1 INTRODUCTION

- Two-component superfluids can be realized by mixtures of two different species at sufficiently low temperatures. Examples are
  - $^3$He-$^4$He mixtures
  - Superfluid Bose-Fermi mixtures of ultra-cold atomic gases
  - Mixture of a neutron superfluid with a proton superconductor in the interior of compact stars

- A counterflow between fluids can be created experimentally (lab) or occurs necessarily (compact stars).

- Instabilities like Landau’s critical velocity or the two-stream instability might be important for astrophysical phenomena like pulsar glitches. These instabilities can serve as a trigger for the collective unpinning of the neutron-superfluid vortices from the neutron lattice, which can be viewed as a second (normal) fluid [1].

2 MICROSCOPIC MODEL

- Two coupled, complex scalar fields with selfinteraction and two types of interspecies coupling: derivative/entrainment coupling and non-entrainment coupling [2, 3].
  \[ \mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 , \]
  \[ \mathcal{L}_1 = \partial_\mu \varphi_i \partial^\mu \varphi_i^* - m^2 |\varphi_i|^2 - \lambda |\varphi_i|^4 , \]
  \[ \mathcal{L}_2 = 2 \hbar \varphi_i^{\dagger} \varphi_j^{\dagger} \partial_\mu \varphi_k \partial^\mu \varphi_j - g \varphi_i^{\dagger} \varphi_j \partial_\mu \varphi_k \partial^\mu \varphi_j . \]

- Superflows $\vec{v}_i$ and the chemical potentials $\mu_i$ are both related to the phases of the condensates (expectation values of the fields, $\langle \varphi_i \rangle = \rho_i e^{i \chi_i}$) [4]:
  \[ \mu_i = \partial_0 \rho_i , \quad \vec{v}_i = \frac{\vec{\nabla} \chi_i}{\mu_i} . \]

- Two-component superfluid: spontaneous symmetry breaking with pattern $U(1) \times U(1) \rightarrow 1$ leads to two Goldstone modes.

3 PHASE DIAGRAMS & LANDAU’S CRITICAL VELOCITY

- Landau’s critical velocity $v_L$: energy of Goldstone mode becomes negative.

- Phase diagrams for positive and negative values of the entrainment coupling $g$ in the plane of the chemical potentials. The colors represent Landau’s critical velocity in units of $c = 1$.
  For simplicity, $\hbar = 0$.

- Four different phases, separated by second order phase transitions except for first order transition between SF$_1$/SF$_2$:
  - NOR: normal phase
  - SF$_1$/SF$_2$: superfluid$_1$/superfluid$_2$ phase
  - COE: two-component superfluid (coexistence phase)

- For $g < 0$, the entrainment term acts as an energy penalty $\Rightarrow$ size of COE phase reduced. For $g > 0$, energy gain $\Rightarrow$ COE phase enlarged.

4 SOUND MODES & INSTABILITIES

- At $T = 0$, the sound modes are given by the slope of the Goldstone modes or can equivalently be calculated using relativistic, linearized hydrodynamics.

- Polar plots show sound modes and development of instabilities for all angles between the superflow $\vec{v}_1$ and the wave vector $\vec{k}$.

  - Two-stream instability starts to develop in downstream direction "after" Landau’s critical velocity.

5 SUMMARY

- Two-component superfluids with non-zero superflow feature several instabilities, which can be calculated by analyzing the sound modes.

- They might serve as a trigger for pulsar glitches.

- It is found that the two-stream instability always takes place in an energetically unstable regime (after Landau’s critical velocity). This is different for a two-component normal fluid (not shown here).

REFERENCES


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