Exercise Sheet 3 (String Theory, LVA Nr. 136.005) due 28th of march

Exercise 5: String equations of motion in conformal gauge

Derive the equations of motion of the fields X_μ from the Polyakov action using light-cone coordinates on the world-sheet. You may assume that that the space-time metric is trivial, $G_{\mu\nu} = \eta_{\mu\nu}$, boundary terms vanish, and that the metric on the worldsheet is in conformal gauge.

Exercise 6: Open Strings

As discussed in the lecturem, the general solution to the equations of motion for closed strings is of the form $X = X_L + X_R$, where

$$X_L = \frac{1}{2}x_L^\mu + \frac{1}{4\pi T}p^\mu\sigma^+ + \frac{i}{\sqrt{4\pi T}}\sum_{n=-\infty,n\neq 0}^{n=\infty}\frac{1}{n}\alpha_n^\mu e^{-in\sigma^+} \tag{1}$$

$$X_R = \frac{1}{2}x_R^\mu + \frac{1}{4\pi T}p^\mu\sigma^- + \frac{i}{\sqrt{4\pi T}}\sum_{n=-\infty,n\neq 0}^{n=\infty}\frac{1}{n}\tilde{\alpha}_n^\mu e^{-in\sigma^-}\,. \tag{2}$$

With

$$\sigma^+ = \tau + \sigma \tag{3}$$

$$\sigma^- = \tau - \sigma \tag{4}$$

we have $X(\tau,\sigma) = X(\tau,\sigma+2\pi)$. Show how to construct a solution describing open strings with von Neumann boundary conditions, $\partial_\sigma X^\mu|_{\sigma=0,\pi} = 0$, from the solution above. Hint: given a periodic function in σ on the domain $0..2\pi$ which is also symmetric under $\sigma \to -\sigma$, we can find a solution respecting Von Neumann boundary conditions by restricting to the interval $0..\pi$.