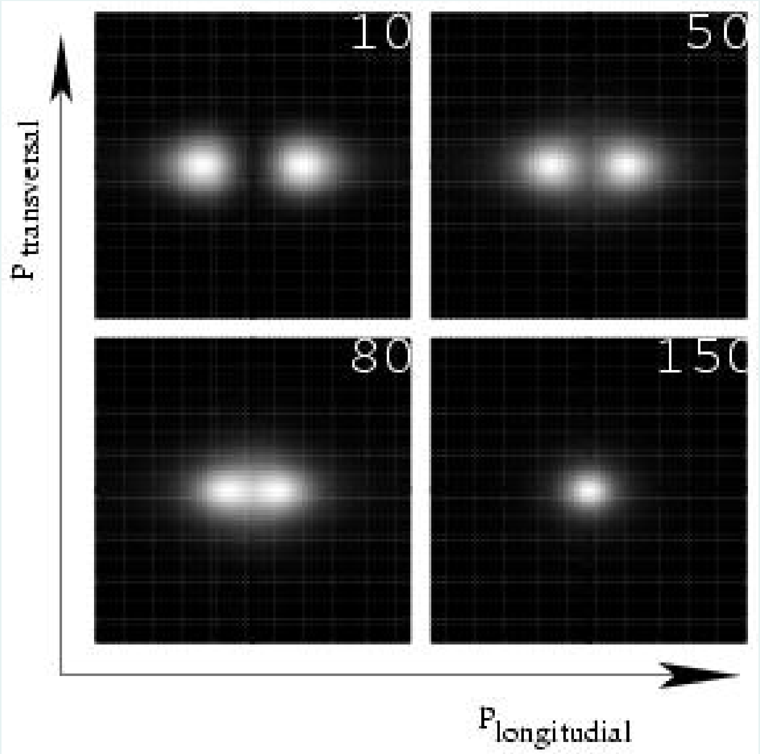




Does isotropization have a rate?

Equal energy densities:



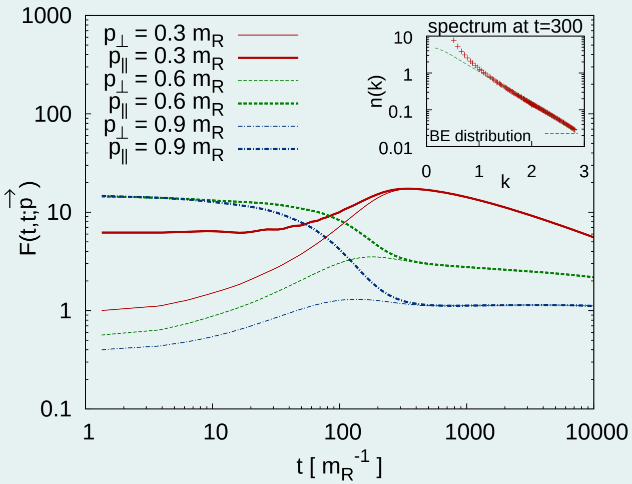


Unique (thermal) rate after distribution “looks” isotropic

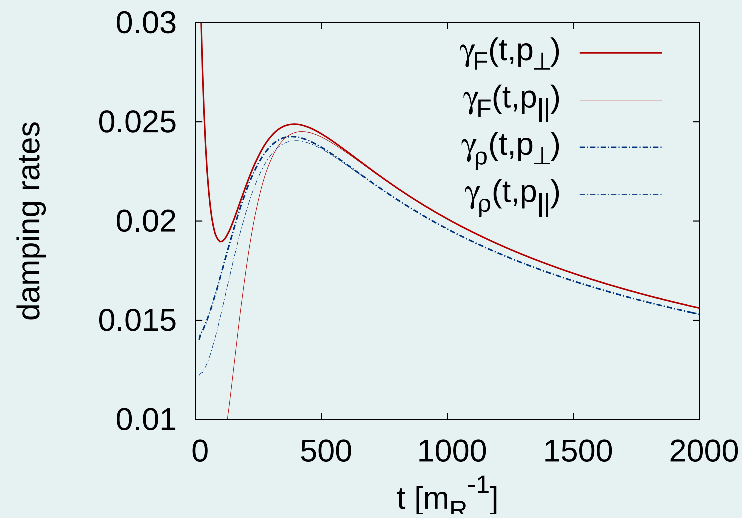
Isotropization of the propagator

$$G(x,y) = F(x,y) - \frac{i}{2}\rho(x,y)\,\mathrm{sgn}_{\mathbb{C}}(x^0 - y^0)$$

Amplitude:



Half width:



- Rate (in the linear regime) $\sim$ pressure isotropization rate, ~ 0.01
- this is **not** the width of the spectral function
- Thermalization longer than isotropization

Wigner transformed observables

$$\langle \mathcal{T}_{\mathcal{C}} \hat{\varphi}(x) \hat{\varphi}(y) \rangle_c \equiv G(x,y) = F(x,y) - \frac{i}{2} \rho(x,y) \, \mathrm{sgn}_{\mathbb{C}}(x^0 - y^0) \, ,$$

$$F(t;\omega,\vec{p}) \quad = \quad \int \mathrm{d}s \, e^{i\omega s} F(t+s/2, t-s/2; \vec{p}) \, ,$$

$$\varrho(t;\omega;\vec{p}) \quad = \quad -i \int \mathrm{d}s \, e^{i\omega s} \rho(t+s/2, t-s/2; \vec{p})$$

For isotropization:

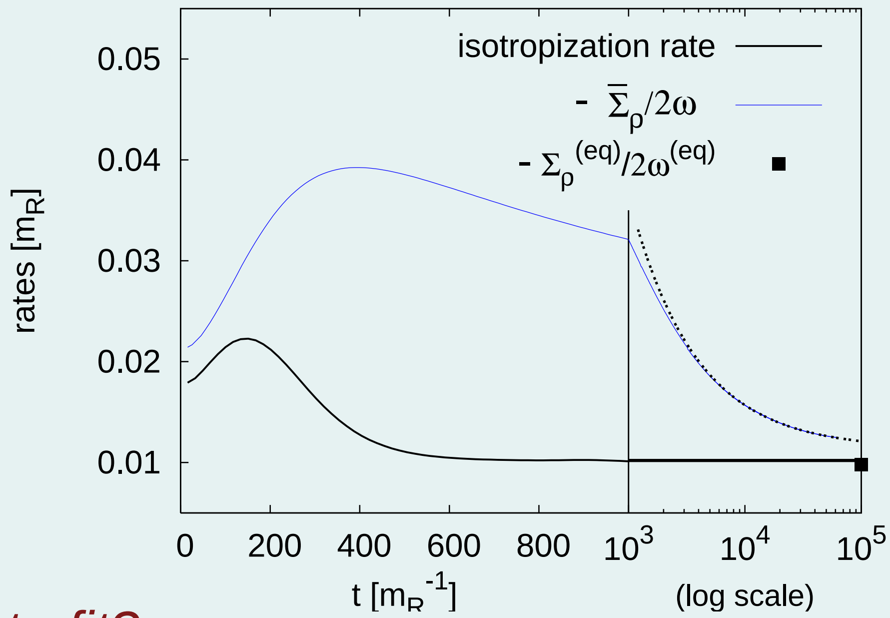
$$\Delta F(t;\omega,\bar{q}) \equiv F(t;\omega,p_\perp = 0, p_\parallel = \bar{q}) - F(t;\omega,p_\perp = \bar{q}, p_\parallel = 0) \, ,$$

For equilibration:

$$\overline{F}(t;\omega,\bar{q}) \equiv \frac{1}{2} \left[F(t;\omega,p_\perp = 0, p_\parallel = \bar{q}) + F(t;\omega,p_\perp = \bar{q}, p_\parallel = 0) \right]$$

Damping rate vs isotropization

isotropization rate:
$$\frac{1}{\Delta F(t)}\frac{d\Delta F(t)}{dt}$$



Does the plasmon rate fit?

$$\gamma(\vec{p})_{\blacksquare} = -\frac{\Sigma_\varrho^{(eq)}(\omega_p^{(eq)},\vec{p})}{2\omega_p^{(eq)}}\,.\qquad \gamma^{(noneq)}(t;\vec{p}) = -\frac{\Sigma_\varrho^{(noneq)}(t;\omega_p^{(eq)},\vec{p})}{2\omega_p^{(eq)}}\,.$$

Gradient expansion

$$\left[\partial_{t_1}^2 + \vec{p}^{\,2} + M^2(t_1)\right] F(t_1, t_2; \vec{p}) = -\int_0^{t_1} \mathrm{d}t'\, \Sigma_\rho(t_1, t'; \vec{p}) F(t', t_2; \vec{p}) + \int_0^{t_2} \mathrm{d}t'\, \Sigma_F(t_1, t'; \vec{p}) \rho(t', t_2; \vec{p}) \,,$$

$$\left[\partial_{t_1}^2 + \vec{p}^{\,2} + M^2(t_1)\right] \rho(t_1, t_2; \vec{p}) = -\int_{t_2}^{t_1} \mathrm{d}t'\, \Sigma_\rho(t_1, t'; \vec{p}) \rho(t', t_2; \vec{p})$$

$$\downarrow$$

$$2\omega \frac{\partial}{\partial t} F(t; \omega, \vec{p}) = \Sigma_\varrho(t; \omega, \vec{p}) F(t; \omega, \vec{p}) - \Sigma_F(t; \omega, \vec{p}) \varrho(t; \omega, \vec{p}) \,,$$

$$2\omega \frac{\partial}{\partial t} \varrho(t; \omega, \vec{p}) = 0$$

Linearized equations

In the equations of motion:

$$\frac{\delta G(p)}{dt} \sim \delta[\Sigma(p) \cdot G(p)] = \delta\Sigma(p) \cdot G^{\mathrm{eq}}(p) + \underbrace{\Sigma^{\mathrm{eq}}(p) \cdot \delta G(p)}_{\text{Damping rate}} = \mathcal{S}_{pr} \delta G_r$$

1st term: off-diagonal in δG:

$$\delta\Sigma(p) \cdot G^{\mathrm{eq}}(p) \sim \int_{qk} G^{\mathrm{eq}}(q) G^{\mathrm{eq}}(k) \delta G(p-k-q) \cdot G^{\mathrm{eq}}(p) = \mathcal{S}_{pr}^{\mathrm{offdiagonal}} \delta G_r$$

For different phenomena (*isotropization*, *kinetic equilibration*) different eigenvectors/eigenvalues are relevant.

Due to the off-diagonal elements small eigenvalues appear.
Final thermalization: **smallest eigenvalue**

Chemical equilibration

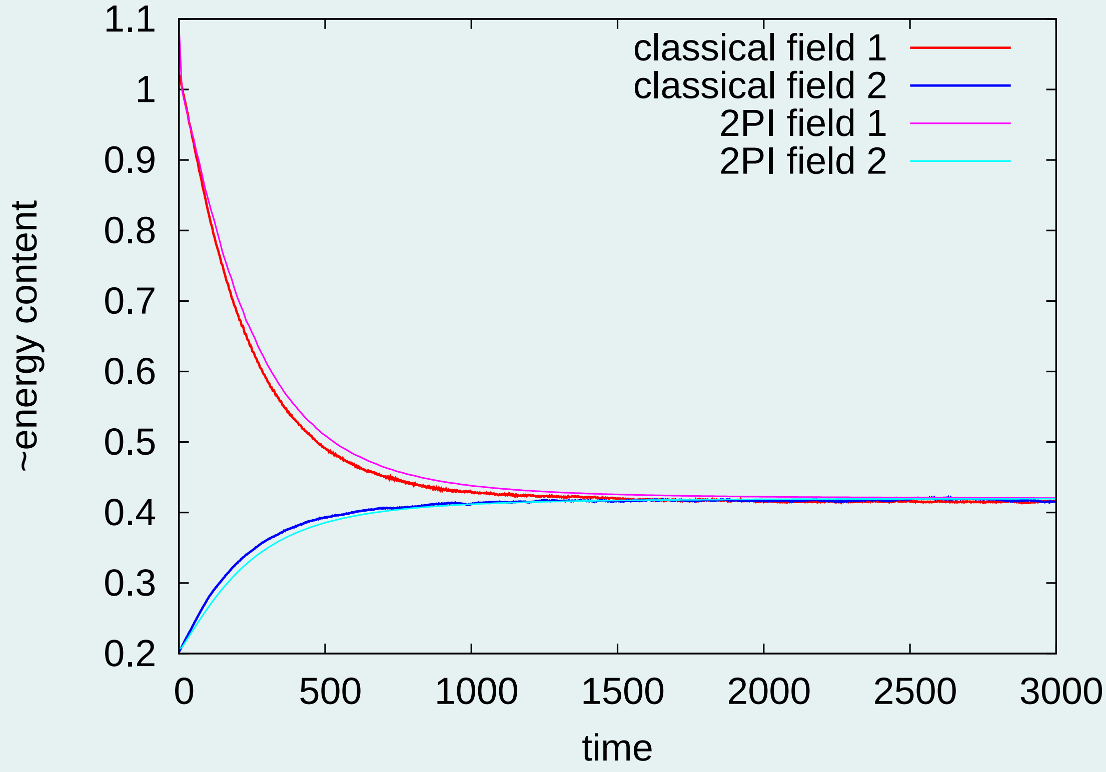
- Toy model: N scalar fields
- Φ_a, $a = 1$ started from an anisotropic state
- Φ_a, $a \neq 1$ started from vacuum
- solve 2PI equations of motion in the *symmetric phase*

Prototype of chemical equilibration:
fields of equal mass should have equal energy

Possible conclusions to heavy ion physics?

- Not the numbers
- Understanding the *time scales* and nonequilibrium phenomena

Equalization of energy/particle content



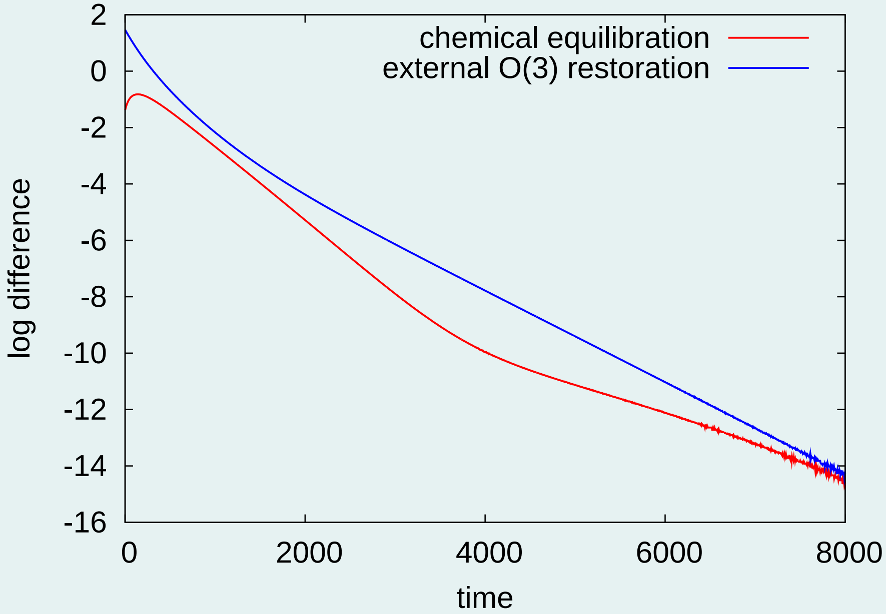
Quantum 2PI results show good agreement to *classical* statistical field theory for high occupation numbers ~ 10.

Isotropization vs particle transfer between field species

Propagators:

chemical: $\Delta_{\rm ch} = F_a(\vec{p}) - F_b(\vec{p}) \qquad a \neq b$

anisotropy: $\Delta F = F_a(\vec{p}) - F_a(\vec{k}) \qquad \vec{p} \perp \vec{k}$

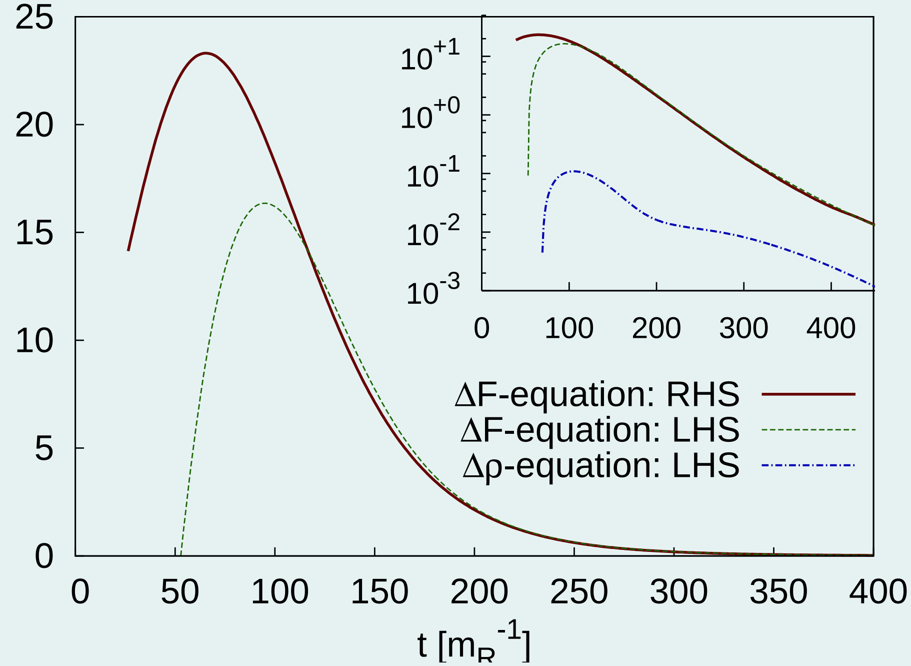


"Chemical" equilibration rate ≈ isotropization rate

Testing the gradient expansion

From the 2PI equations for (F=Re G, ϱ=Im G)
After Wigner transformation and LO gradient expansion:

$$
\begin{aligned}
2\omega\frac{\partial}{\partial t}\Delta F(t;\omega,p) &= \Delta\left[\Sigma_\varrho(t;\omega,p)F(t;\omega,p)-\Sigma_F(t;\omega,p)\varrho(t;\omega,p)\right], \\
2\omega\frac{\partial}{\partial t}\Delta\varrho(t;\omega,p) &= 0\,.
\end{aligned}
$$

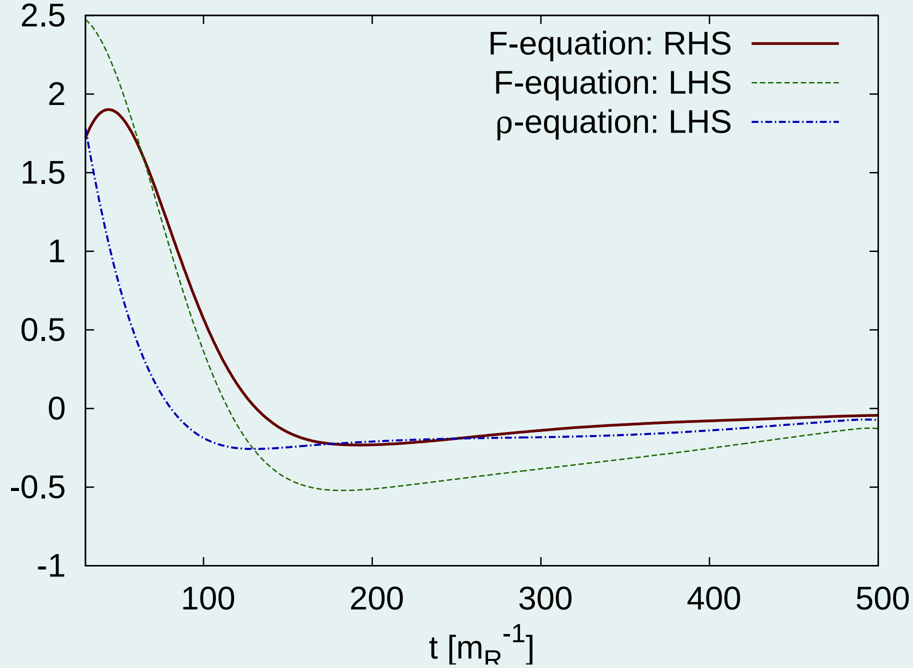


Gradient expansion and thermalization

From the 2PI equations for (F=Re G, ϱ=Im G)
After Wigner transformation and LO gradient expansion:

$$2\omega \frac{\partial}{\partial t}\overline{F(t;\omega,p)} = \overline{[\Sigma_\varrho(t;\omega,p)F(t;\omega,p) - \Sigma_F(t;\omega,p)\varrho(t;\omega,p)]},$$

$$2\omega \frac{\partial}{\partial t}\overline{\varrho(t;\omega,p)} = 0\,.$$



Summary

Kinetic Equilibration much longer than Isotropization

even without plasma instabilities.

Gradient expansion vs. 2PI

- Linearized dynamics is determined by the analytical structure of the equilibrium propagator: cuts (off-diagonal) and poles (diagonal).

- For isotropization gradient expansion works well
unfortunately not from the earliest nonlinear part.
Gradient expansion starts working from the time scale it is about to describe

Time scales:
– microscopical dynamics $\longrightarrow$ *prethermalization*
– elastic scattering rate $\longrightarrow$ *isotropization*
– global particle number changing processes $\longrightarrow$ *final equilibration*