Stressed pairing in conventional color superconductors is unavoidable

Krishna Rajagopal, Andreas Schmitt, Phys. Rev. D73, 045003 (2006)

Motivation:

(Astro)physical version:

What is the ground state of quark matter in the interior of a neutron star?

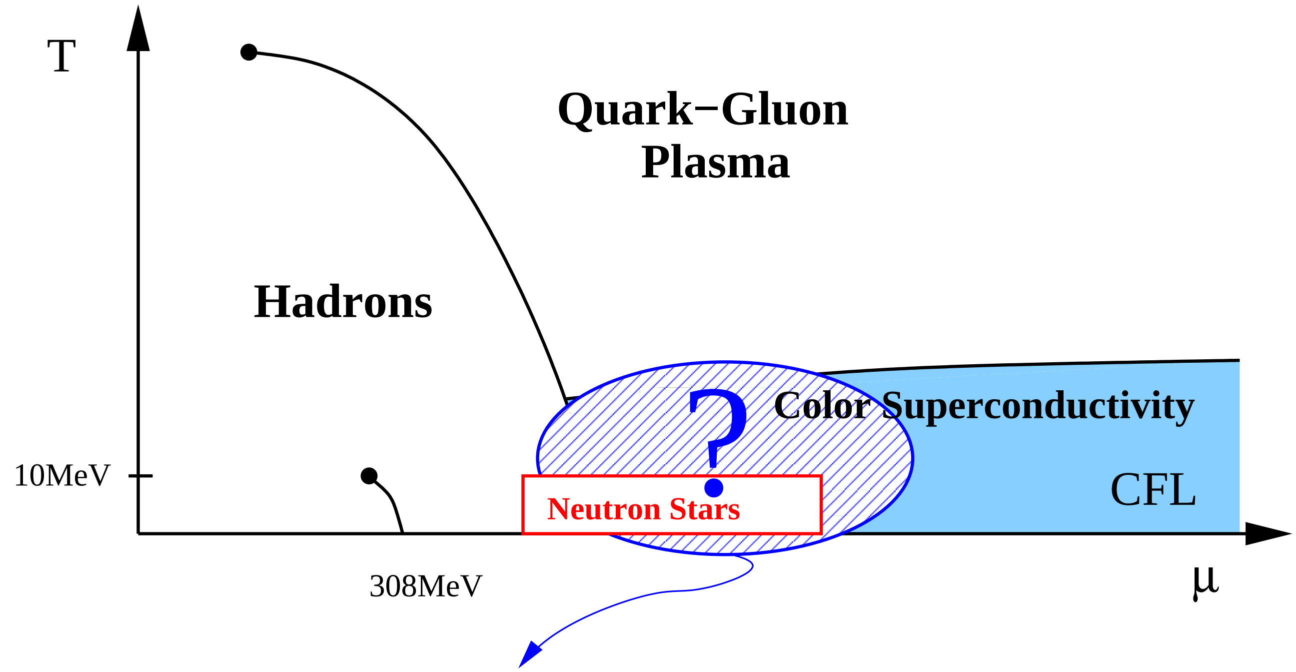
Theoretical version:

How does quark matter at large (but not asymptotically large) densities react on the conditions of electric and color neutrality?

Mathematical version:

Proof of statement in title

- **Color superconductivity in the QCD phase diagram**



- One-flavor pairing: Color-Spin-Locking, A phase, polar phase, planar phase

  T. Schäfer, PRD 62, 094007 (2000)

  A. Schmitt, Q. Wang and D. H. Rischke, Phys. Rev. D **66**, 114010 (2002)

- Gapless superconductors: g2SC, gCFL

  I. Shovkovy, M. Huang, PLB 564, 205 (2003)

  M. Alford, C. Kouvaris, K. Rajagopal, PRL **92**, 222001 (2004)

- Counter-propagating currents: LOFF, crystalline phases, meson current

  M. Alford, J. Bowers, K. Rajagopal, PRD 63, 074016 (2001)

  T. Schäfer, PRL 96, 012305 (2006)

• On safe grounds: Asymptotically large density (page 1/2)

$0 \simeq m_s \simeq m_u \simeq m_d \ll \mu$ all quark masses negligible

Cooper pairing in color and flavor triplet channel:

$$\left.\begin{array}{l}[3] \otimes [3] = [\bar{3}]_a \oplus [6]_s \\ \\ [3] \otimes [3] = [\bar{3}]_a \oplus [6]_s\end{array}\right\} \quad \textbf{gap matrix} \quad \mathcal{M} \in [\bar{3}]_a \otimes [\bar{3}]_a$$

→ In general:

$$\mathcal{M} = \Delta_{ij}\, J_i \otimes I_j = \begin{pmatrix} 0 & 0 & 0 & 0 & -\Delta_{33} & \Delta_{32} & 0 & \Delta_{23} & -\Delta_{22} \\ 0 & 0 & 0 & \Delta_{33} & 0 & -\Delta_{31} & -\Delta_{23} & 0 & \Delta_{21} \\ 0 & 0 & 0 & -\Delta_{32} & \Delta_{31} & 0 & \Delta_{22} & -\Delta_{21} & 0 \\ 0 & \Delta_{33} & -\Delta_{32} & 0 & 0 & 0 & 0 & -\Delta_{13} & \Delta_{12} \\ -\Delta_{33} & 0 & \Delta_{31} & 0 & 0 & 0 & \Delta_{13} & 0 & -\Delta_{11} \\ \Delta_{32} & -\Delta_{31} & 0 & 0 & 0 & 0 & -\Delta_{12} & \Delta_{11} & 0 \\ 0 & -\Delta_{23} & \Delta_{22} & 0 & \Delta_{13} & -\Delta_{12} & 0 & 0 & 0 \\ \Delta_{23} & 0 & -\Delta_{21} & -\Delta_{13} & 0 & \Delta_{11} & 0 & 0 & 0 \\ -\Delta_{22} & \Delta_{21} & 0 & \Delta_{12} & -\Delta_{11} & 0 & 0 & 0 & 0 \end{pmatrix}$$

→ pairing pattern characterized by (complex) 3×3 matrix Δ

- **On safe grounds: Asymptotically large density (page 2/2)**

$$0 \simeq m_s \simeq m_u \simeq m_d \ll \mu \qquad \rightarrow \qquad SU(3)_c \times SU(3)_f \text{ symmetry}$$

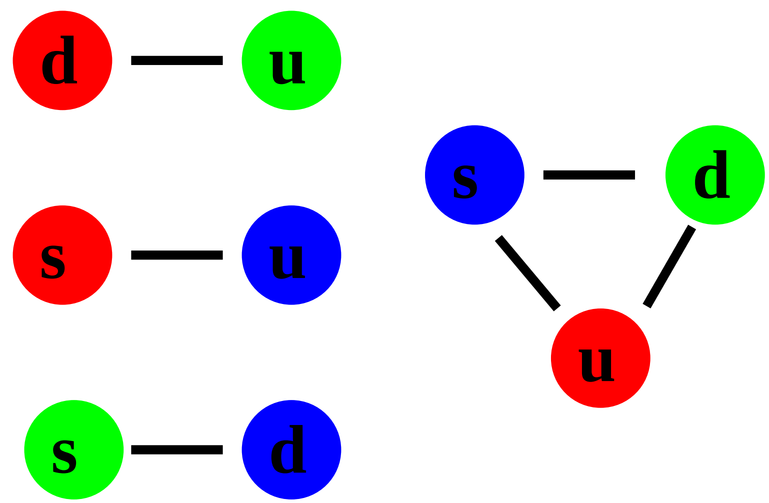
$$\forall\Delta \quad \exists U \in SU(3)_c, V \in SU(3)_f: \quad U^T \, \Delta V \, \text{diagonal}$$

$\rightarrow$ Δ and $U^T \, \Delta \, V$ are physically equivalent $\rightarrow$ **diagonal Δ's sufficient**

$$\rightarrow \qquad \Delta_{\text{CFL}} = \Delta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

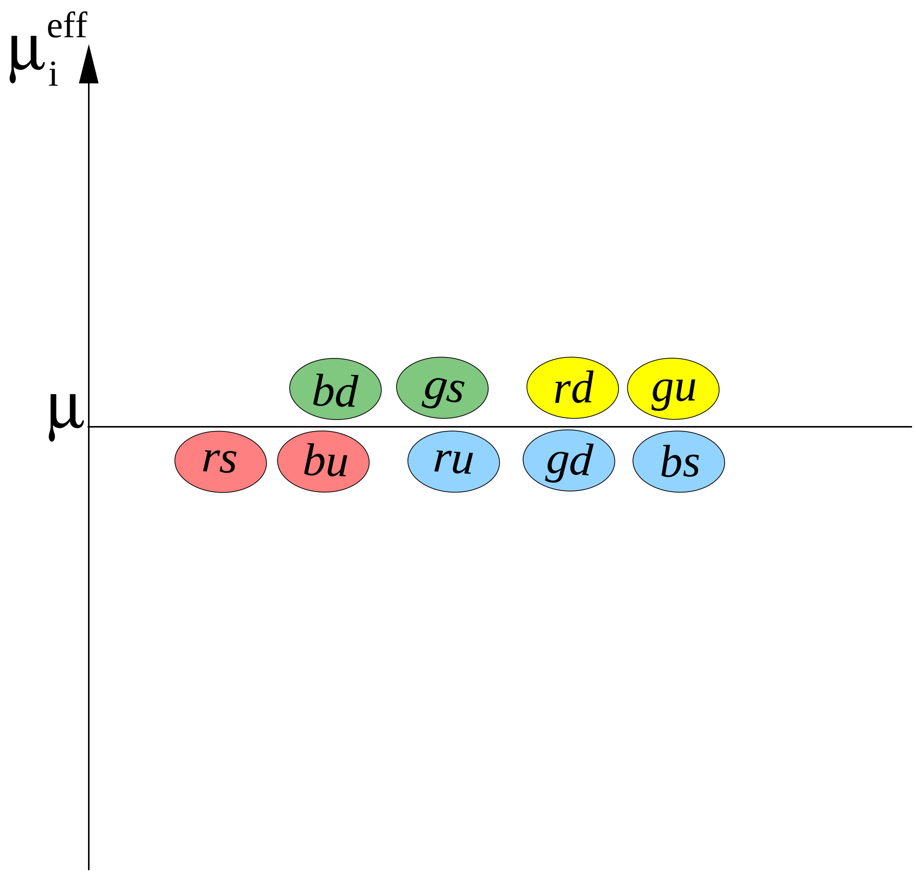
"color-flavor locked phase (CFL)"
M. Alford, K. Rajagopal, F. Wilczek, Nucl. Phys. B537, 443 (1999)

$$\mathcal{M}_{\text{CFL}} = \begin{pmatrix} 0 & 0 & 0 & 0 & -\Delta & 0 & 0 & 0 & -\Delta \\ 0 & 0 & 0 & \Delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Delta & 0 & 0 \\ 0 & \Delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\Delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Delta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta & 0 \\ 0 & 0 & \Delta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta & 0 & 0 & 0 \\ -\Delta & 0 & 0 & 0 & -\Delta & 0 & 0 & 0 & 0 \end{pmatrix}$$



- **CFL phase at not asymptotically large densities (page 1/3)**

(1) $m_s = 0$**:**



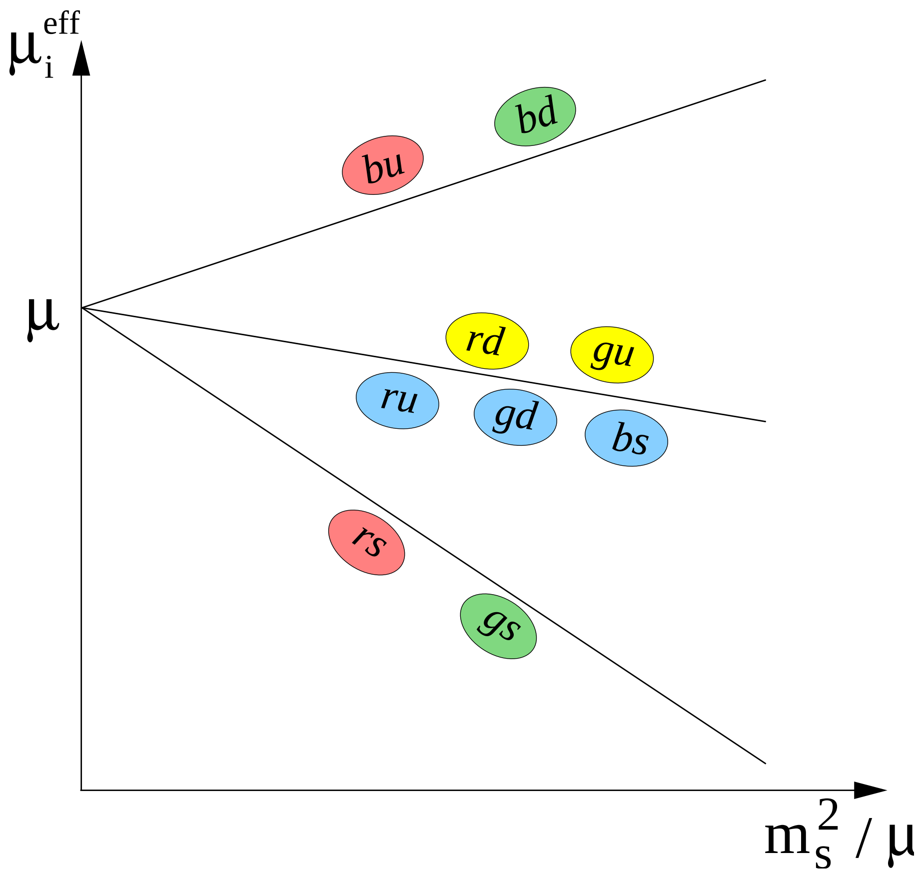
same effective Fermi surface for all quarks

Smaller densities: m_s no longer negligible: $0 \simeq m_u \simeq m_d \ll m_s \simeq \mu$

And: Require electric and color neutrality $\rightarrow$

- **CFL phase at not asymptotically large densities (page 2/3)**

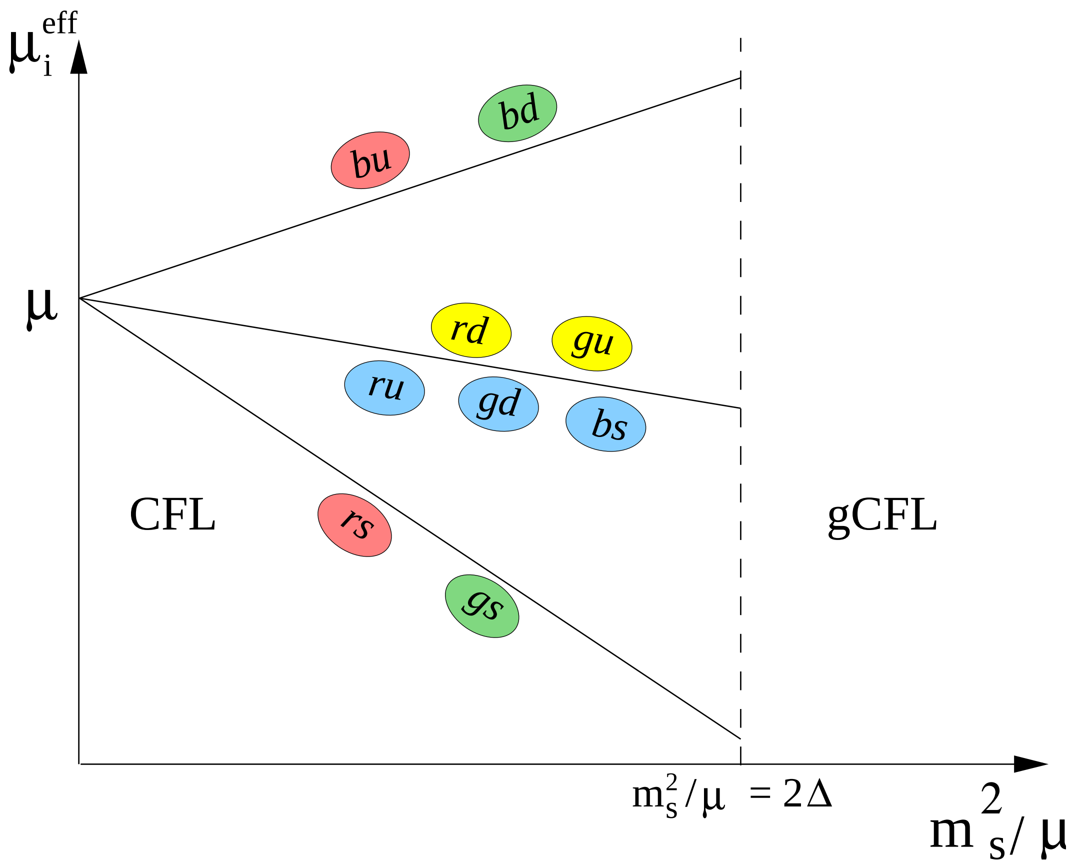
(2) switch on m_s:



Fermi surfaces split apart

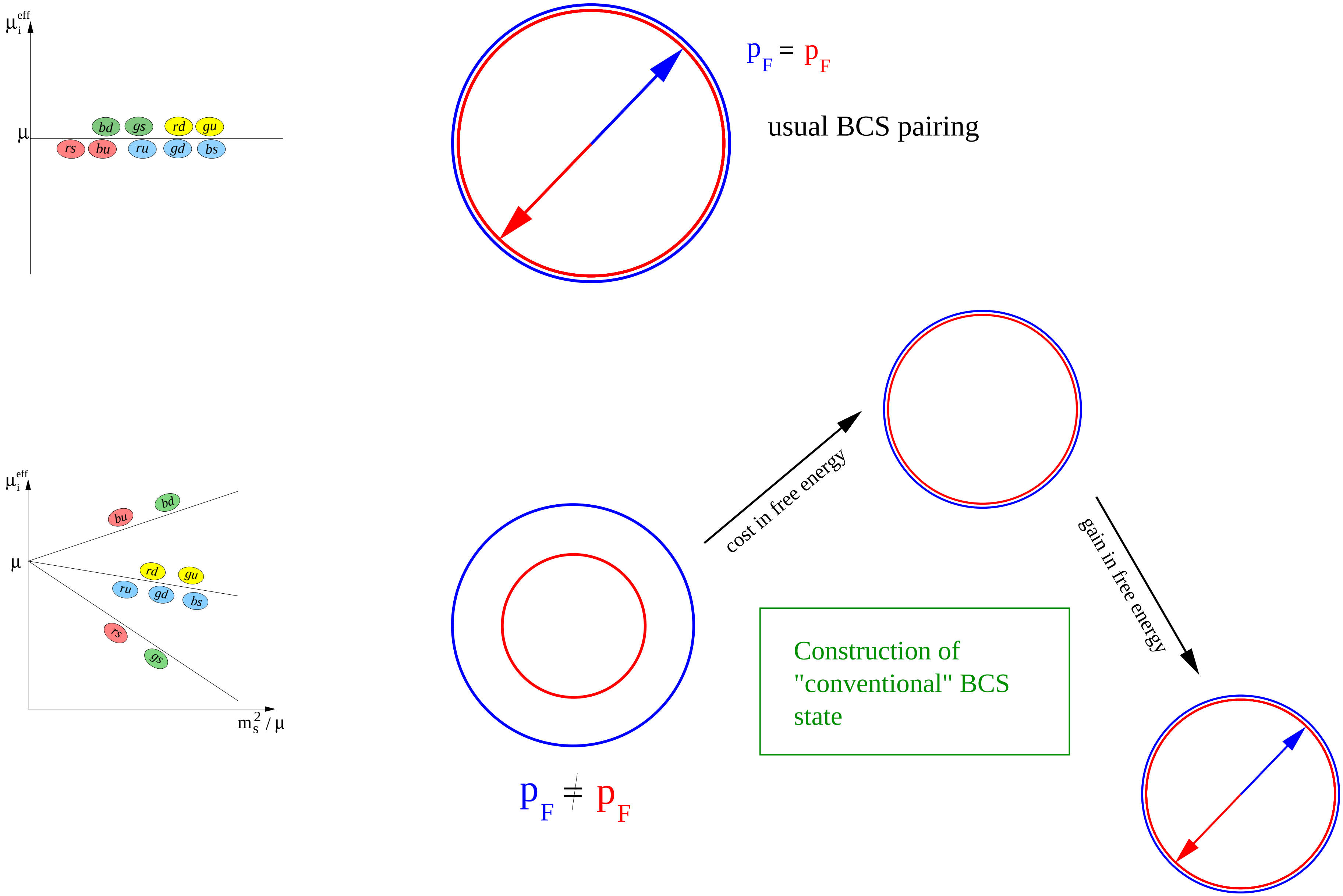
• **CFL phase at not asymptotically large densities (page 3/3)**

(3) even larger m_s:

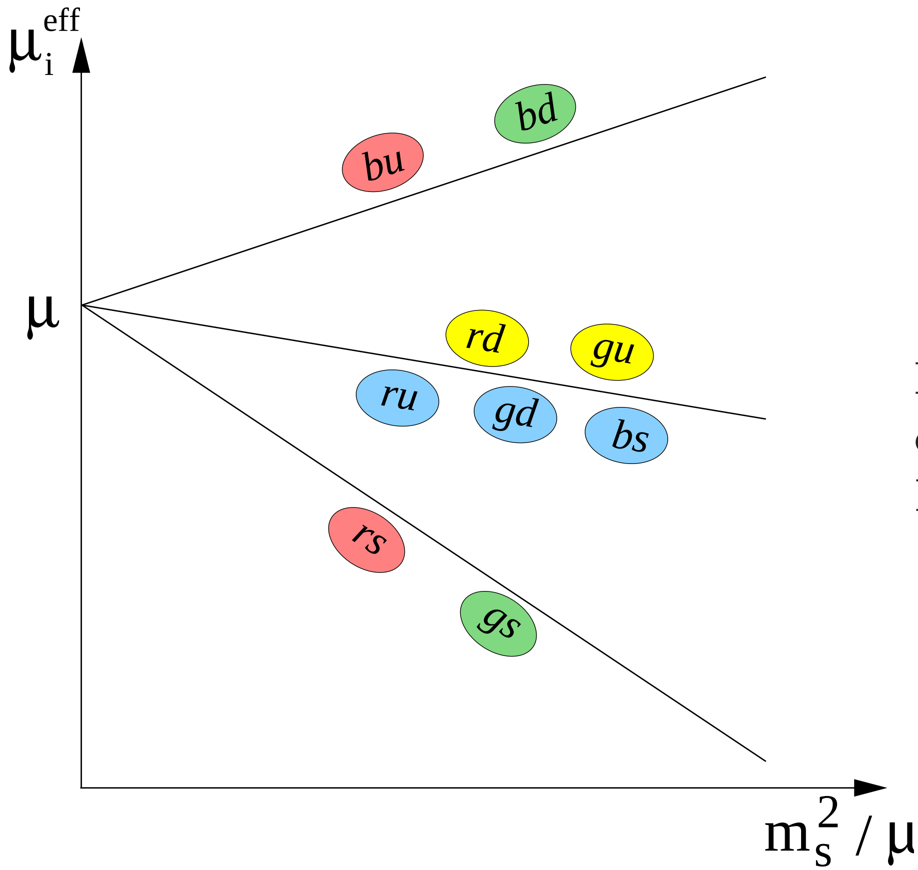


CFL phase breaks down

• Stressed pairing



- **Is stressed pairing avoidable?**



Is there any pattern such that fermions of the same "color" appear at the same line?

In other words:

Is there a pattern of pairing in which, once electric and color neutrality is imposed, pairing only occurs among those quarks whose Fermi momenta would be equal in the absence of pairing?

- **How to capture all possible patterns? (page 1/3)**

Remember: Asymptotically large densities $\rightarrow SU(3)_c \times SU(3)_f$
$\rightarrow$ sufficient to consider diagonal Δ's

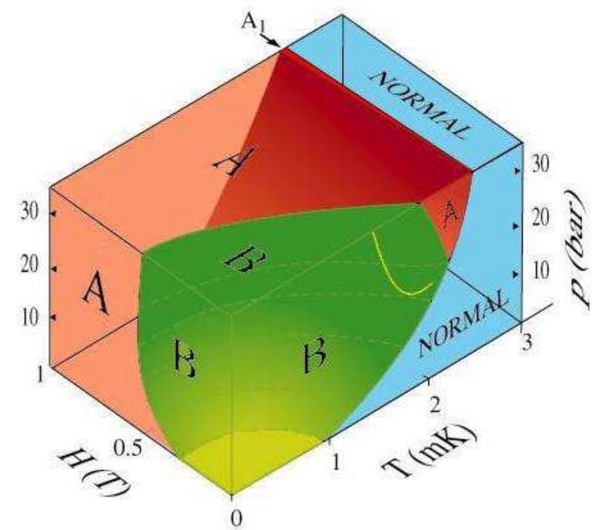
However: Nonzero $m_s \rightarrow SU(3)_c \times SU(2)_f$

$\rightarrow$ **Not true:** $\forall\, \Delta \quad \exists U \in SU(3)_c, V \in SU(2)_f$: $U^T \Delta V$ diagonal

$\rightarrow$ **non-diagonal**, physically distinct, Δ's! **See analogous systems:**

- **superfluid ^{3}He:** $SO(3)_S \times SO(3)_L$

A phase: $\quad \Delta \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & i & 0 \end{pmatrix}$

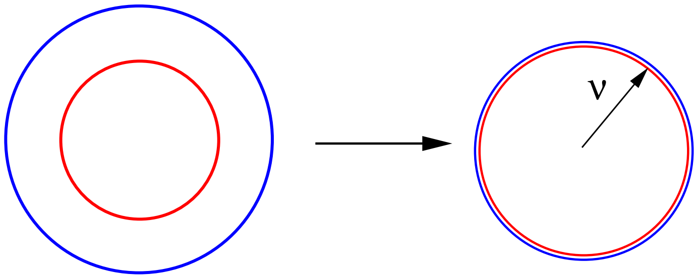


- **Spin-1 color superconductors:** $SU(3)_c \times SU(2)_J$
  A. Schmitt, PRD 71, 054016 (2005)

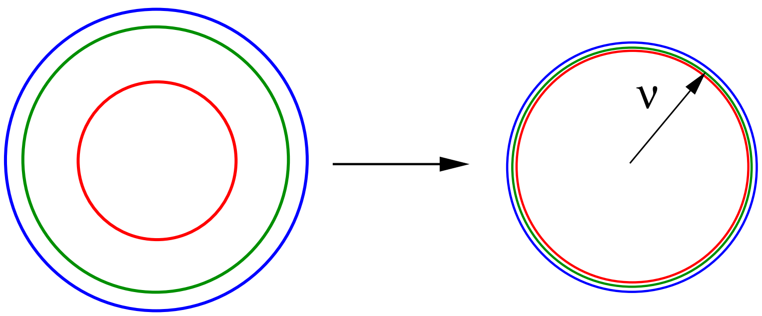
- **How to capture all possible patterns? (page 2/3)**

- In principle: Huge task to study (even classify) all possible patterns

- However: For our purpose all we need to know is **“Who pairs with whom?”**



$$\nu_1 = \nu_2 = \frac{\mu_1^{\rm eff} + \mu_2^{\rm eff}}{2}$$



$$\nu_4 = \nu_5 = \nu_6 = \frac{\mu_4^{\rm eff} + \mu_5^{\rm eff} + \mu_6^{\rm eff}}{3}$$

$$\frac{\partial\Omega(\{\nu_i\})}{\partial\mu_\ell} = 0\,, \quad \mu_\ell = \mu_e, \mu_3, \mu_8$$

Neutrality

$$\nu_i = \mu_i^{\rm eff}\,, \quad i = 1, \ldots 9$$

Stress-free pairing

$\longrightarrow$ Check all patterns for simultaneous solution of **Neutrality** and **Stress-free pairing**

- **How to capture all possible patterns? (page 3/3)**

- Start from

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & 0 & -\Delta_{33} & \Delta_{32} & 0 & \Delta_{23} & -\Delta_{22} \\ 0 & 0 & 0 & \Delta_{33} & 0 & -\Delta_{31} & -\Delta_{23} & 0 & \Delta_{21} \\ 0 & 0 & 0 & -\Delta_{32} & \Delta_{31} & 0 & \Delta_{22} & -\Delta_{21} & 0 \\ 0 & \Delta_{33} & -\Delta_{32} & 0 & 0 & 0 & 0 & -\Delta_{13} & \Delta_{12} \\ -\Delta_{33} & 0 & \Delta_{31} & 0 & 0 & 0 & \Delta_{13} & 0 & -\Delta_{11} \\ \Delta_{32} & -\Delta_{31} & 0 & 0 & 0 & 0 & -\Delta_{12} & \Delta_{11} & 0 \\ 0 & -\Delta_{23} & \Delta_{22} & 0 & \Delta_{13} & -\Delta_{12} & 0 & 0 & 0 \\ \Delta_{23} & 0 & -\Delta_{21} & -\Delta_{13} & 0 & \Delta_{11} & 0 & 0 & 0 \\ -\Delta_{22} & \Delta_{21} & 0 & \Delta_{12} & -\Delta_{11} & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Set Δ_{ij} either 0 or $\Delta \rightarrow 512$ patterns

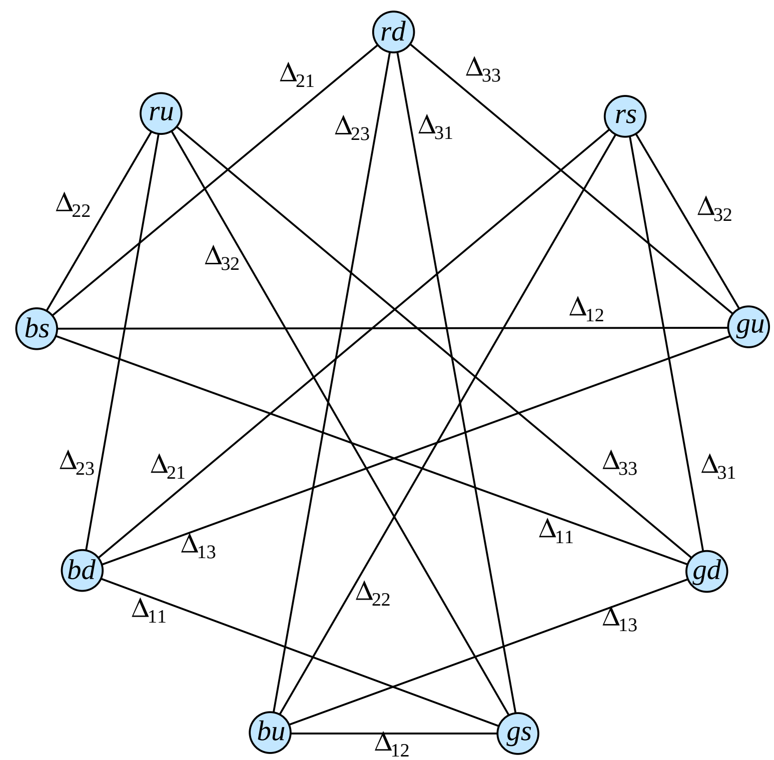
- Not all physically distinct (color symmetry!)

- Every single one may contain different phases (e.g. CFL/gCFL)

- **Automatize calculation (computer aided proof)**

1. Create a list of all 512 gap matrices (patterns)
2. Translate each pattern into graph

$$\Delta = \begin{pmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} \end{pmatrix} \quad \longrightarrow$$



3. Determine components of each graph $\longrightarrow$ 149 different sets of components
4. Compute common Fermi momenta $\nu_1, \ldots, \nu_{c_i}$, $i = 1, \ldots, 149$
5. Search for simultaneous solution (μ_e, μ_3, μ_8) of **Neutrality** and **Stress-free pairing**

- **Result**

None of the patterns fulfills **Neutrality** and **Stress-free pairing** simultaneously

Stressed pairing is unavoidable

→ all patterns can be expected to become unstable to gapless modes at densities comparable to that at which the CFL phase becomes unstable

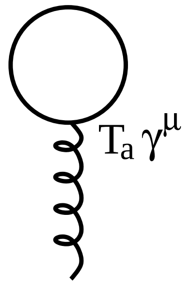
• Color neutrality (page 1/2)

color neutrality in QCD:

D. D. Dietrich and D. H. Rischke, Prog. Part. Nucl. Phys. 53, 305 (2004)

Yang-Mills equation for gluon field (mean field approximation)

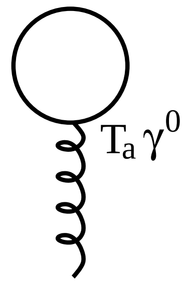
$$\mathcal{D}_\nu F_a^{\mu\nu} = -\frac{g}{2}\,\mathrm{Tr}[\gamma^\mu\, T_a\, \mathcal{S}(\Delta)] =$$



$\mathcal{S}(\Delta)$: quark propagator

constant expectation value of gluon field

$A_a^\mu(x) = g^{0\mu}\, A_a \Rightarrow\ \mathcal{D}_\nu\, F_a^{\mu\nu} = 0$



$\overset{!}{=}$ 0 **color neutrality**

“tadpoles” $\mathrm{Tr}[\gamma^0\, T_a\, \mathcal{S}(\Delta)] \leftrightarrow$ **color charge density** n_a

gluon expectation value $A_a \leftrightarrow$ **color chemical potential** μ_a

$\rightarrow$ **Adjust gluon fields** (or μ_a's) **such that tadpoles** (or n_a's) **vanish**

- **Color neutrality (page 2/2)**

"Usual" procedure: Adjust "color chemical potentials" μ_3, μ_8 such that

$$n_3(\mu_e, \mu_3, \mu_8) = 0 \qquad n_8(\mu_e, \mu_3, \mu_8) = 0$$

However: In general $\mu_1, \ldots, \mu_8 \neq 0$
M. Buballa, I. Shovkovy, PRD 72, 097501 (2005)

Buballa/Shovkovy: "We found a Δ that needs μ_2, μ_4, μ_8!"

Rajagopal/Schmitt: "Well, there are certainly such Δ's."

Buballa/Shovkovy: "But you only considered μ_3, μ_8 and claim to treat *all* phases?!"

Rajagopal/Schmitt: "Take your Δ and do this:
- determine values of μ_2, μ_4, μ_8 (color neutralize the state)
- apply color rotation $U(\mu_2, \mu_4, \mu_8)$:

$$U\,(\mu_2\, T_2 + \mu_4\, T_4 + \mu_8\, T_8)\, U^\dagger = \mu_3'\, T_3 + \mu_8'\, T_8$$

The equivalent gap matrix $U^T\Delta$ *is* correctly included in our analysis!"

Buballa/Shovkovy: "So if you had found a stress-free pattern,
you couldn't say much about this state."

Rajagopal/Schmitt: "That's correct. But the negative result is rigorous."

- **Summary/Conclusion**

  - Cold and *asymptotically* dense quark matter: Only a handful possible phases; CFL favored
  - Cold and not so dense quark matter: Many more possible phases
  - Rigorous proof that *all* phases feel stress
  - Hence they all become unstable to gapless modes
  - Still: Not excluded that less symmetric phases appear in phase diagram

- **Outlook**

Besides theoretical approach, use astrophysics to confirm/exclude possible phases:

- Cooling of neutron stars – neutrino emissivity
  A. Schmitt, I. A. Shovkovy and Q. Wang, PRD 73, 034012 (2006)
- R-mode instabilities – bulk/shear viscosity
  M. Alford, A. Schmitt, in preparation
- Magnetic fields, glitches, etc.