Exercise Sheet 6 (String Theory, LVA Nr. 136.005) due 9th of may

Exercise 10: BRST and Yang-Mills Theory

Consider the FP Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \frac{1}{2\xi} (\partial^\mu A^a_\mu)^2 - \bar{c}^a \partial^\mu D^{ac}_\mu c^c \,. \tag{1}$$

Here, the fields $c^a, \bar{c}^a$ are anticommuting scalars which transform in the adjoint representation of the gauge group. Correspondingly,

$$D^{ac}_\mu c^c = \left(\delta^{ac} \partial_\mu + g f^{abc} A^b_\mu\right) c^c \,. \tag{2}$$

- Show that the above Lagrangian is obtained by integrating out the auxiliary field B^a from

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - \frac{\xi}{2} (B^a)^2 + B^a \partial^\mu A^a_\mu - \bar{c}^a \partial^\mu D^{ac}_\mu c^c \,. \tag{3}$$

- Show that the Lagrangian eq. (3) is invariant under the BRST transformation Q which acts on the fields as

$$\delta_Q A^a_\mu = \epsilon D^{ac}_\mu c^c \tag{4}$$

$$\delta_Q c^a = -g \frac{1}{2} \epsilon f^{abc} c^b c^c \tag{5}$$

$$\delta_Q \bar{c}^a = \epsilon B^a \tag{6}$$

$$\delta_Q B^a = 0 \tag{7}$$

  where ϵ is a Grassmann valued constant.

- Show that $Q^2 = 0$ by considering the action of this on any field