Exercise Sheet 8 (String Theory, LVA Nr. 136.005) due 23th of may

Exercise 12: (Co)-Homology

- a)

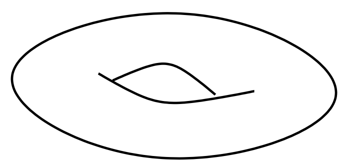
  Let $\mathbb{R}^2$ be a Hilbert space and take $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \in \mathbb{R}^2$. Consider the operator Q which acts on v as

$$Q\vec{v} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \vec{w}\,. \tag{1}$$

  - Check that $Q^2 = 0$.
  - Describe the space of closed and exact vectors.
  - Describe the cohomology

$$\frac{\mathcal{H}_{\text{closed}}}{\mathcal{H}_{\text{exact}}}\,. \tag{2}$$

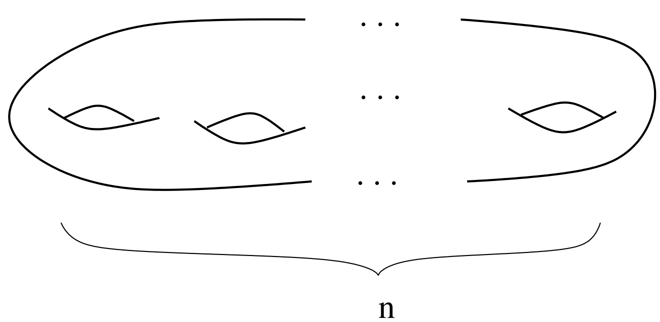
- b) Consider the torus.



The boundary operator ∂ maps each submanifold to its boundary. Can you see why $\partial^2 = 0$? Describe

$$\frac{\ker(\partial)}{\text{im}(\partial)}\,. \tag{3}$$

Can you see what happens when we replace the torus by a surface of genus n ?



Exercise 13: Grassmann variables

Consider the Grassmann variables θ_i, $\bar{\theta}_j$. Any of these anticommutes with any other, so in particular $\theta_i^2 = 0$ and $\bar{\theta}_i^2 = 0$ holds for any i. Integration is defined by

$$\int \mathrm{d}\theta_i = 0 \tag{4}$$

and

$$\int \mathrm{d}\theta_i \theta_i = 1 (\text{no summation}) \tag{5}$$

so that e.g.

$$\int \mathrm{d}\theta_1 \mathrm{d}\theta_2 \cdots \mathrm{d}\theta_n \theta_n \cdots \theta_2 \theta_1 = 1\,, \tag{6}$$

and

$$\int \mathrm{d}\bar{\theta}_1 \mathrm{d}\theta_1 \bar{\theta}_1 \theta_1 = -1\,. \tag{7}$$

Show that for a hermitian matrix B

$$\int \mathrm{d}\bar{\theta}_1 \mathrm{d}\theta_1 \mathrm{d}\bar{\theta}_2 \mathrm{d}\theta_2 \cdots \mathrm{d}\bar{\theta}_n \mathrm{d}\theta_n e^{-\bar{\theta}_i B_{ij} \theta_j} = \det(B)\,. \tag{8}$$