Instabilities in Relativistic Two-Component Superfluids

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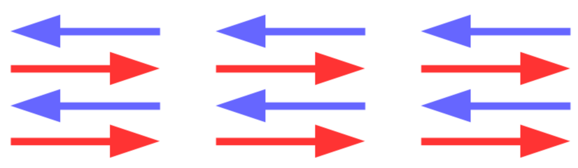
$\int dk\,\Pi$ Doktoratskolleg Particles and Interactions

1 Introduction

- **Two-component superfluids** can be realized by mixtures of two different species at sufficiently low temperatures. Examples are
  - ^{3}He- ^{4}He mixtures
  - Superfluid Bose-Fermi mixtures of ultra-cold atomic gases
  - Mixture of a neutron superfluid with a proton superconductor in the interior of **compact stars**
- A **counterflow** between fluids can be created experimentally (lab) or occurs necessarily (**compact stars**).



- Instabilities like **Landau's critical velocity** or the **two-stream instability** might be important for astrophysical phenomena like **pulsar glitches**. These instabilities can serve as a **trigger** for the collective unpinning of the neutron-superfluid vortices from the neutron lattice, which can be viewed as a second (normal) fluid [1].

2 Microscopic Model

- Two coupled, complex scalar fields with selfinteraction and two types of interspecies coupling: derivative/entrainment coupling and non-entrainment coupling [2, 3].

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_I,\\ \mathcal{L}_i &= \partial_\mu\varphi_i\partial^\mu\varphi_i^* - m^2|\varphi_i|^2 - \lambda|\varphi_i|^4,\\ \mathcal{L}_I &= 2h|\varphi_1|^2|\varphi_2|^2 - g\varphi_1\varphi_2\partial_\mu\varphi_1^*\partial^\mu\varphi_2^*.\end{aligned}$$

- Superflows $\vec{v}_i$ and the chemical potentials μ_i are both related to the phases of the condensates (expectation values of the fields, $\langle\varphi_i\rangle = \rho_i e^{i\psi_i}$) [4]:

$$\mu_i = \partial_0\psi_i, \quad \vec{v}_i = \frac{\vec{\nabla}\psi_i}{\mu_i}.$$

- **Two-component superfluid:** spontaneous symmetry breaking with pattern $U(1)\times U(1)\rightarrow \mathbf{1}$ leads to **two Goldstone modes**

References

[1] N. Andersson, G.L. Comer, R. Prix, Phys.Rev.Lett. **90**, 091101 (2002), arXiv: astro-ph/0210486

[2] S. Stetina, PhD thesis, arXiv: 1502.00122 (2015)

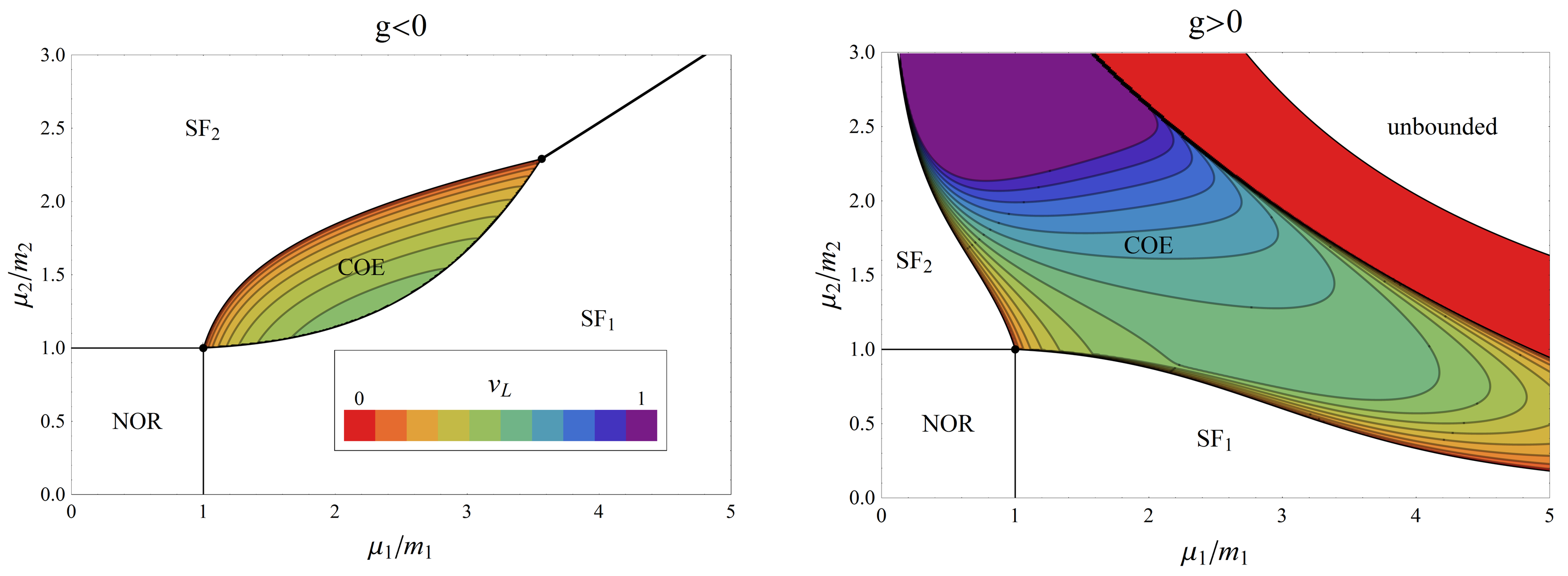
[3] A. Haber, A. Schmitt, S. Stetina in preparation

[4] A. Schmitt, Lect.Notes Phys. **888**, (2015), arXiv: 1404.1284

3 Phase Diagrams & Landau's Critical Velocity

Landau's critical velocity v_L: **energy of Goldstone mode** becomes **negative**

Phase diagrams for positive and negative values of the entrainment coupling g in the plane of the chemical potentials. The colors represent Landau's critical velocity in units of $c = 1$. For simplicity, $h = 0$.



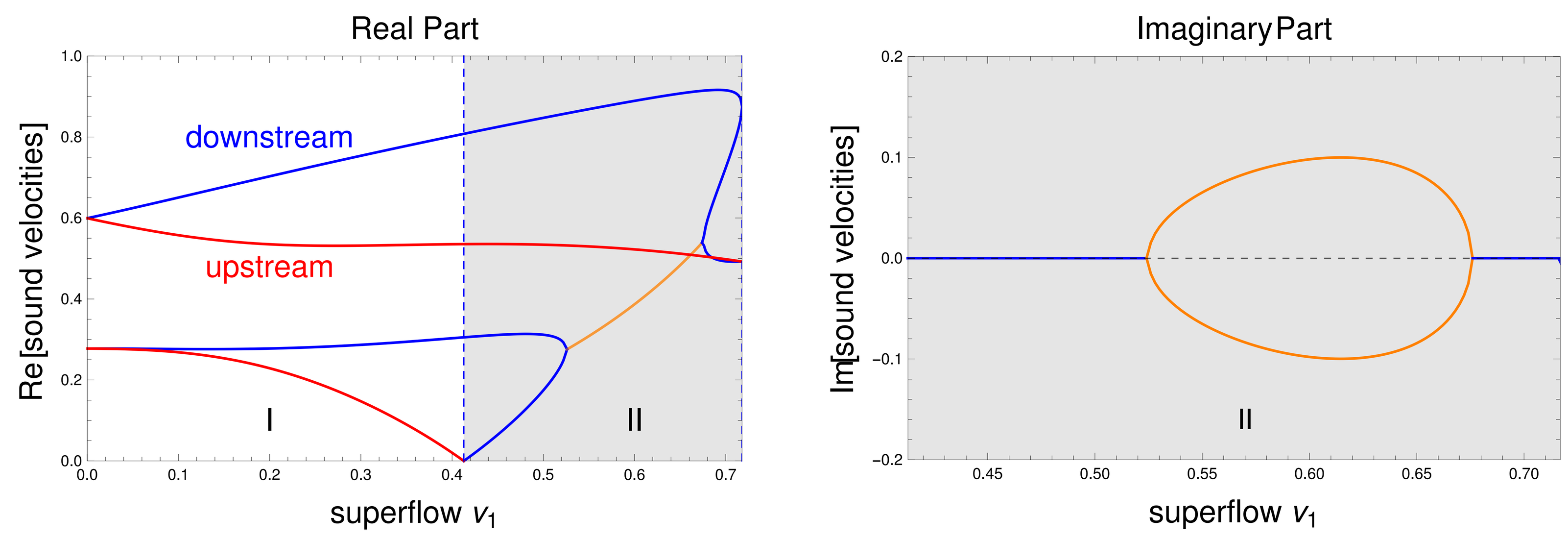
Four different phases, separated by second order phase transitions exept for first order transition between $\mathrm{SF}_1/\mathrm{SF}_2$:

- NOR: **normal phase**
- $\mathrm{SF}_{1/2}$: **superfluid$_{1/2}$ phase**
- COE: **two-component superfluid** (coexistence phase)
  For $g < 0$, the entrainment term acts as an energy penalty $\Rightarrow$ size of COE phase reduced. For $g > 0$: energy gain $\Rightarrow$ COE phase enlarged.

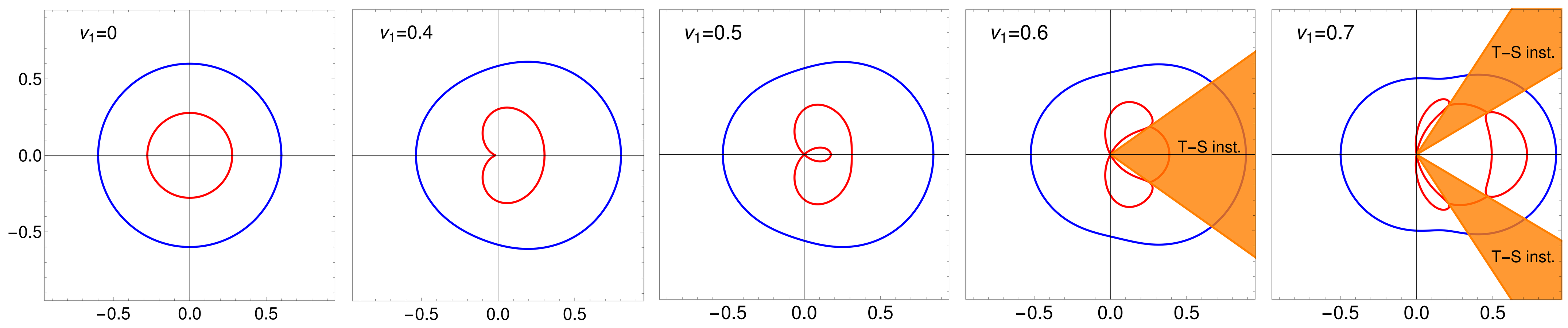
4 Sound Modes & Instabilities

- At $T = 0$, the sound modes are given by the slope of the Goldstone modes or can equivalently be calculated using relativistic, linearized hydrodynamics.

Real and imaginary parts of the sound velocities for $g < 0$ in up- and downstream direction.



- Two regions:
  - I: Stable regime, two sound modes in upstream direction (superflow antiparallel to sound wave) and two in downstream direction
  - II: Energetically unstable regime (shaded region), one upstream mode flips to downstream (analogy: r-mode instability in rotating star). **Orange line**: two solutions merge and become complex (see right panel) $\Rightarrow$ onset of **two-stream instability**
- **Polar plots** show sound modes and development of instabilities for all angles between the superflow $\vec{v}_1$ and the wave vector $\vec{k}$



- **Two-stream instability** starts to develop in downstream direction "**after**" Landau's critical velocity

5 Summary

- **Two-component superfluids** with non-zero superflow feature several instabilities, which can be calculated by analyzing the sound modes.
- They might serve as a **trigger for pulsar glitches**.
- It is found that the **two-stream instability** always takes place in an **energetically unstable** regime (after Landau's critical velocity). This is different for a two-component **normal fluid** (not shown here).

Contact & Funding

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