Exercise Sheet 3 (String Theory, LVA Nr. 136.005) due 18th of April

Exercise 7: Poisson brackets

Show that the Poisson brackes of the coordinates x^μ with themselves and the oscillators α_n^μ and $\tilde{\alpha}_n^\mu$ vanish by appropriately Fourier transforming the canonical commutation relations

$$\{\Pi^\mu(\sigma), X^\nu(\sigma')\}_{PB} = \delta(\sigma - \sigma')\eta^{\mu\nu}, \quad \{X^\mu(\sigma), X^\nu(\sigma')\}_{PB} = 0, \quad \{\Pi^\mu(\sigma), \Pi^\nu(\sigma')\}_{PB} = 0 \quad (1)$$

Hint: try to consider only those expression that pick out the right terms for you !

Exercise 7: The Hamiltonian in terms of momentum and oscillators

Use the mode expansions of $X = X_L + X_R$ to show that the Hamiltonian, which in the two-dimensional field theory can be written as

$$H = \int_0^{2\pi} \mathrm{d}\sigma (\Pi^\mu \dot{X}_\mu - \mathcal{L}) \, , \quad (2)$$

is given by

$$H = \frac{-p^\mu p_\mu}{4\pi T} - \frac{1}{2} \sum_{n=-\infty, n\neq o}^{\infty} (\alpha_n \cdot \alpha_{-n} + \tilde{\alpha}_n \cdot \tilde{\alpha}_{-n}) = L_0 + \tilde{L}_0 \quad (3)$$

Hint: The Lagrangian is $\mathcal{L} = -\frac{T}{2}(\partial_\tau X^\mu \partial_\tau X_\mu - \partial_\sigma X^\mu \partial_\sigma X_\mu)$, so that $\Pi^\mu = -T\dot{X}^\mu$. Furthermore, you can compute the Hamiltonian at fixed $\tau = 0$.