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Inverse magnetic catalysis in dense holographic matter

F. Preis, A. Rebhan, A. Schmitt, JHEP 1103, 033 (2011)

- **(Chiral) magnetic catalysis and its relevance for QCD**
- **Chiral transition in the Sakai-Sugimoto model**
- ***Inverse* magnetic catalysis at finite chemical potential**

• Magnetic catalysis (page 1/2)

K. G. Klimenko, Theor. Math. Phys. 89, 1161-1168 (1992)

V. P. Gusynin, V. A. Miransky, I. A. Shovkovy, PLB 349, 477-483 (1995)

- massless fermions in background magnetic field
- e.g., **Nambu-Jona-Lasinio (NJL)** model ($A_\mu = -Bx_2\delta_{\mu 1}$):

$$\mathcal{L} = \bar{\psi}\, i\gamma^\mu(\partial_\mu - ieA_\mu)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\, i\gamma^5\psi)^2\right]$$

dimensionless coupling $g \equiv G\Lambda^2/(4\pi^2)$

Zero magnetic field:	**Nonzero magnetic field:**
dynamical fermion mass	$M \neq 0$ for *arbitrarily small* g,
$M \propto \langle\bar{\psi}\psi\rangle \neq 0$	$M \propto \sqrt{eB}\, e^{-\text{const.}/eBg}$
for coupling $g > g_c = 1$	at weak coupling $g \ll 1$

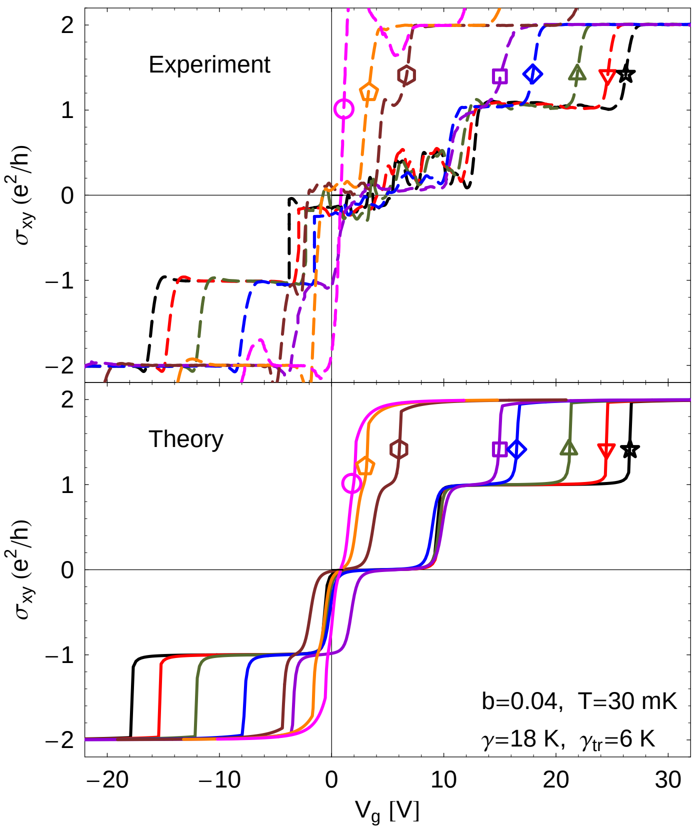
• Magnetic catalysis (page 2/2)

Analogy to BCS Cooper pairing:

BCS superconductor	Magnetic catalysis
$\langle\psi\psi\rangle$	$\langle\overline{\psi}\psi\rangle$
$\Delta \propto \mu\, e^{-\text{const.}/G\nu_F}$ (ν_F: d.o.s. at $E=\mu$ Fermi surface)	$M \propto \sqrt{eB}\, e^{-\text{const.}/G\nu_0}$ (ν_0: d.o.s. at $E=0$ surface)
pairing dynamics effectively (1+1)-dimensional because of Fermi surface	effectively (1+1)-dimensional in lowest Landau level (LLL) ($\epsilon = \sqrt{k_z^2 + 2\lvert eB\rvert n}$)

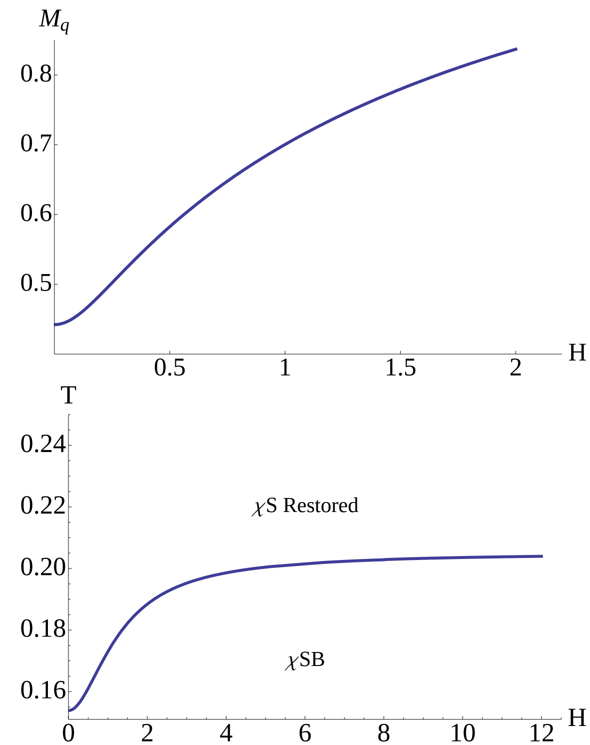
Magnetic catalysis
= magnetic field favors/enhances $\bar{\psi} - \psi$ pairing

• Magnetic catalysis in the real world and in holography



V.P.Gusynin *et al.*, PRB 74, 195429 (2006)

- **graphene**: appearance of additional plateaus in strong magnetic fields

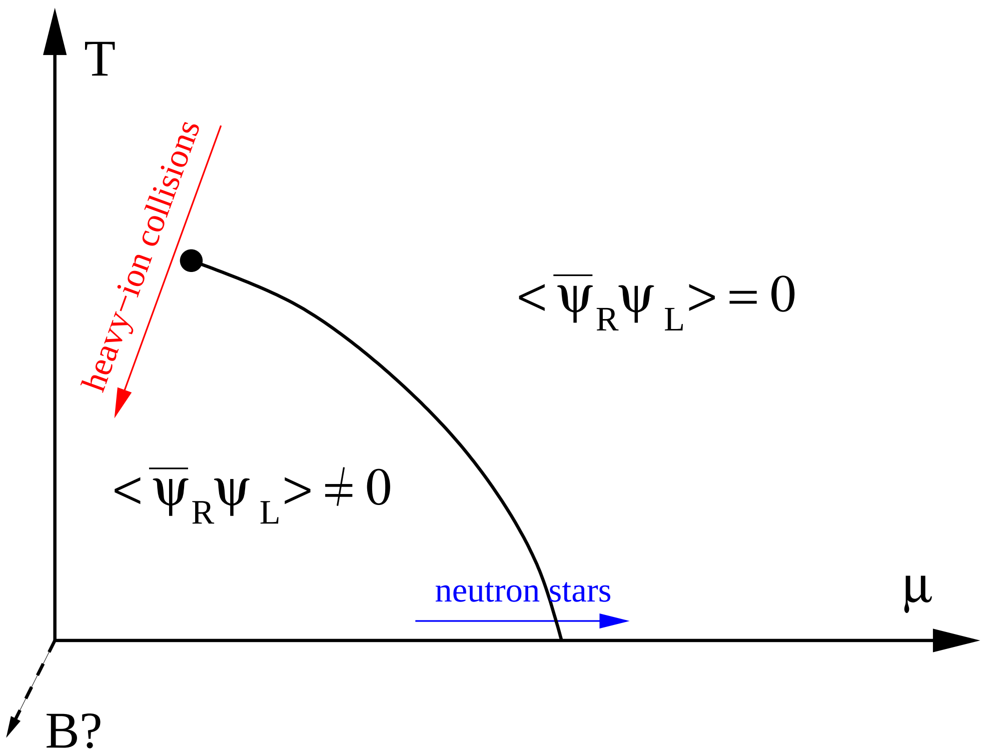


C.V.Johnson, A.Kundu, JHEP 0812, 053 (2008)

- **Sakai-Sugimoto**: magnetic field enhances dynamical mass M_q and critical temperature T_c

see also: J. Erdmenger *et al.*, JHEP 0712, 091 (2007)
V.G.Filev, R.C.Rashkov, AHEP 473206 (2010)

• (Chiral) magnetic catalysis in QCD?



chiral & deconfinement transitions in strong magnetic fields:

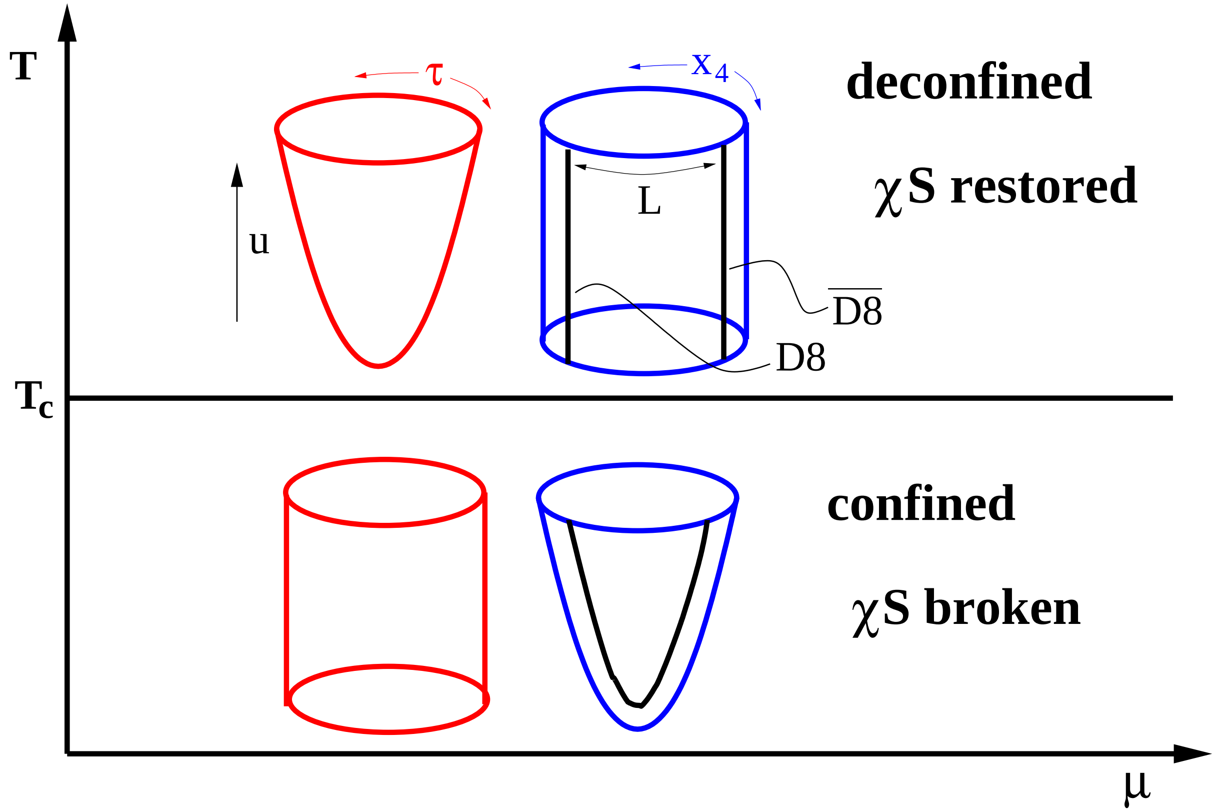
lattice QCD M. D'Elia, S. Mukherjee, F. Sanfilippo, PRD 82, 051501 (2010)

"PNJL" model R. Gatto, M. Ruggieri, PRD 83, 034016 (2011)

- need large magnetic fields $\Lambda_{QCD}^2 \sim (200\,\mathrm{MeV})^2 \sim 2 \times 10^{18}\,\mathrm{G}$
- possibly reached in
  - heavy-ion collisions: temporarily $B \sim 10^{18}\,\mathrm{G}$
  - neutron stars (magnetars): at surface $B \sim 10^{15}\,\mathrm{G}$, larger in the interior

• Chiral transition in the Sakai-Sugimoto model (p. 1/3)

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)



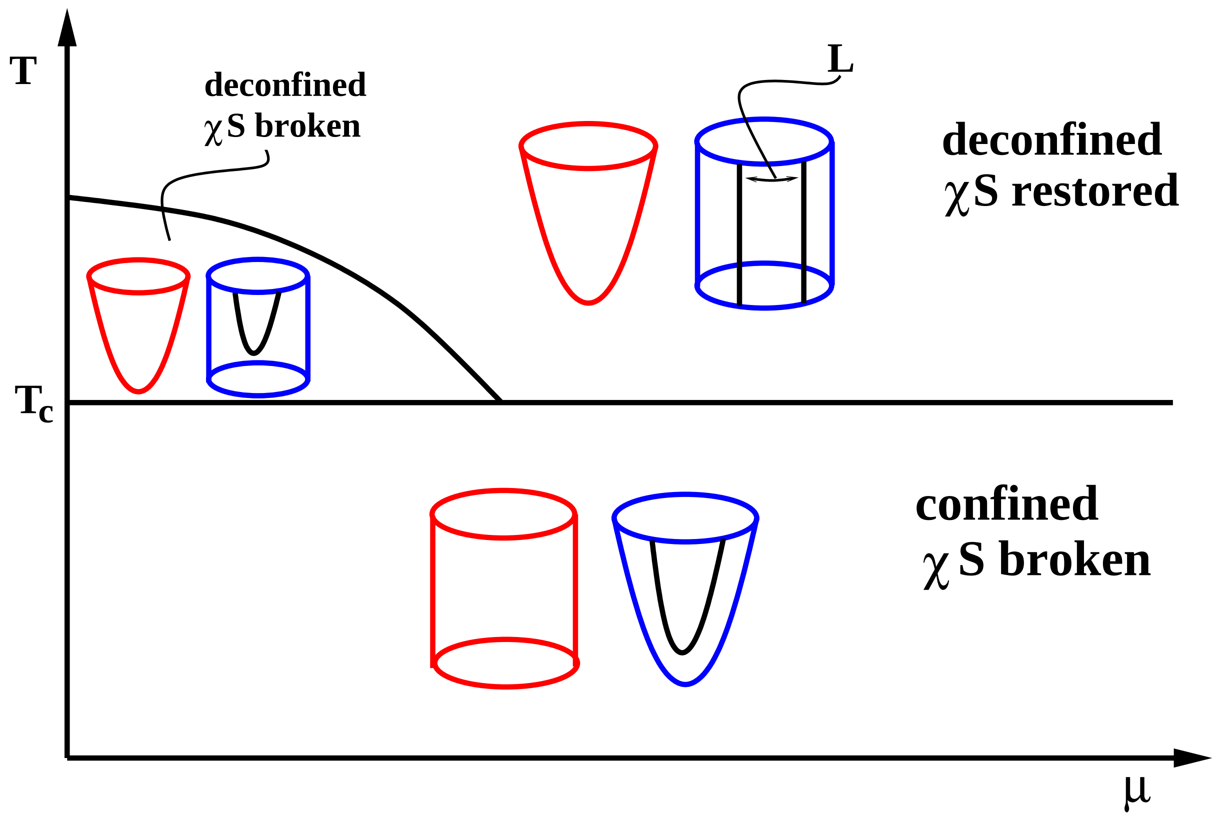
- not unlike expectation from large-N_c QCD
  L. McLerran, R. D. Pisarski, Nucl. Phys. A 796, 83 (2007)
- in probe brane approximation: **chiral transition** unaffected by quantities on flavor branes (μ, B, $\ldots$)

• Chiral transition in the Sakai-Sugimoto model (p. 2/3)

- less "rigid" behavior for smaller L
- deconfined, chirally broken phase for $L < 0.3\,\pi/M_{\rm KK}$

O. Aharony, J. Sonnenschein, S. Yankielowicz, Annals Phys. 322, 1420 (2007)

N. Horigome, Y. Tanii, JHEP 0701, 072 (2007)

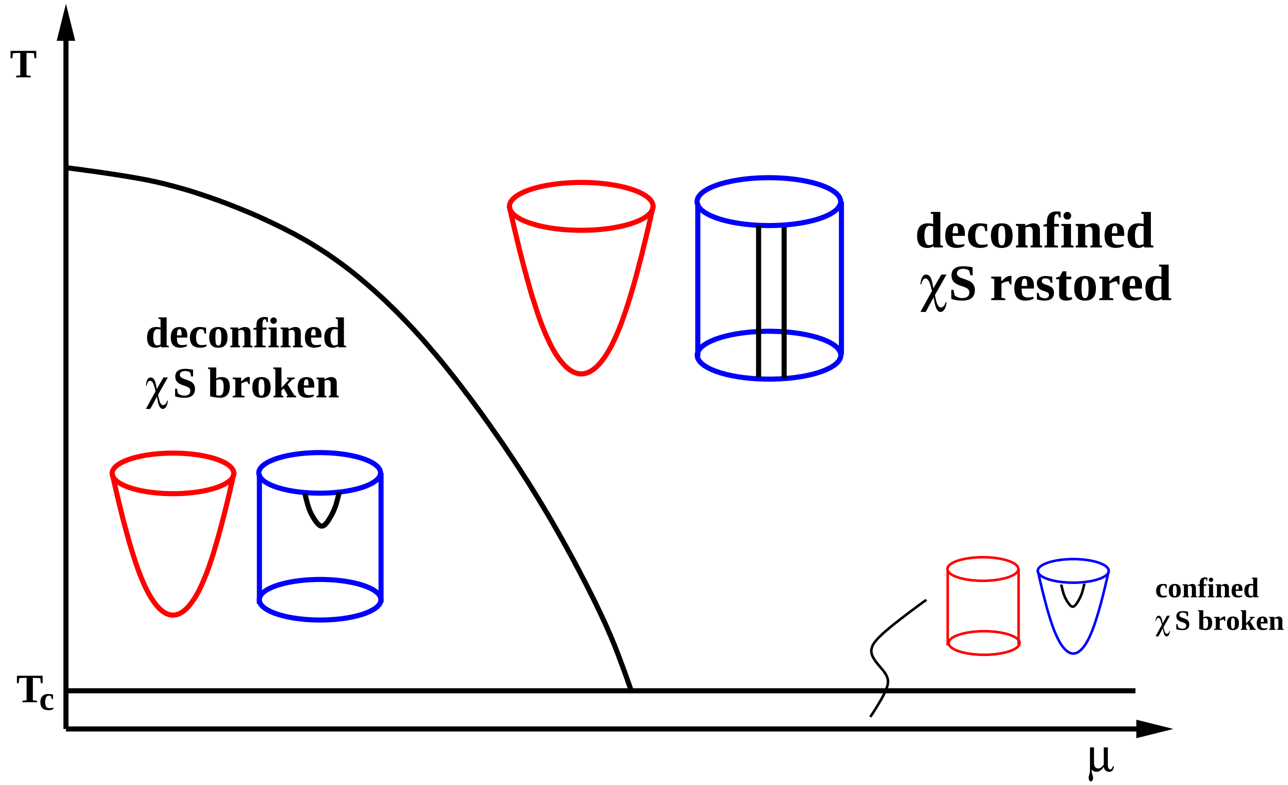


• Chiral transition in the Sakai-Sugimoto model (p. 3/3)

- $L \ll \pi / M_{\rm KK}$ corresponds to (non-local) NJL model

  E. Antonyan, J. A. Harvey, S. Jensen, D. Kutasov, hep-th/0604017

  J. L. Davis, M. Gutperle, P. Kraus, I. Sachs, JHEP 0710, 049 (2007)



- this limit is considered in the following calculation ...

• Setup of our calculation

- D8-brane action in deconfined geometry for $N_f = 1$
  O. Bergman, G. Lifschytz, M. Lippert, PRD 79, 105024 (2009)

$$S = S_{\rm DBI} + S_{\rm CS}$$
$$= \mathcal{N}\int_{u_T}^{\infty} du\,\sqrt{u^5+b^2u^2}\sqrt{1+fa_3'^2-a_0'^2+u^3fx_4'^2} + \frac{3\mathcal{N}}{2}b\int_{u_T}^{\infty} du\,(a_3a_0'-a_0a_3')$$

- a_0 accounts for μ
- magnetic field $b = F_{12}$
- b, μ induce a_3
  ($\longrightarrow$ anisotropic condensate $\nabla\pi^0$)
  E.G.Thompson, D.T.Son, PRD 78, 066007 (2008)
  A. Rebhan *et al.*, JHEP 0905, 084 (2009)
- $x_4(u) = \begin{cases} \text{const.} & \chi S \\ \text{nontrivial} & \chi Sb \end{cases}$

- equations of motion:

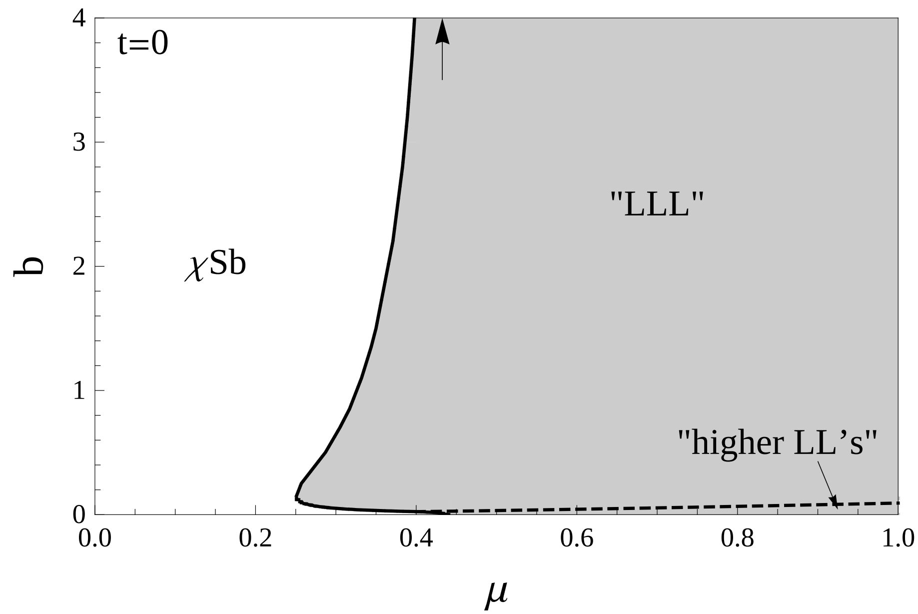
$$\partial_u\left(\frac{a_0'\sqrt{u^5+b^2u^2}}{\sqrt{1+fa_3'^2-a_0'^2+u^3fx_4'^2}}\right) = 3ba_3'$$

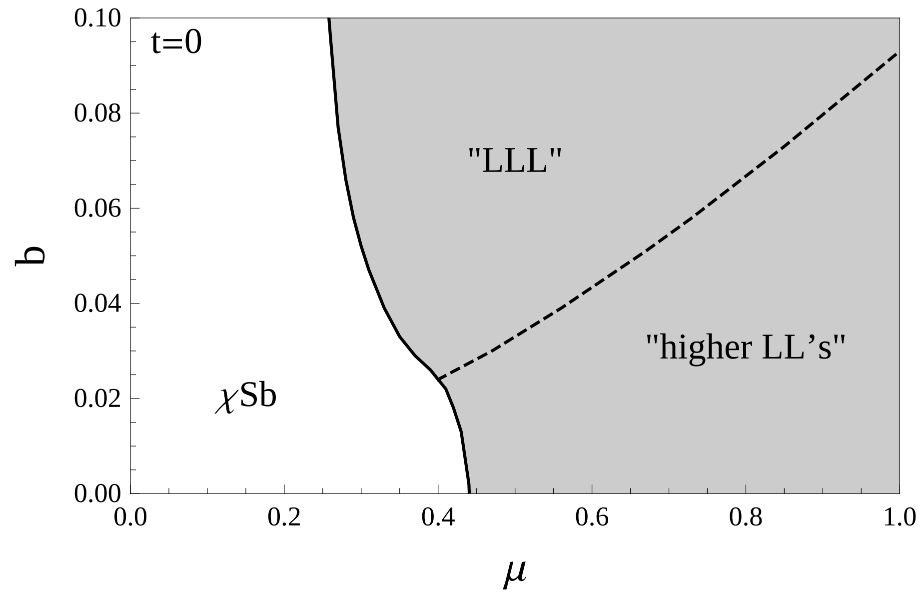
$$\partial_u\left(\frac{f\,a_3'\sqrt{u^5+b^2u^2}}{\sqrt{1+fa_3'^2-a_0'^2+u^3fx_4'^2}}\right) = 3ba_0'$$

$$\partial_u\left(\frac{u^3f\,x_4'\sqrt{u^5+b^2u^2}}{\sqrt{1+fa_3'^2-a_0'^2+u^3fx_4'^2}}\right) = 0$$

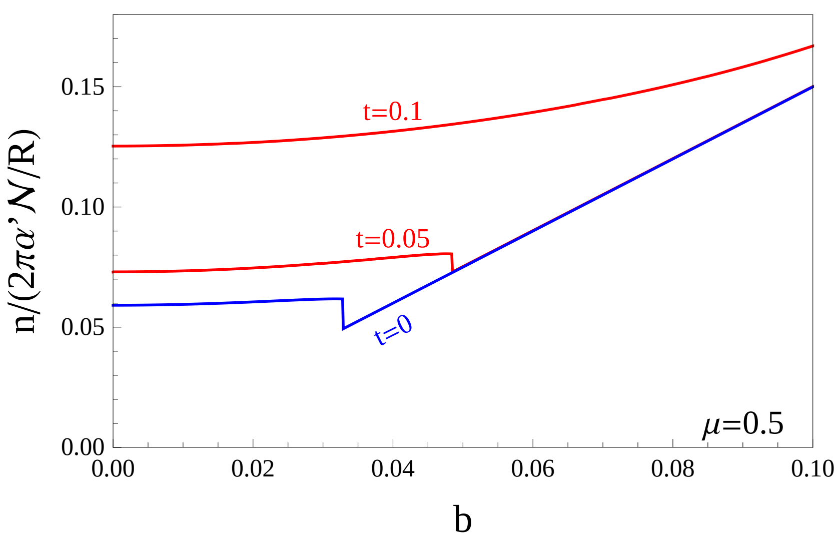
$\longrightarrow$ solve for $a_0(u)$, $a_3(u)$, $x_4(u)$

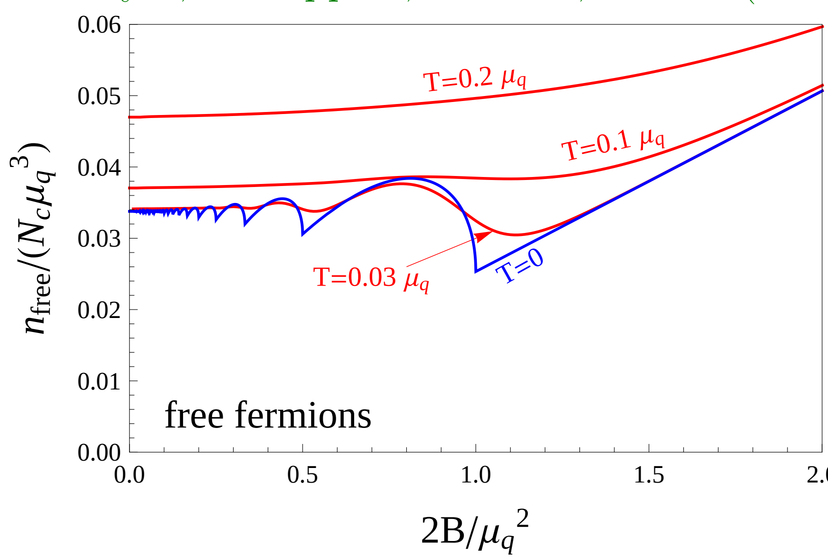
• $T = 0$ phase diagram





Apparent Landau level transition G. Lifschytz, M. Lippert, PRD 80, 066007 (2009):





- **Inverse magnetic catalysis**

Why does B *restore* chiral symmetry for certain μ?

- free energy *gain* from $\bar{\psi}-\psi$ pairing increases with B (magnetic catalysis)
- μ induces free energy *cost* for pairing; this cost depends on B!

weak coupling (NJL):

E. V. Gorbar *et al.*, PRC 80, 032801 (2009)

$$\Delta\Omega \propto B(\mu^2 - M^2/2)$$

analogous to Cooper pairing with mismatched Fermi surfaces

A. Clogston, PRL 9, 266 (1962)

B. Chandrasekhar, APL 1, 7 (1962)

Sakai-Sugimoto:

large B:

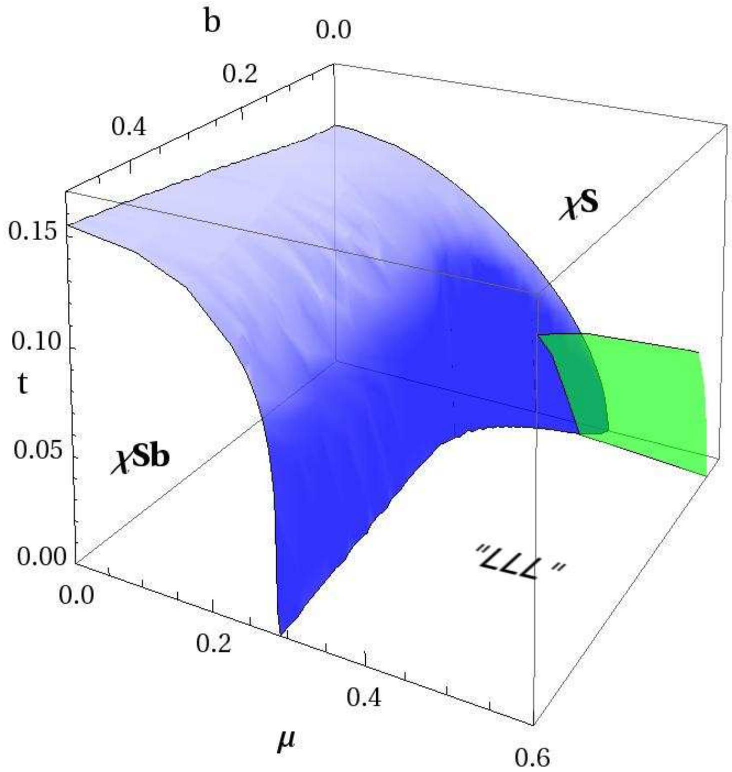
$$\Delta\Omega \propto B(\mu^2 - 0.12\, M^2)$$

small B:

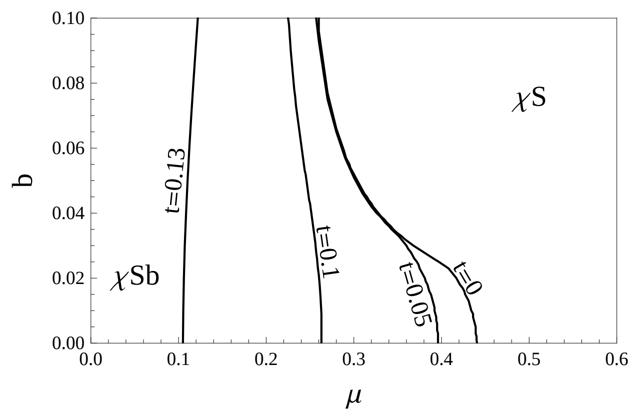
$$\Delta\Omega \propto \mu^2 B - \text{const} \times M^{7/2}$$

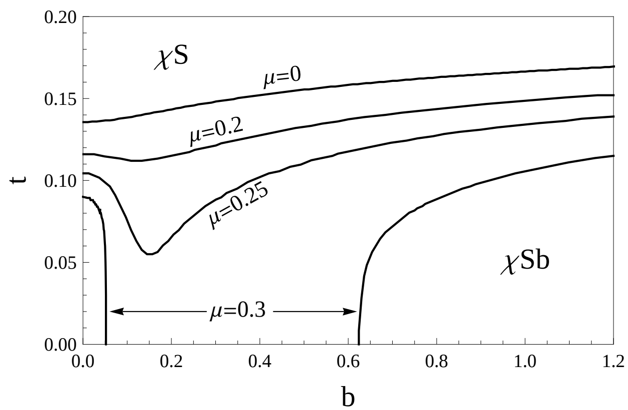
"Inverse magnetic catalysis"

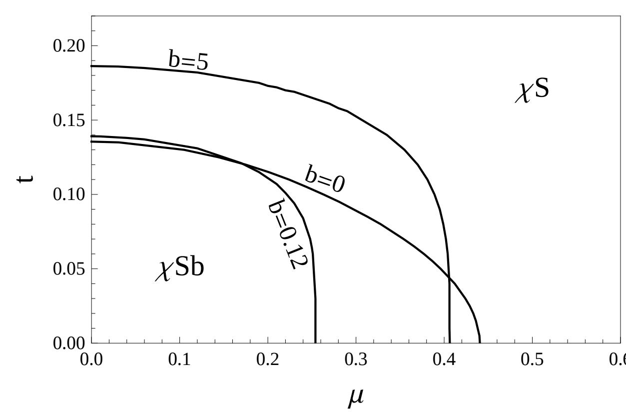
• Phase structure at nonzero temperature



blue: chiral phase transition
green: "LLL" transition

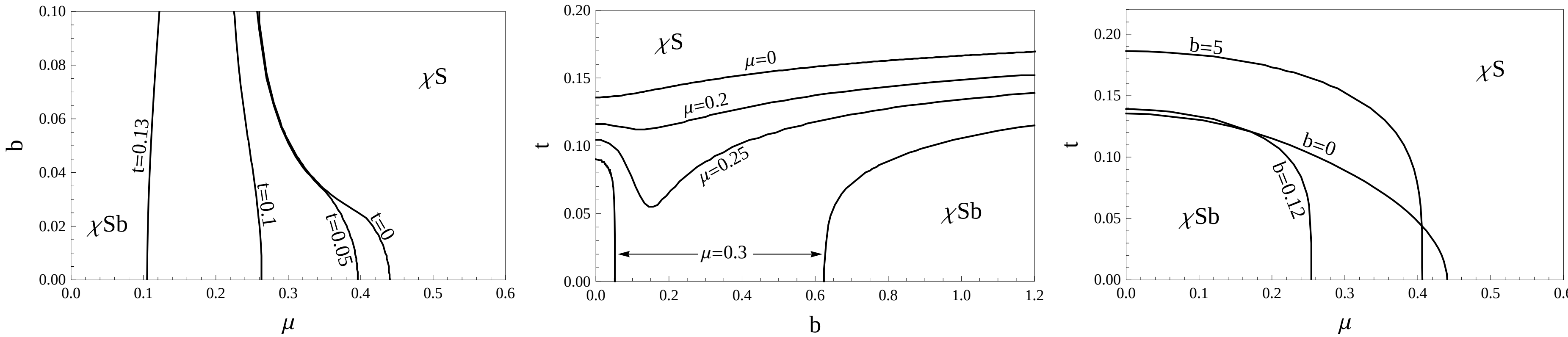






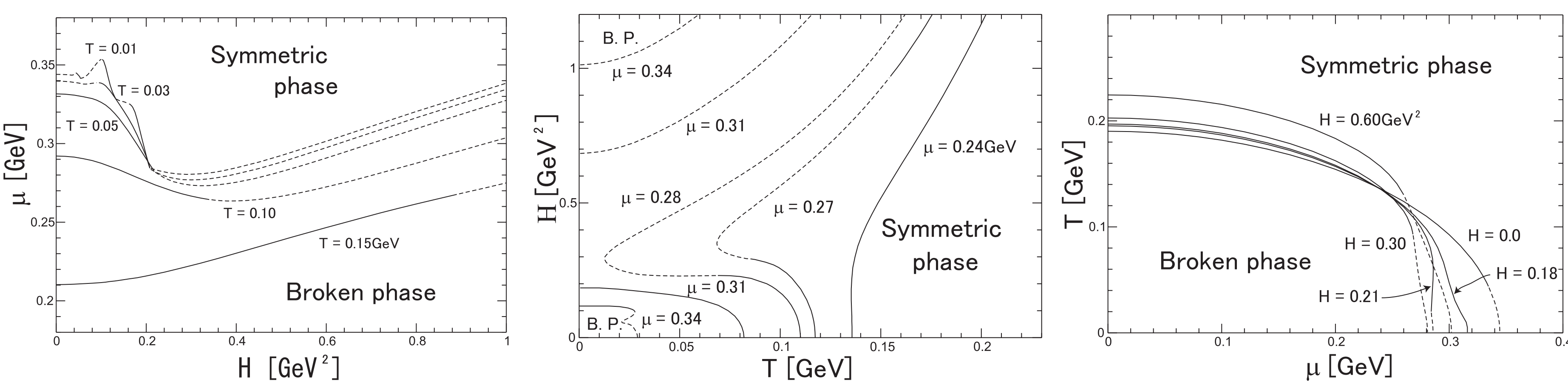
- # Agreement with NJL calculation

Our work



NJL

T. Inagaki, D. Kimura, T. Murata, Prog. Theor. Phys. 111, 371-386 (2004)



• Conclusions

- Dense matter in the "NJL limit" of the Sakai-Sugimoto model...
  ... shows a **transition reminiscent of LLL**
  (but no de Haas – van Alphen oscillations)
  ... shows a chiral phase transition with
  MC at large B and **IMC at small** B

- Fixing $T_c \simeq 150\,\mathrm{MeV}$, $\mu_c \sim 400\,\mathrm{MeV}$, **IMC occurs for**

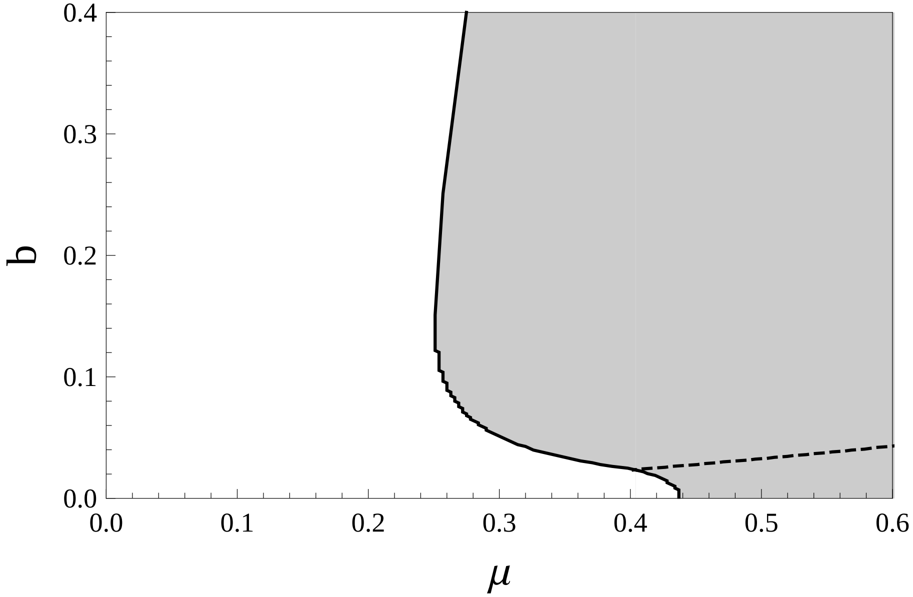
$$|qB| \lesssim 1.0 \times 10^{19}\,\mathrm{G}$$

  and reduces μ_c from $\sim 400\,\mathrm{MeV}$ to $\sim 230\,\mathrm{MeV}$
  ($\longrightarrow$ relevant for magnetars?)

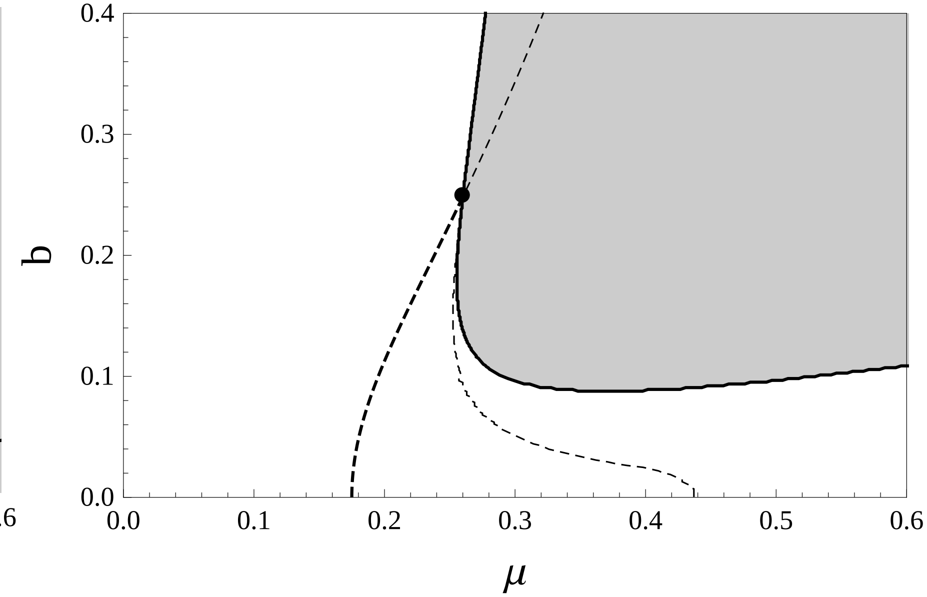
• Outlook: including nuclear matter

F. Preis, A. Rebhan, A. Schmitt, work in progress

without nuclear matter



with nuclear matter (prelim.)



- No symmetry restoration at small B (?)

  $B = 0$: O. Bergman, G. Lifschytz, M. Lippert, JHEP 0711, 056 (2007)

- How physical is this nuclear matter?
  - Sakai-Sugimoto model: nuclear matter unbound

    V. S. Kaplunovsky, J. Sonnenschein, JHEP 1105, 058 (2011)
  - (pure) large-N_c effect?