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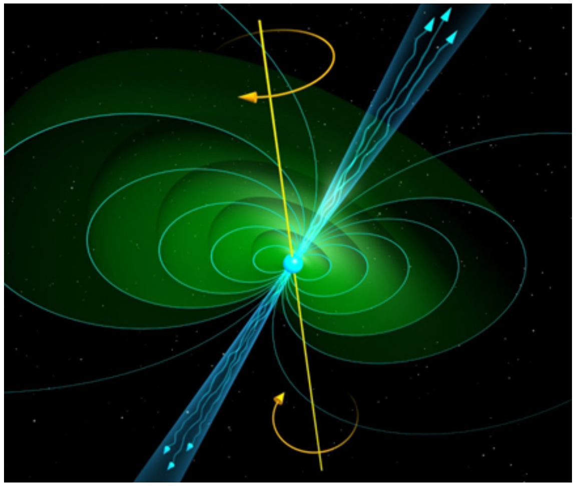
1040 Vienna, Austria

Strongly interacting matter in a magnetic field



... from a field theoretical and a holographic point of view

• Outline

1. Setting the stage: equilibrium phases of QCD
   - QCD phase transitions at nonzero temperature T and chemical potential μ
   - chiral symmetry breaking in QCD
   - laboratories for probing QCD phase transitions: heavy-ion collisions & compact stars
   - QCD at nonzero T, μ, and magnetic field B

2. Effect of a magnetic field on chiral symmetry breaking
   - "*magnetic catalysis*" in the Nambu-Jona Lasinio (NJL) model

3. Magnetic effects in holographic QCD (Sakai-Sugimoto model)
   - the Sakai-Sugimoto model (and how chiral symmetry breaking is realized)
   - phase diagrams in the Sakai-Sugimoto model
   - "*magnetic catalysis*" and "*inverse magnetic catalysis*"
   - comparison to field-theoretical (NJL) results

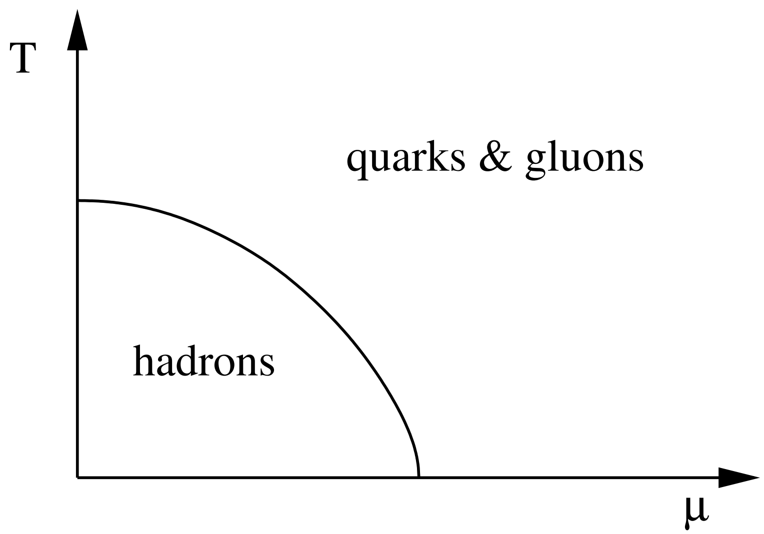
- **Outline**

1. **Setting the stage: equilibrium phases of QCD**
2. Effect of a magnetic field on chiral symmetry breaking
3. Magnetic effects in holographic QCD (Sakai-Sugimoto model)

• QCD phase transitions at nonzero T and μ (page 1/2)

1. quarks & gluons at large T and/or μ are weakly coupled due to asymptotic freedom
   D.J. Gross, F. Wilczek, PRL 30, 1343 (1973); H.D. Politzer, *ibid.* 1346
2. at small T, μ we observe hadrons rather than quarks & gluons

$\Rightarrow$ naive guess of the phase diagram:



N. Cabibbo, G. Parisi, PLB 59, 67 (1975)

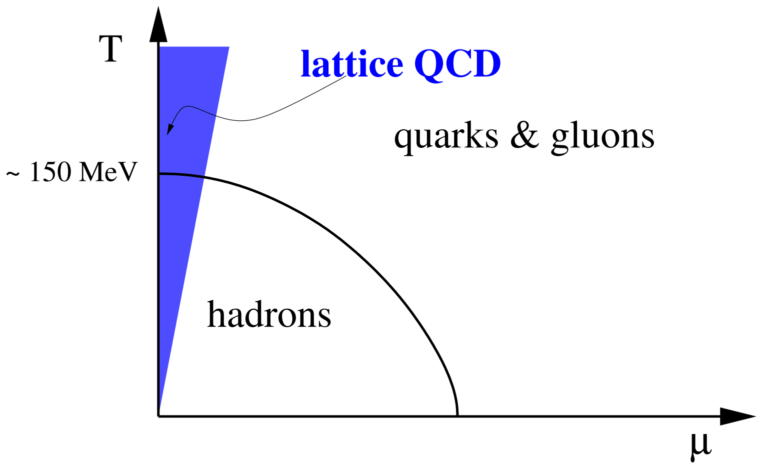
- Nature of transition?
- Order parameter?
- How to observe it?
- How to compute it?

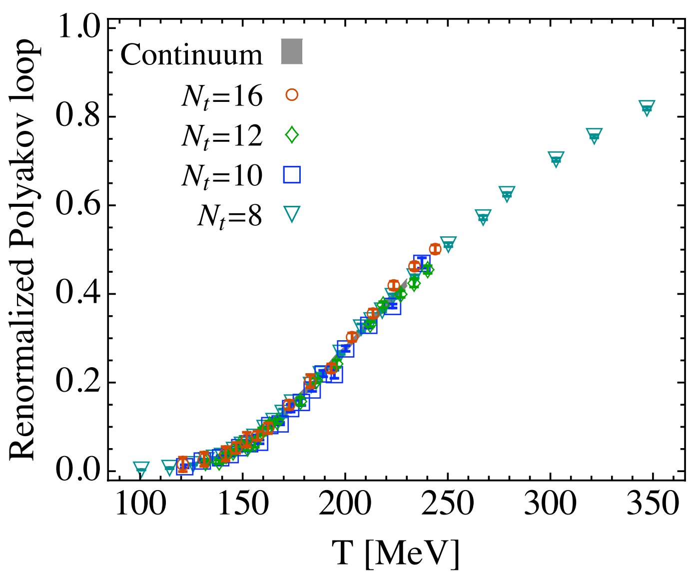
• QCD phase transitions at nonzero T and μ (page 2/2)

- zero chemical potential:

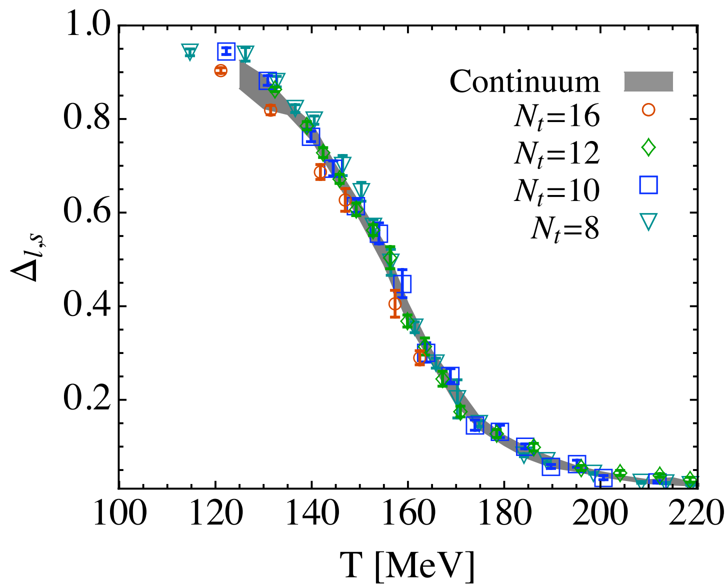
use lattice QCD
to compute transition

S. Borsanyi *et al.* JHEP 1009, 073 (2010)





deconfinement transition
(crossover)



chiral transition
(crossover)

• Chiral symmetry (breaking) in QCD (page 1/3)

QCD Lagrangian	chiral fermions
$\mathcal{L}_{\rm QCD} = \bar\psi(i\gamma^\mu D_\mu - M)\psi - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu}$ $= \bar\psi_R i\gamma^\mu D_\mu \psi_R + \bar\psi_L i\gamma^\mu D_\mu \psi_L$ $-\bar\psi_R M\psi_L - \bar\psi_L M\psi_R - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu}$	$\psi_R \equiv P_R\psi\,, \quad \psi_L \equiv P_L\psi$ $P_R = \frac{1+\gamma^5}{2}\,, \quad P_L = \frac{1-\gamma^5}{2}$

$\Rightarrow M = 0$: $\mathcal{L}_{\rm QCD}$ invariant under $\psi_R \to \underbrace{e^{i\phi_R^a t_a}}_{\in U(N_f)_R}\psi_R\,,\ \psi_L \to \underbrace{e^{i\phi_L^a t_a}}_{\in U(N_f)_L}\psi_L$

$\Rightarrow$ *global* symmetry group

$$U(N_f)_R \times U(N_f)_L \cong \boxed{\underbrace{SU(N_f)_R \times SU(N_f)_L}_{\text{"chiral symmetry"}}} \times U(1)_B \times U(1)_A$$

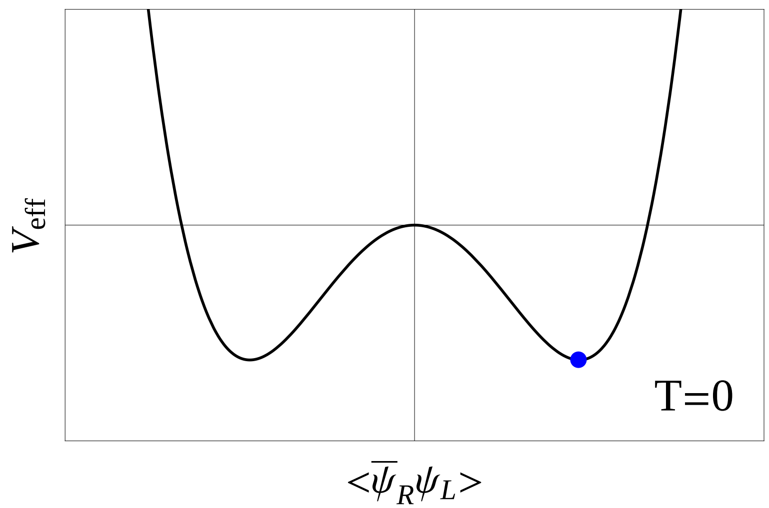
• Chiral symmetry (breaking) in QCD (page 2/3)

- quark mass(es) break chiral symmetry explicitly
- chiral condensate $\langle \bar{\psi}_R \psi_L \rangle$ breaks chiral symmetry spontaneously

$$\boxed{SU(N_f)_R \times SU(N_f)_L \to SU(N_f)_{R+L}}$$

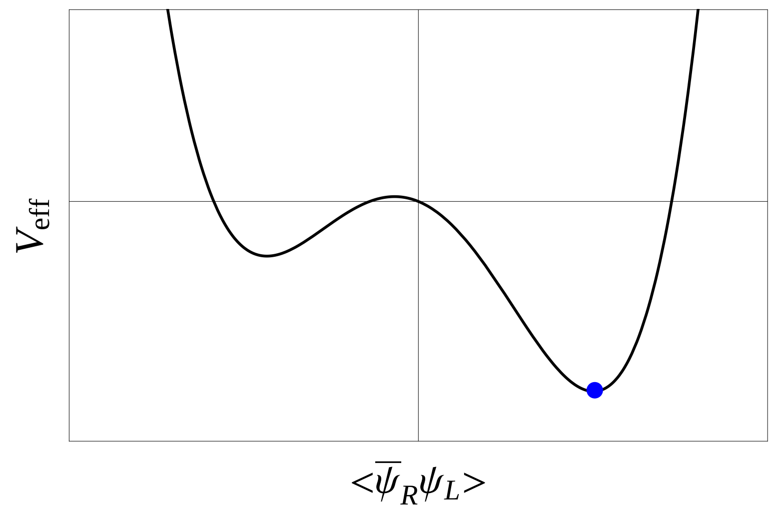
$M = 0$

$\langle \bar{\psi}_R \psi_L \rangle = 0$ for $T \geq T_c$



$M \neq 0$

$\langle \bar{\psi}_R \psi_L \rangle$ always nonzero



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  (possibly 1st order transition at $\mu \neq 0$)

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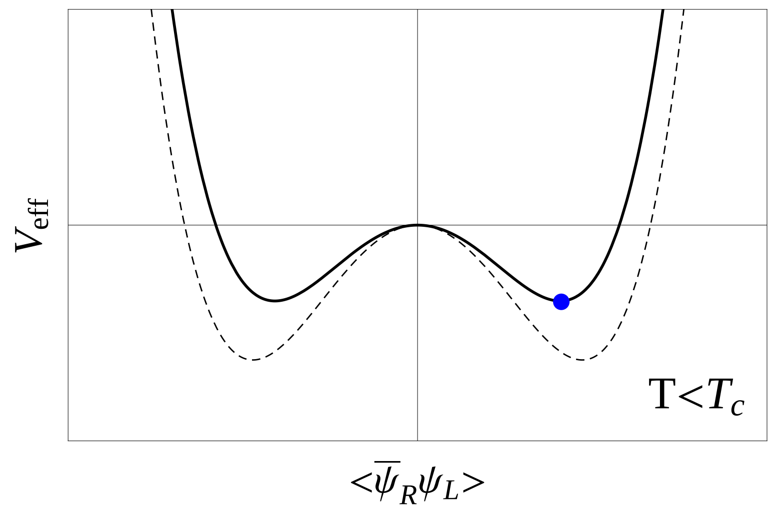

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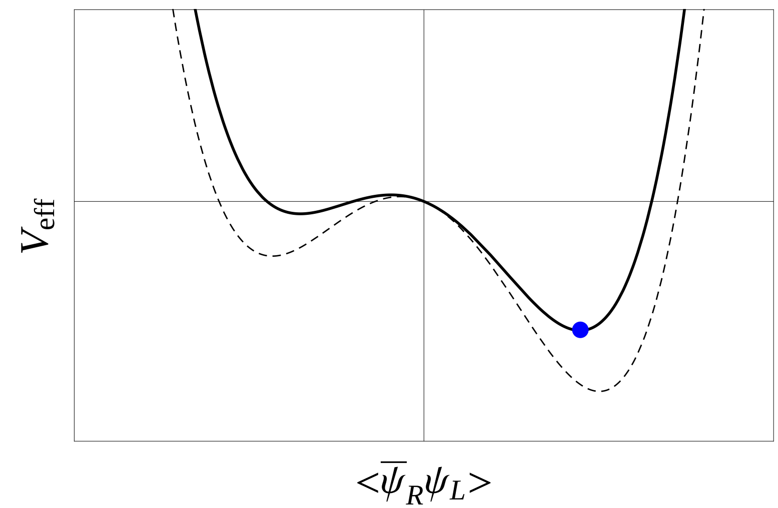

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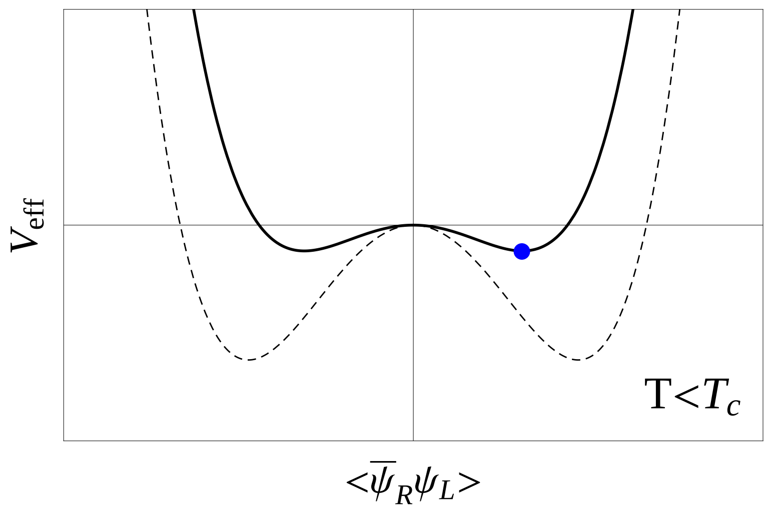

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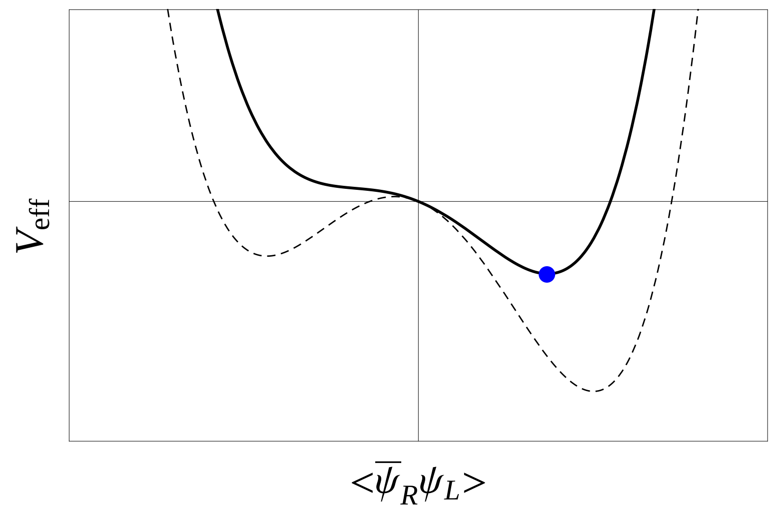

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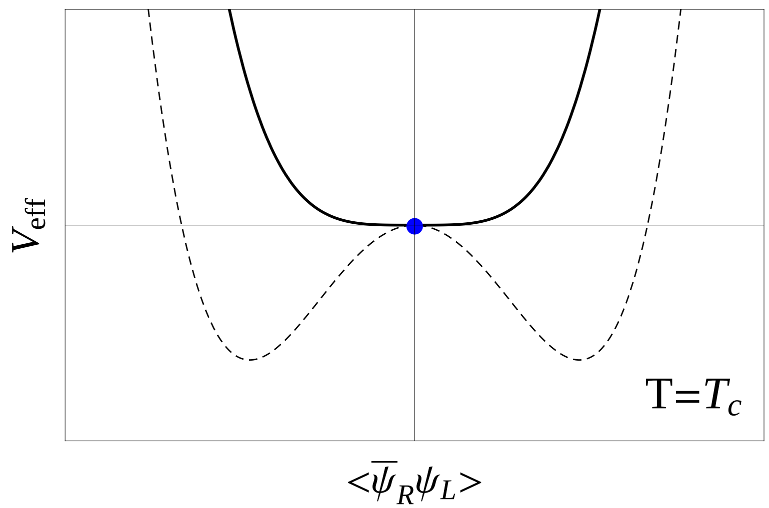

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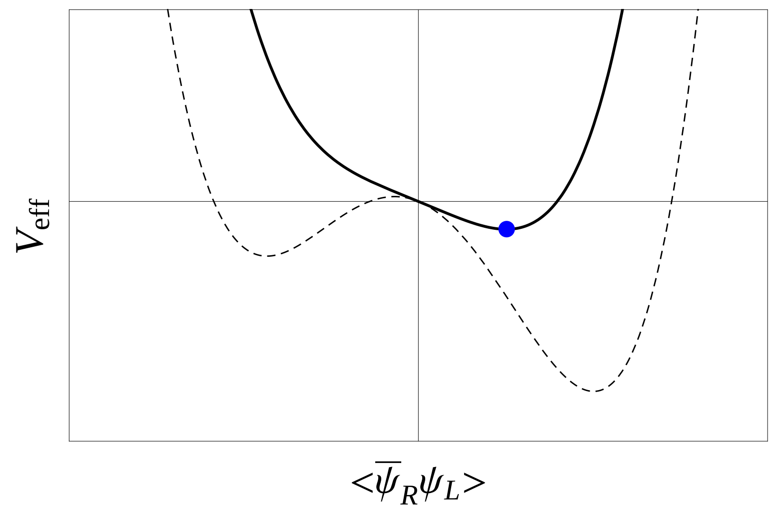

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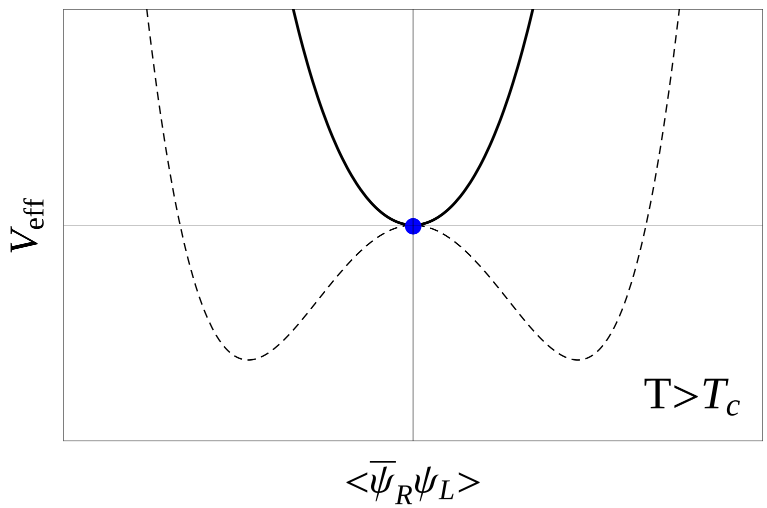

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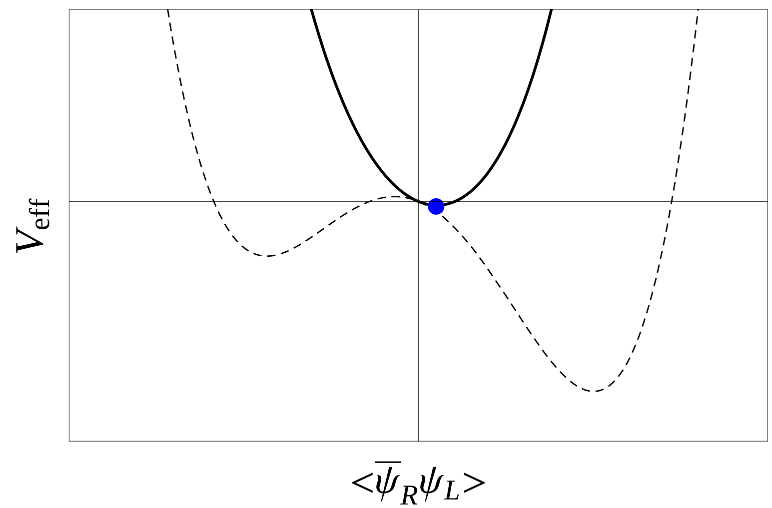
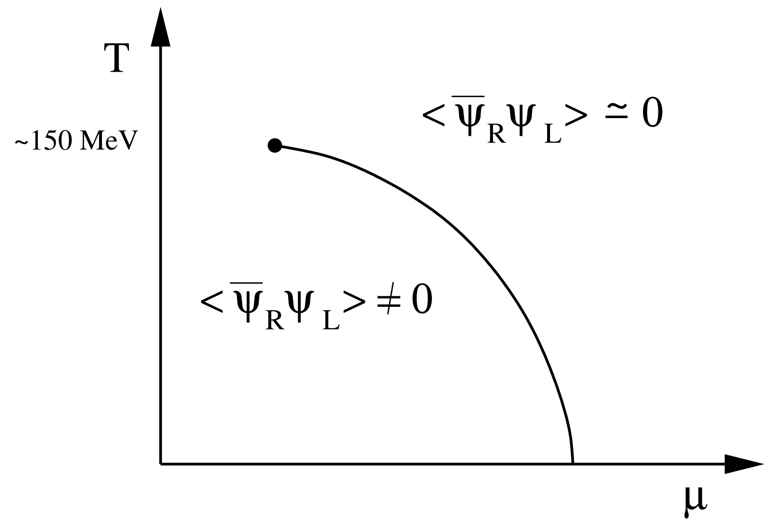


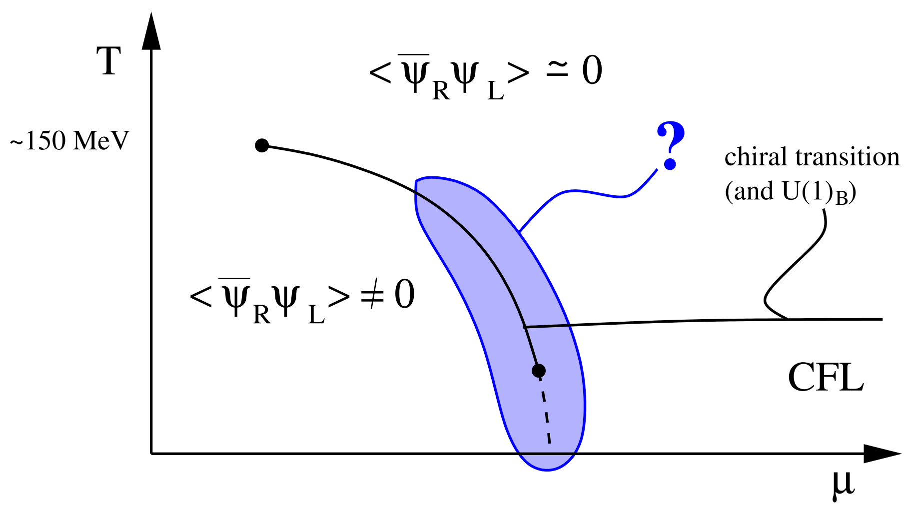
- nonzero quark masses in real world $\longrightarrow$ crossover at $\mu = 0$ (possibly 1st order transition at $\mu \neq 0$)

• Chiral symmetry (breaking) in QCD (page 3/3)

→ refined guess of phase diagram

- no first-principle calculation for intermediate μ



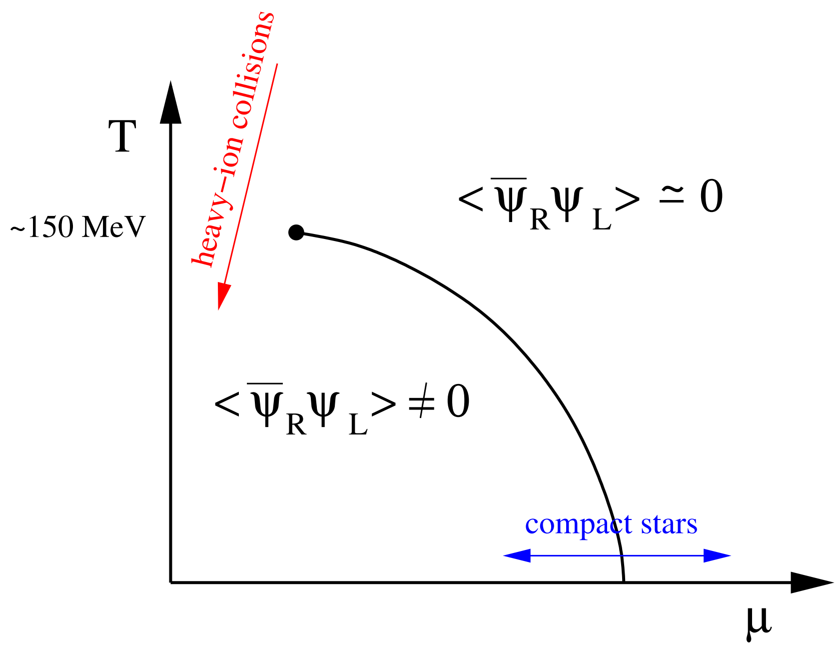


- chiral symmetry also broken spontaneously at asymptotically large μ by color-flavor locking (CFL) ($N_f = 3$)

→ CFL will be ignored for the remainder of the lecture

• "Laboratories" for probing QCD phase transitions (page 1/3)

- theoretically, "intermediate" regions very challenging:
  - energies too small to use perturbation theory (strong coupling!)
  - energies too large to use conventional nuclear physics
- how about experiments?

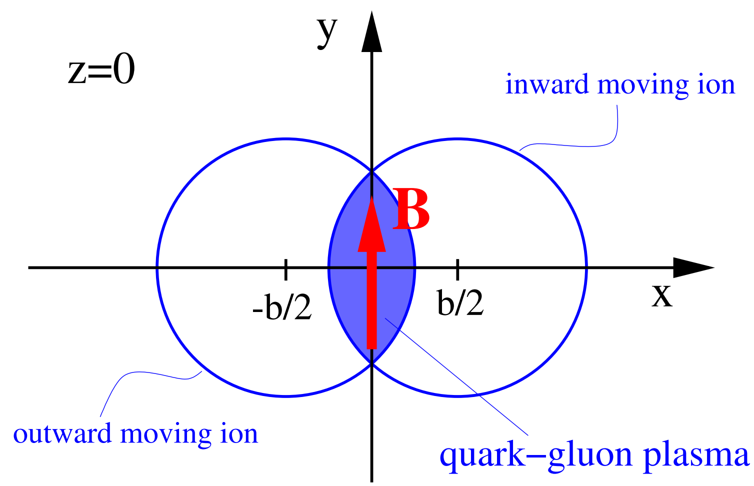


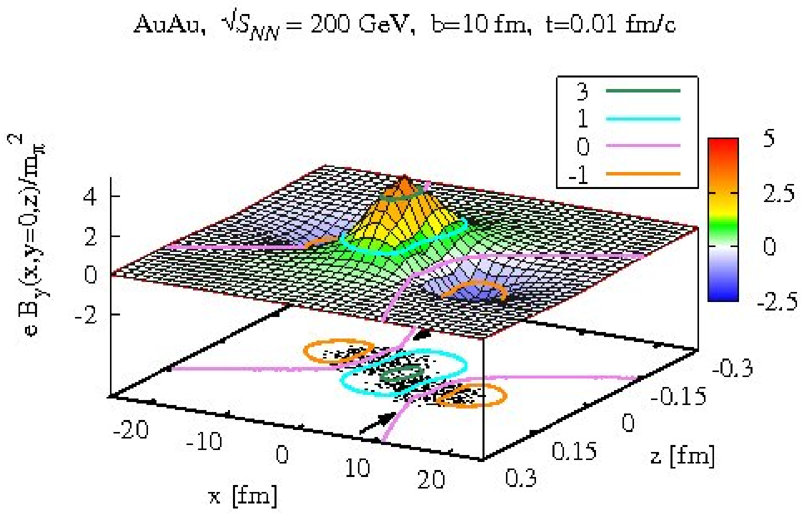
- Heavy-ion collisions: signatures of quark-gluon plasma? (large $T \gtrsim T_c$, small $\mu \ll T$)
- Compact stars: neutron stars or quark stars or hybrid stars? (large $\mu \sim 400\,\mathrm{MeV}$, small $T \ll \mu$)

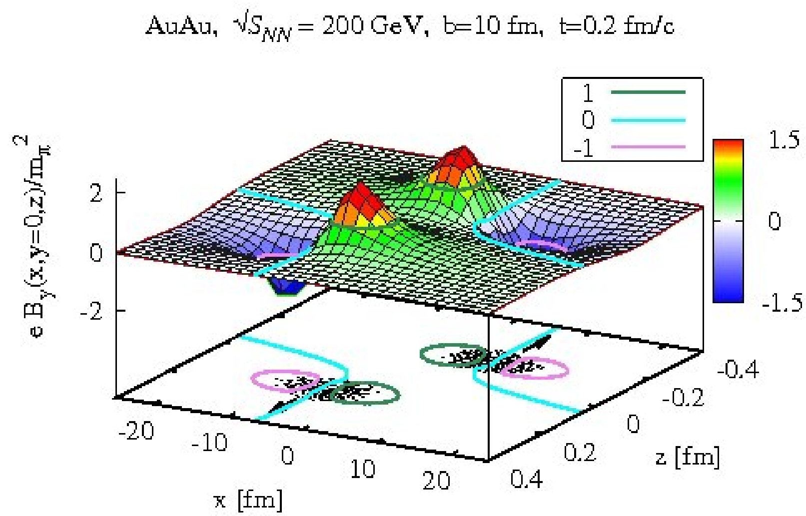
- **In both instances large magnetic fields are present!**

• **“Laboratories” for probing QCD phase transitions (page 2/3)**

(1) Non-central heavy-ion collisions:

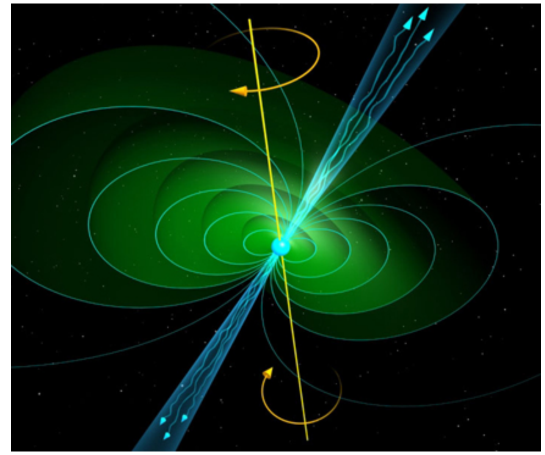




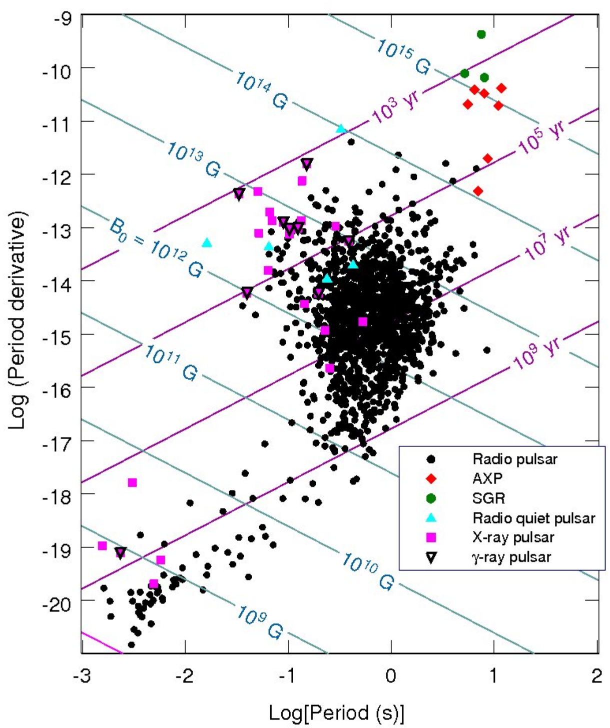


V. Voronyuk, *et al.* PRC 83, 054911 (2011)

• "Laboratories" for probing QCD phase transitions (page 3/3)



(2) Compact stars ("Magnetars"):



- magnetic fields from star's progenitor, strongly enhanced (flux conserved)
- surface magnetic field measured via

$$B \propto (P\dot{P})^{1/2}$$

(magn. dipole radiation)

A. K. Harding, D. Lai, Rept. Prog. Phys. 69, 2631 (2006)

• QCD at nonzero T, μ, and B (page 1/3)

- heavy-ion collisions:

  temporarily $B \lesssim 10^{19}\,\mathrm{G}$

  Skokov, Illarionov, Toneev,
  Int. J. Mod. Phys. A 24, 5925 (2009)

  (*compare:*
  earth's magn. field: $B \simeq 0.6\,\mathrm{G}$
  LHC supercond. magnets: $B \simeq 10^5\,\mathrm{G}$)

- magnetars:

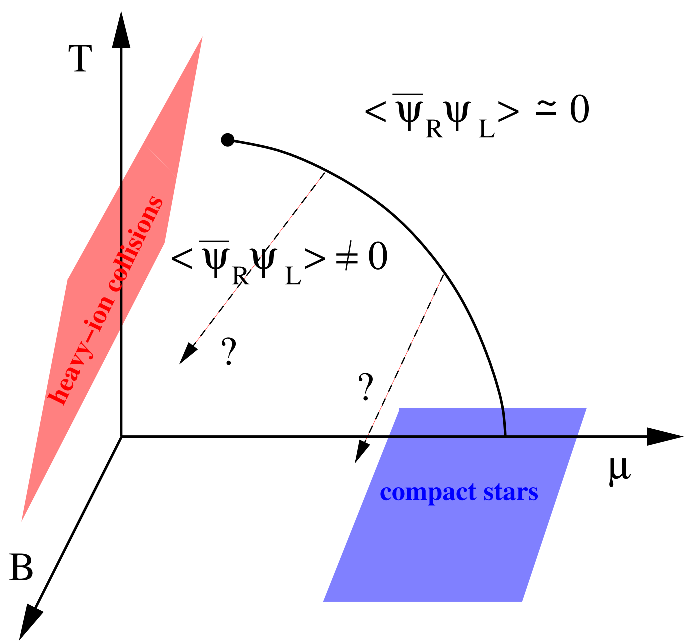
  at surface $B \lesssim 10^{15}\,\mathrm{G}$

  Duncan, Thompson, Astrophys.J. 392, L9 (1992)

  larger in the interior,

  $B \sim 10^{18-20}\,\mathrm{G}$?

  Lai, Shapiro, Astrophys.J. 383, 745 (1991)
  E. J. Ferrer *et al.*, PRC 82, 065802 (2010)



effect on QCD phase transitions?

$$\Lambda_{\mathrm{QCD}}^2 \sim (200\,\mathrm{MeV})^2 \sim 2 \times 10^{18}\,\mathrm{G}$$

$(1\,\mathrm{eV}^2 \simeq 51.189\,\mathrm{G})$

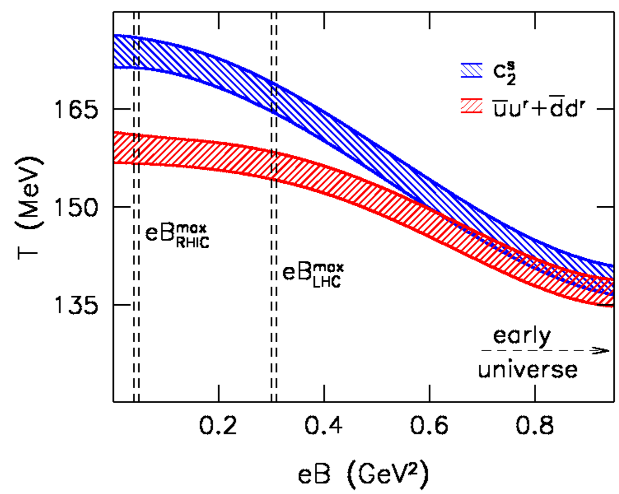
• QCD at nonzero T, μ, and B (page 2/3)

A (very incomplete) collection of recent "magnetic activities":

- QCD phase transitions in a magnetic field on the lattice

  M. D'Elia, S. Mukherjee, F. Sanfilippo, PRD 82, 051501 (2010)

  G.S. Bali, *et al.*, JHEP 1202, 044 (2012) (see plot)



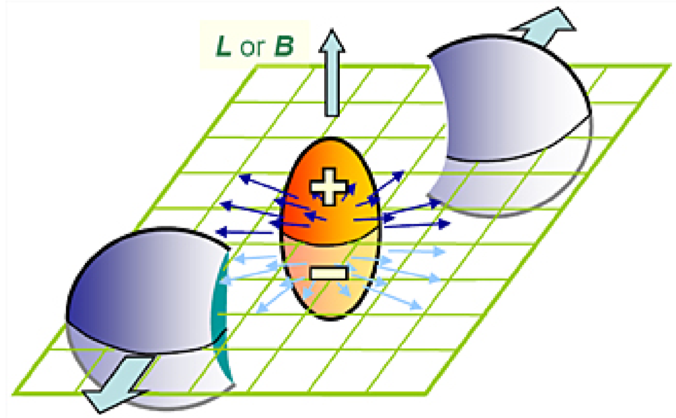
- "splitting" of deconfinement and chiral symmetry breaking

  R. Gatto, M. Ruggieri, PRD 83, 034016 (2011)

  A. J. Mizher, M. N. Chernodub, E. S. Fraga, PRD 82, 105016 (2010)

  holographically: F. Preis, A. Rebhan and A. Schmitt, JHEP 1103, 033 (2011)

• QCD at nonzero T, μ, and B (page 3/3)



- chiral magnetic effect
  Kharzeev, McLerran, Warringa, NPA 803, 227 (2008)
  holographically: H. -U. Yee, JHEP 0911, 085 (2009)
  Rebhan, Schmitt, Stricker, JHEP 1001, 026 (2010)
  A. Gynther, K. Landsteiner, F. Pena-Benitez
  and A. Rebhan, JHEP 1102, 110 (2011)

- ρ meson condensation through magnetic field
  M. N. Chernodub, PRD 82, 085011 (2010)
  holographically: N. Callebaut, D. Dudal, H. Verschelde, arXiv:1105.2217 [hep-th]

- anomalous hydrodynamics
  D. T. Son and P. Surowka, PRL 103, 191601 (2009)
  K. Landsteiner, E. Megias, F. Pena-Benitez, PRL 107, 021601 (2011)

$\longrightarrow$ D. Kharzeev, K. Landsteiner, A. Schmitt, H.-U. Yee (Eds.),
"Strongly interacting matter in magnetic fields", Lect. Notes Phys., to appear in late 2012

- **Summary part 1**

- QCD phase structure is very difficult to compute (especially at finite μ)
- both instances that probe QCD phase transitions involve huge magnetic fields
- also theoretically, nonzero B might help to understand QCD phases (B as another "knob" like N_c, μ_I etc.)

• Outline

1. Setting the stage: equilibrium phases of QCD
2. **Effect of a magnetic field on chiral symmetry breaking**
3. Magnetic effects in holographic QCD (Sakai-Sugimoto model)

• Magnetic catalysis (page 1/5)

K. G. Klimenko, Theor. Math. Phys. 89, 1161-1168 (1992)

V. P. Gusynin, V. A. Miransky, I. A. Shovkovy, PLB 349, 477-483 (1995)

• (massless) fermions in **Nambu-Jona-Lasinio (NJL)** model

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\gamma^\mu\partial_\mu - \mu\gamma^0)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2]$$

Mean-field approximation:

$$\bar{\psi}\psi = \langle\bar{\psi}\psi\rangle + \underbrace{(\bar{\psi}\psi - \langle\bar{\psi}\psi\rangle)}_{\text{small fluctuation}} \Rightarrow (\bar{\psi}\psi)^2 \simeq -\langle\bar{\psi}\psi\rangle^2 + 2\langle\bar{\psi}\psi\rangle\bar{\psi}\psi$$

$$\Rightarrow \quad \mathcal{L}_{\text{mean field}} = \bar{\psi}(i\gamma^\mu\partial_\mu - M - \mu\gamma^0)\psi - \frac{M^2}{4G}$$

⇒ chiral condensate induces "constituent quark mass"

$$M = -2G\langle\bar{\psi}\psi\rangle$$

• Magnetic catalysis (page 2/5)

- determine M from minimizing free energy

$$\frac{\partial\Omega}{\partial M} = 0 \qquad \Rightarrow \qquad M = 2G\sum_e \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{M}{E_k}\tanh\frac{E_k - e\mu}{2T}$$

"gap equation" $(B = 0)$

$$E_k = \sqrt{k^2 + M^2}$$

- gap equation at $T = \mu = 0$

$$1 - \frac{1}{g} = \frac{M^2}{\Lambda^2}\ln\frac{\Lambda}{M}$$

- Λ momentum cutoff
- $g \equiv G\Lambda^2/\pi^2$ dimensionless coupling

Zero magnetic field:

dynamical fermion mass

$$M \propto \langle\bar\psi\psi\rangle \neq 0$$

only for coupling $g > g_c = 1$

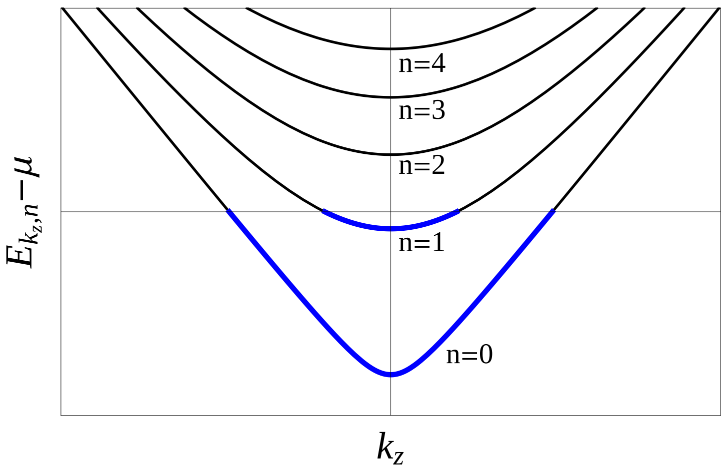
• Magnetic catalysis (page 3/5)

• include magnetic field $\vec{B} = (0, 0, B)$

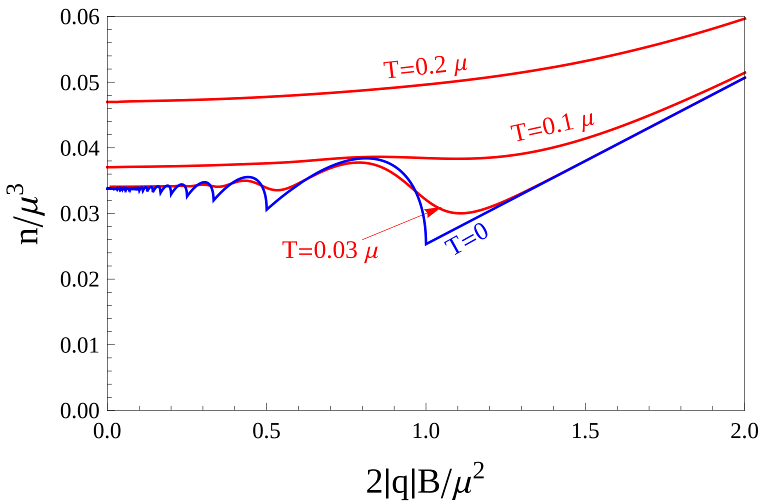
$$2\int \frac{d^3\mathbf{k}}{(2\pi)^3} \to \frac{|q|B}{2\pi}\sum_{n=0}^{\infty}(2-\delta_{n0})\int_{-\infty}^{\infty}\frac{dk_z}{2\pi}$$

$$E_k \to E_{k_z,n} = \sqrt{k_z^2 + 2n|q|B + M^2}$$

• remember Landau levels n:



fermion excitations



density (massless fermions)

• Magnetic catalysis (page 4/5)

- gap equation with magnetic field ($\mu = T = 0$), $x \equiv \frac{M^2}{2|q|B}$

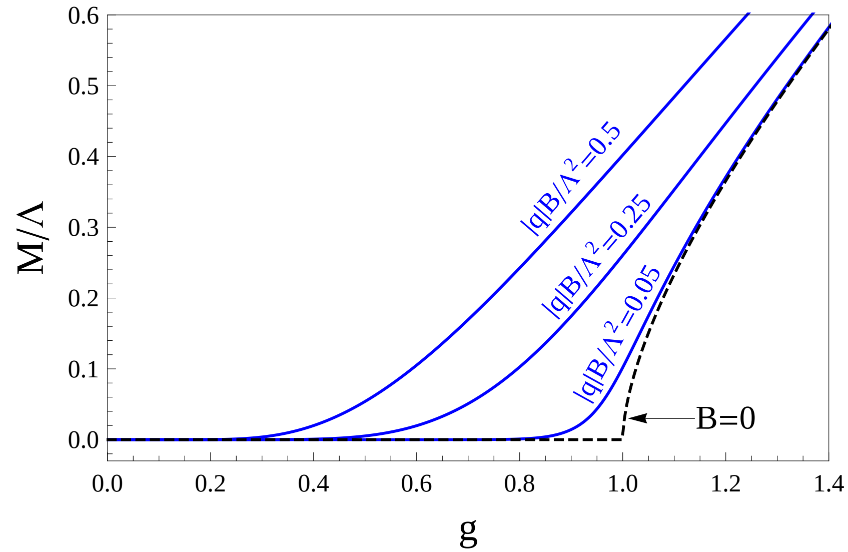
$$1 - \frac{1}{g} = \frac{M^2}{\Lambda^2} \ln \frac{\Lambda}{M} - \underbrace{\frac{|q|B}{\Lambda^2}\left[\left(\frac{1}{2} - x\right)\ln x + x - \frac{1}{2}\ln 2\pi + \ln \Gamma(x)\right]}_{\simeq \frac{|q|B}{\Lambda^2} \ln \frac{\sqrt{|q|B}}{M\sqrt{\pi}} \quad (M^2 \ll |q|B)} .$$

Nonzero magnetic field:

$M \neq 0$ for *arbitrarily small* g,

$$M \simeq \sqrt{\frac{|q|B}{\pi}}\, e^{-\Lambda^2/(|q|Bg)}$$

at weak coupling $g \ll 1$

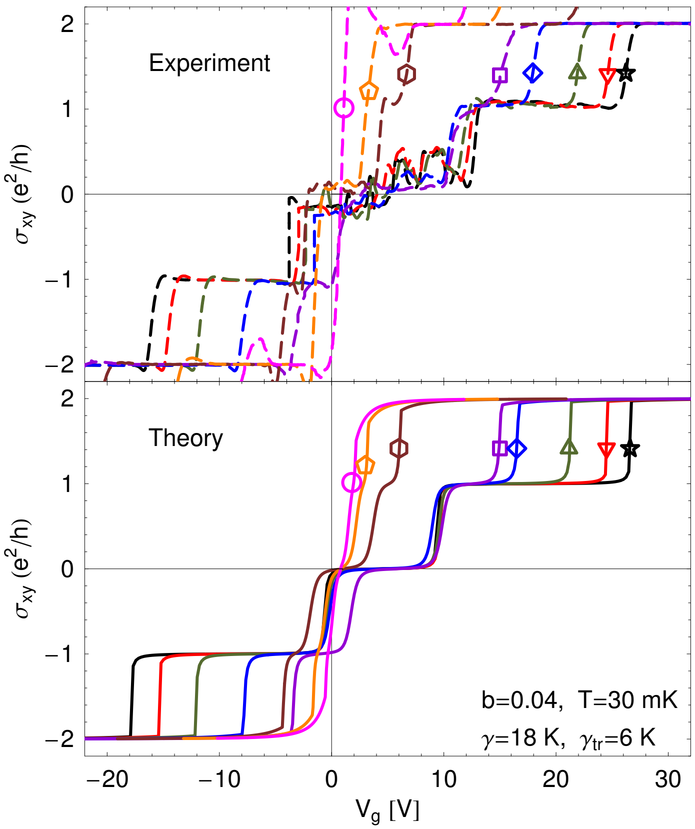


- **Magnetic catalysis (page 5/5)**

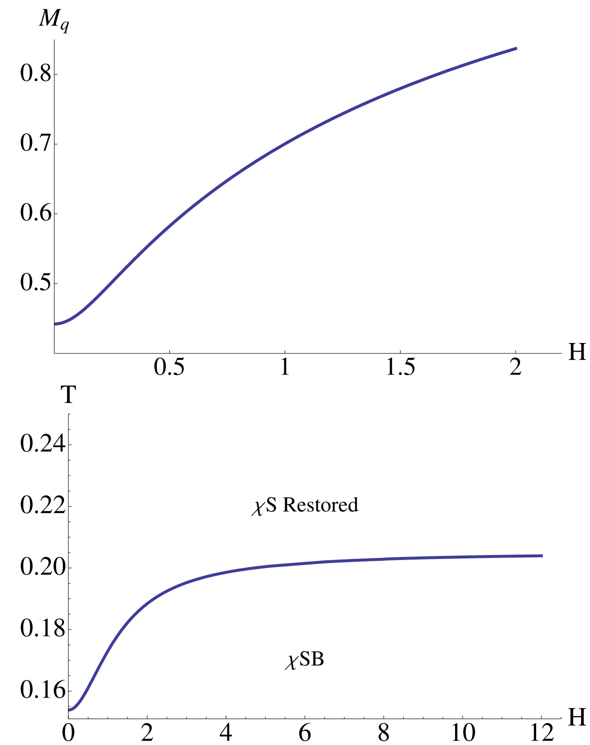
Analogy to BCS Cooper pairing:

BCS superconductor	**Magnetic catalysis**
Cooper pair condensate $\langle\psi\psi\rangle$	chiral condensate $\langle\overline{\psi}\psi\rangle$
$\Delta \propto \mu\, e^{-\text{const.}/G\nu_F}$ (ν_F: d.o.s. at $E = \mu$ Fermi surface)	$M \propto \sqrt{eB}\, e^{-\text{const.}/G\nu_0}$ (ν_0: d.o.s. at $E = 0$ surface)
pairing dynamics effectively (1+1)-dimensional because of Fermi surface	effectively (1+1)-dimensional in lowest Landau level (LLL) because of magn. field
gap equation $\Delta = \frac{\mu^2 G}{2\pi^2}\int_0^\infty dk\, \frac{\Delta}{\sqrt{(k-\mu)^2+\Delta^2}}$	gap equation (LLL) $M = \frac{\lvert q\rvert BG}{2\pi^2}\int_{-\infty}^\infty dk_z\, \frac{M}{\sqrt{k_z^2+M^2}}$

• Magnetic catalysis in the real world and in holography



V.P.Gusynin *et al.*, PRB 74, 195429 (2006)



C.V.Johnson, A.Kundu, JHEP 0812, 053 (2008)

- **graphene**: appearance of additional plateaus in strong magnetic fields [$B = 9\,\mathrm{T}$ (pink), $B = 45\,\mathrm{T}$ (black)]

- **Sakai-Sugimoto**: magnetic field enhances dynamical mass M_q and critical temperature T_c

$\rightarrow$ *see next part of this lecture*

- **Summary part 2**

Magnetic catalysis

=

magnetic field favors/enhances $\bar{\psi} - \psi$ pairing

- **Outline**

1. Setting the stage: equilibrium phases of QCD

2. Effect of a magnetic field on chiral symmetry breaking

3. **Magnetic effects in holographic QCD (Sakai-Sugimoto model)**

• Applications of the gauge/gravity duality to QCD

a "pedestrian's guide": S. S. Gubser and A. Karch, Ann. Rev. Nucl. Part. Sci. 59, 145 (2009)

- compare with $\mathcal{N} = 4$ SYM

  typically in the context of heavy-ion collisions

  see for instance the review
  Casalderrey-Solana, Liu, Mateos, Rajagopal, Wiedemann, arXiv:1101.0618 [hep-th]

  – viscosity G. Policastro, D. T. Son, A. O. Starinets, PRL 87, 081601 (2001)
  – jet quenching H. Liu, K. Rajagopal, U. A. Wiedemann, PRL 97, 182301 (2006)
  – expanding plasma R. A. Janik, R. B. Peschanski, PRD 73, 045013 (2006)

- towards a gravity dual of QCD

  – add flavor to AdS/CFT A. Karch, E. Katz, JHEP 0206, 043 (2002)
  – "bottom-up" approach Erlich, Katz, Son, Stephanov, PRL 95, 261602 (2005)
  – Sakai-Sugimoto model ("top-down")
  T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

• The Sakai-Sugimoto model in two steps

1. Background geometry with D4-branes

   E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998)

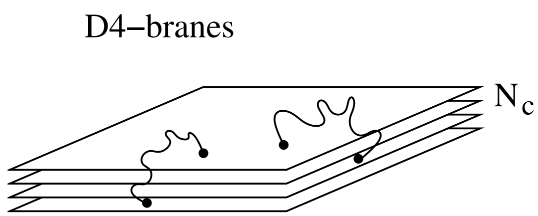
   M. Kruczenski, D. Mateos, R. C. Myers, D. J. Winters, JHEP 0405, 041 (2004)

2. Add flavor D8-branes

   T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

• Sakai-Sugimoto model: background geometry (p. 1/3)

N_c D4-branes compactified on circle $x_4 \equiv x_4 + 2\pi/M_{\rm KK}$



- 4-4 strings → adjoint scalars & fermions, gauge fields
- periodic x_4 → break SUSY by giving mass $\sim M_{\rm KK}$ to scalars & fermions

⇒ $SU(N_c)$ gauge theory

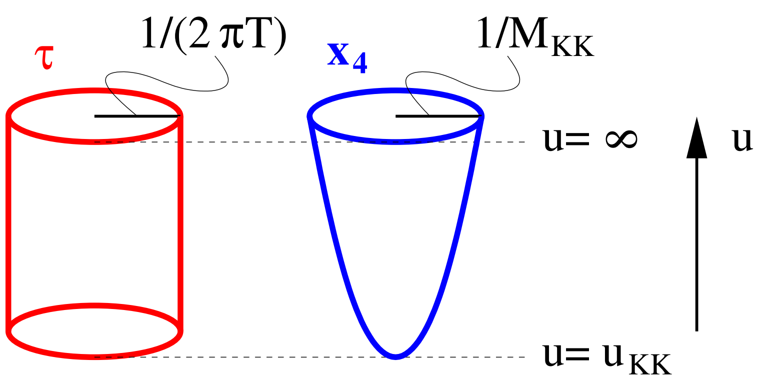
$$\lambda = \frac{g_5^2 N_c}{2\pi/M_{\rm KK}}$$

	$\lambda \ll 1$	$\lambda \gg 1$
dual to large-N_c QCD (at energies $\ll M_{\rm KK}$)	√ $\Lambda_{\rm QCD} \ll M_{\rm KK}$	**x** $\Lambda_{\rm QCD} \sim M_{\rm KK}$
gravity approximation	**x**	√

• Background geometry (page 2/3): two solutions

Confined phase

$$ds^2_{\rm conf} = \left(\frac{u}{R}\right)^{3/2}\left[d\tau^2 + d\mathbf{x}^2 + f(u)dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$

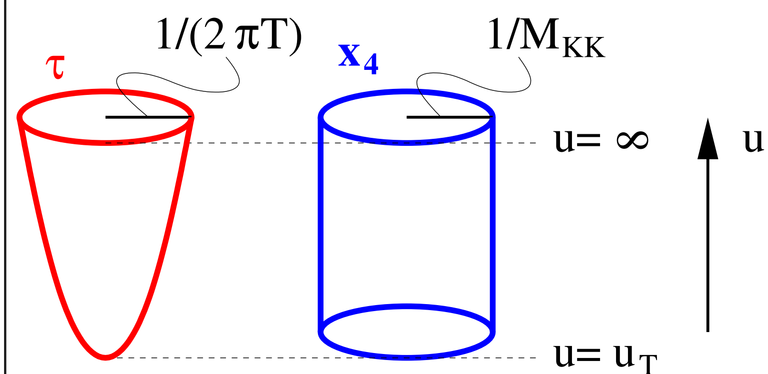


$$M_{\rm KK} = \frac{3}{2}\frac{u_{\rm KK}^{1/2}}{R^{3/2}} \qquad f(u) \equiv 1 - \frac{u_{\rm KK}^3}{u^3}$$

Wick rotated regular geometry

Deconfined phase

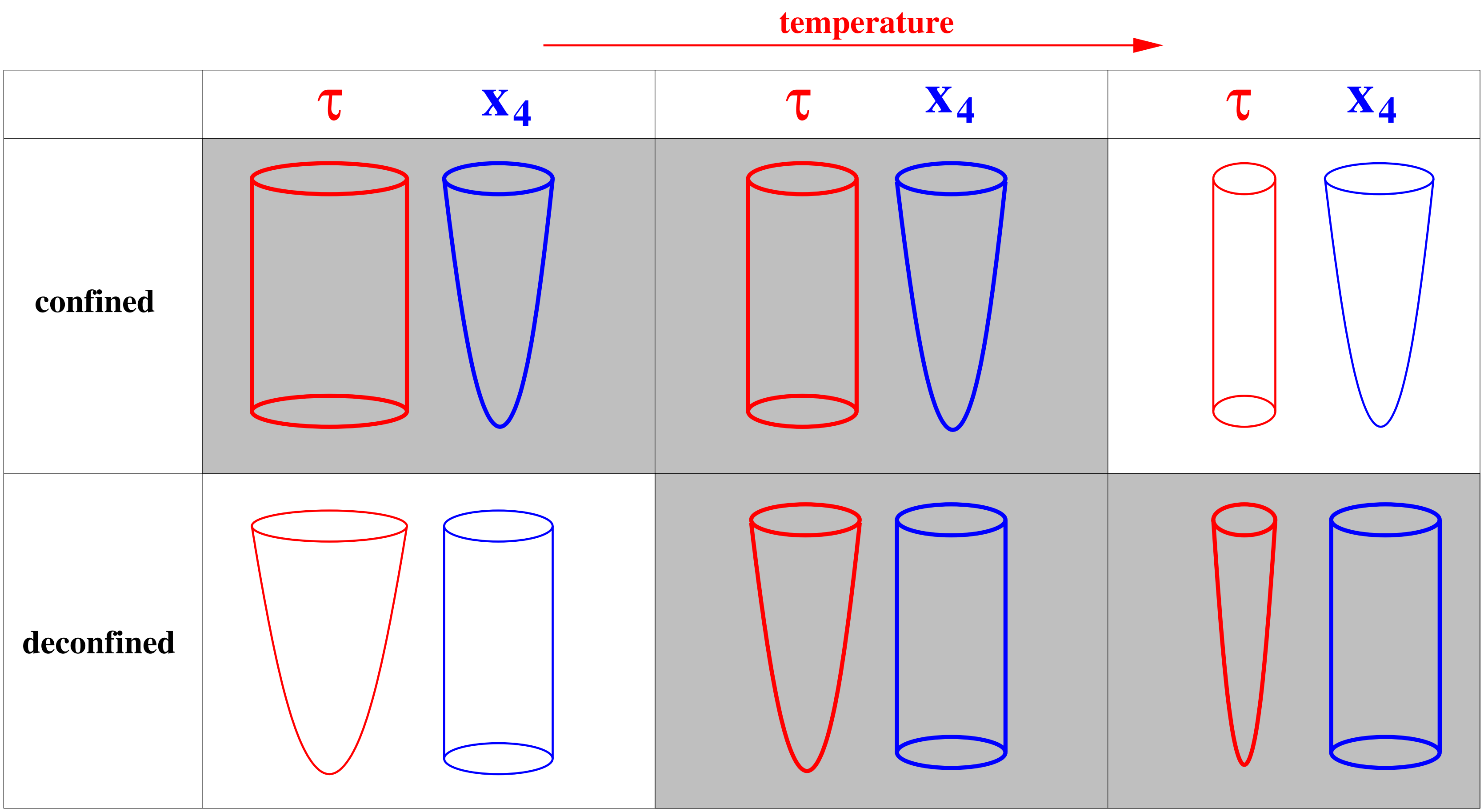
$$ds^2_{\rm deconf} = \left(\frac{u}{R}\right)^{3/2}\left[\tilde{f}(u)d\tau^2 + \delta_{ij}d\mathbf{x}^2 + dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{\tilde{f}(u)} + u^2 d\Omega_4^2\right]$$



$$T = \frac{3}{4\pi}\frac{u_T^{1/2}}{R^{3/2}} \qquad \tilde{f}(u) \equiv 1 - \frac{u_T^3}{u^3}$$

Wick rotated black brane

- **Background geometry (page 3/3): deconfinement phase transition**



$$\boxed{T_c = \frac{M_{\mathrm{KK}}}{2\pi}}$$

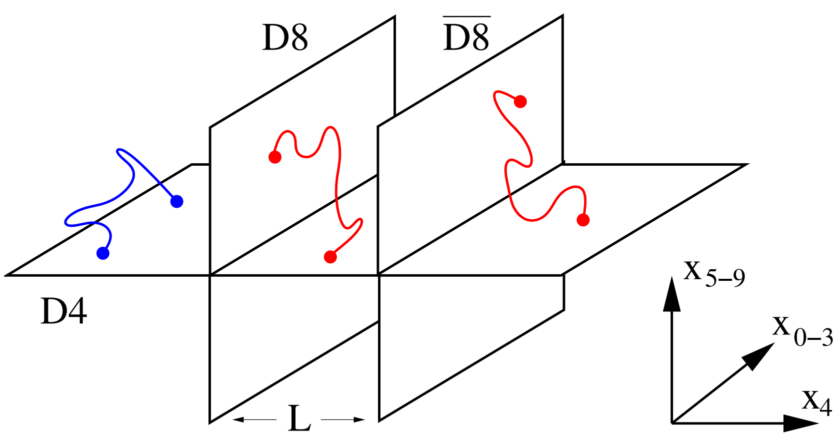
$$\text{fit } M_{\mathrm{KK}} = 949\,\mathrm{MeV} \text{ to reproduce } \rho \text{ mass}$$
$$\Rightarrow T_c \simeq 150\,\mathrm{MeV}$$

• Add flavor (page 1/2)

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

- add N_f D8- and $\overline{\text{D8}}$-branes, separated in x_4

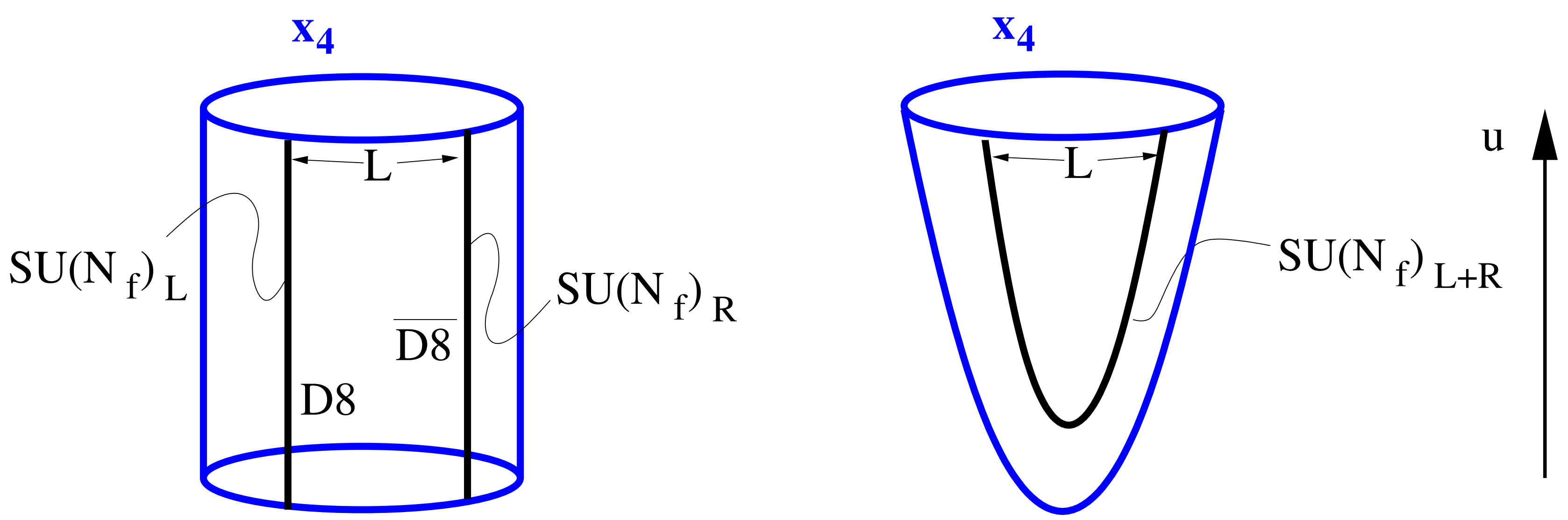
	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8/$\overline{\text{D8}}$	x	x	x	x		x	x	x	x	x



- 4-8, 4-$\bar{8}$ strings
  $\rightarrow$ fundamental, massless
  chiral fermions
  under $U(N_f)_L \times U(N_f)_R$
  $\Rightarrow$ quarks & gluons

• Add flavor (page 2/2): Chiral symmetry breaking

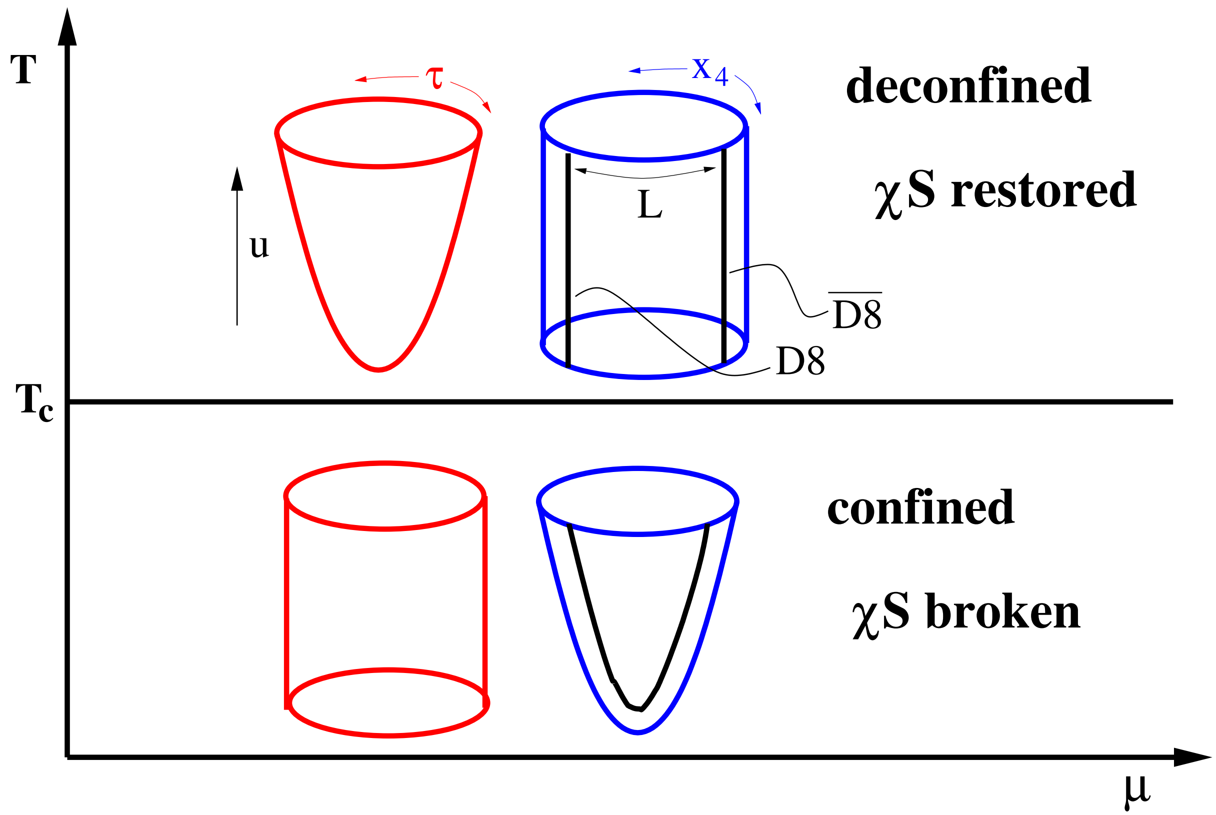
- background geometry unchanged if $N_f \ll N_c$ ("probe branes") $\rightarrow$ "quenched" approximation
- gauge symmetry on the branes $\rightarrow$ global symmetry at $u = \infty$



- chiral symmetry breaking

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_{L+R}$$

• Chiral transition in the Sakai-Sugimoto model (p. 1/3)



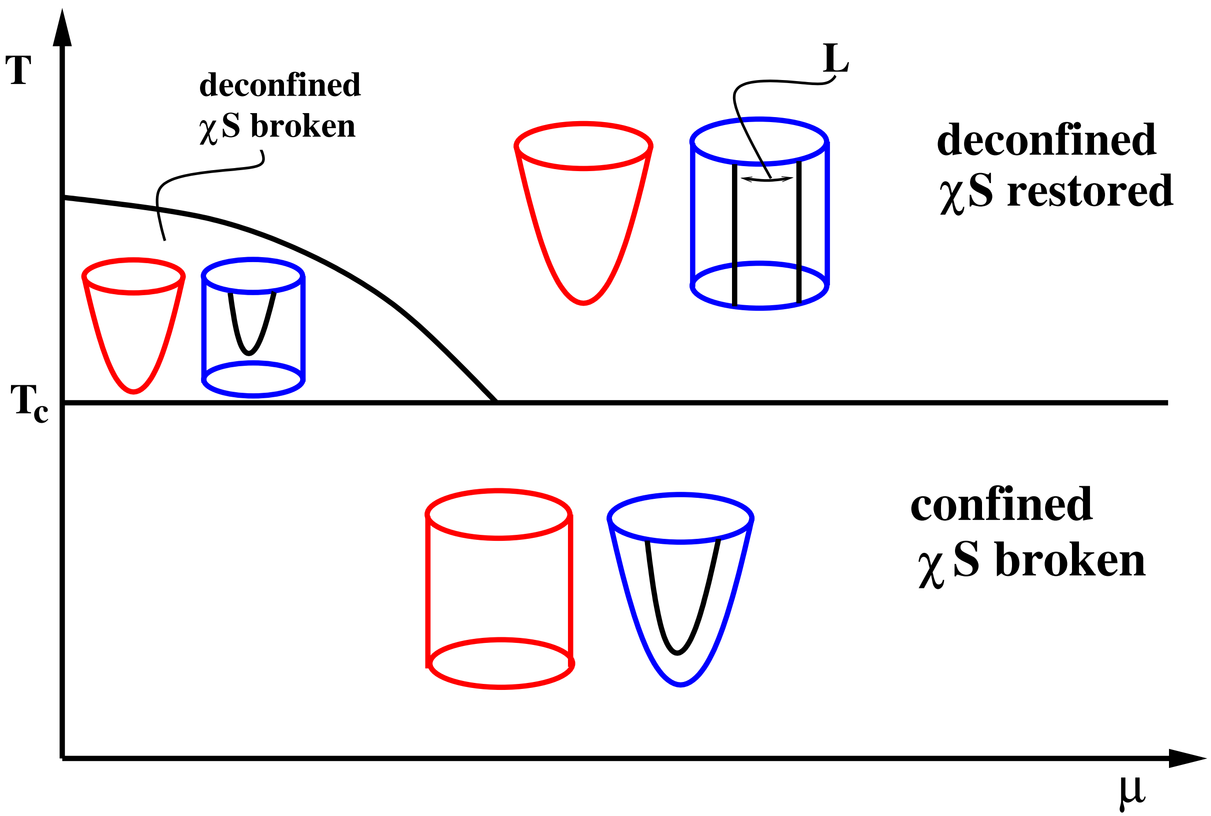
- not unlike expectation from large-N_c QCD
- in probe brane approximation: **chiral transition** unaffected by quantities on flavor branes (μ, B, ...)

• Chiral transition in the Sakai-Sugimoto model (p. 2/3)

- less "rigid" behavior for smaller L
- deconfined, chirally broken phase for $L < 0.3\,\pi/M_{\mathrm{KK}}$

O. Aharony, J. Sonnenschein, S. Yankielowicz, Annals Phys. 322, 1420 (2007)

N. Horigome, Y. Tanii, JHEP 0701, 072 (2007)

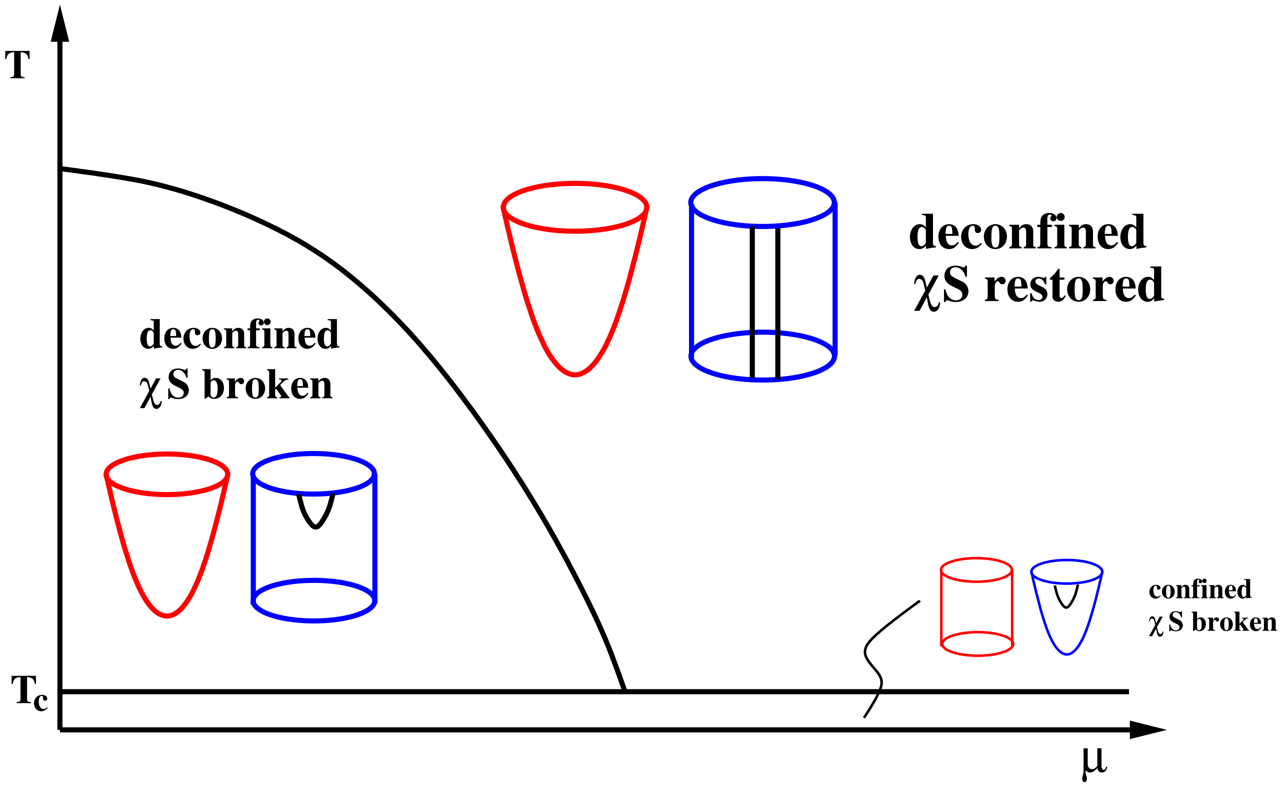


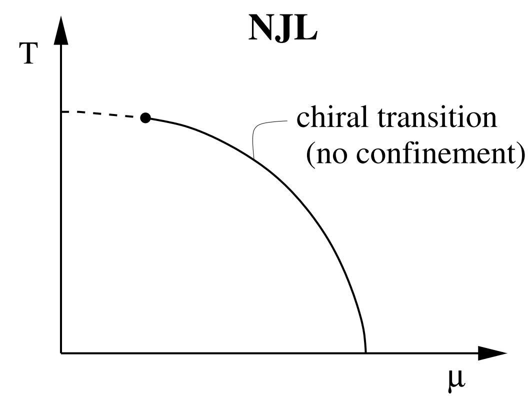
• Chiral transition in the Sakai-Sugimoto model (p. 3/3)

- $L \ll \pi/M_{\text{KK}}$ corresponds to (non-local) NJL model

  E. Antonyan, J. A. Harvey, S. Jensen, D. Kutasov, hep-th/0604017

  J. L. Davis, M. Gutperle, P. Kraus, I. Sachs, JHEP 0710, 049 (2007)





- "decompactified" limit → gluon dynamics decouple
- this limit is considered in the following calculation ...

- **Sketch of the holographic calculation (page 1/3)**

- D8-brane action

$$S = \underbrace{T_8 V_4 \int d^4x \int dU\, e^{-\Phi} \sqrt{\det(g + 2\pi\alpha' F)}}_{\text{Dirac-Born-Infeld (DBI)}} + \underbrace{\frac{N_c}{24\pi^2} \int d^4x \int \; A_\mu F_{u\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}}_{\text{Chern-Simons (CS)}} ,$$

- deconfined geometry, $N_f = 1$

$$S \;=\; \mathcal{N} \int du \sqrt{u^5 + b^2 u^2} \sqrt{1 + f a_3'^2 - a_0'^2 + u^3 f x_4'^2} + \frac{3\mathcal{N}}{2} b \int du\, (a_3 a_0' - a_0 a_3')$$

(dimensionless quantities, $a_\mu = \frac{2\pi\alpha'}{R} A_\mu$, $b = 2\pi\alpha' B$)

- chemical potential $\mu = a_0(\infty)$
- magnetic field in 3-direction $b = F_{12}(\infty)$
- $a_3(u)$ induced $\rightarrow$ anisotropic condensate $a_3(\infty) = \nabla\pi^0$

• Sketch of the holographic calculation (page 2/3)

- equations of motion:

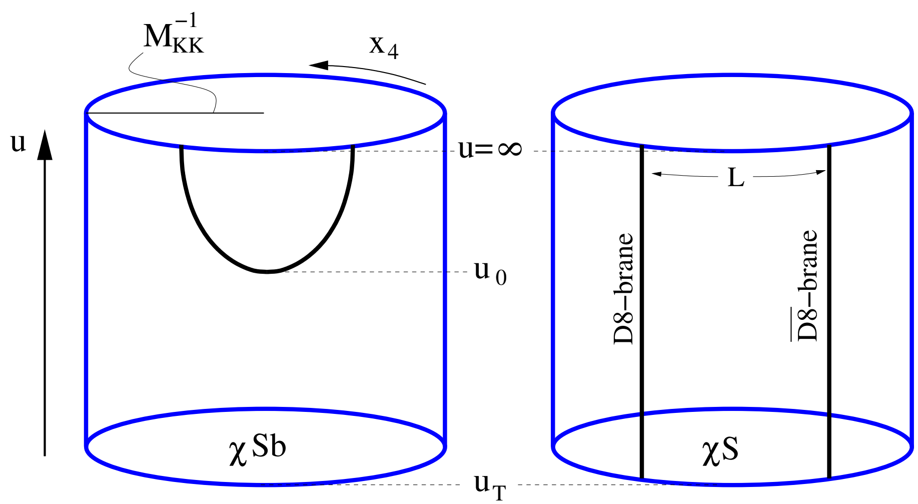
$$\partial_u\left(\frac{a_0'\sqrt{u^5+b^2u^2}}{\sqrt{1+fa_3'^2-a_0'^2+u^3fx_4'^2}}\right) = 3ba_3'$$

$$\partial_u\left(\frac{f\,a_3'\sqrt{u^5+b^2u^2}}{\sqrt{1+fa_3'^2-a_0'^2+u^3fx_4'^2}}\right) = 3ba_0'$$

$$\partial_u\left(\frac{u^3f\,x_4'\sqrt{u^5+b^2u^2}}{\sqrt{1+fa_3'^2-a_0'^2+u^3fx_4'^2}}\right) = 0$$

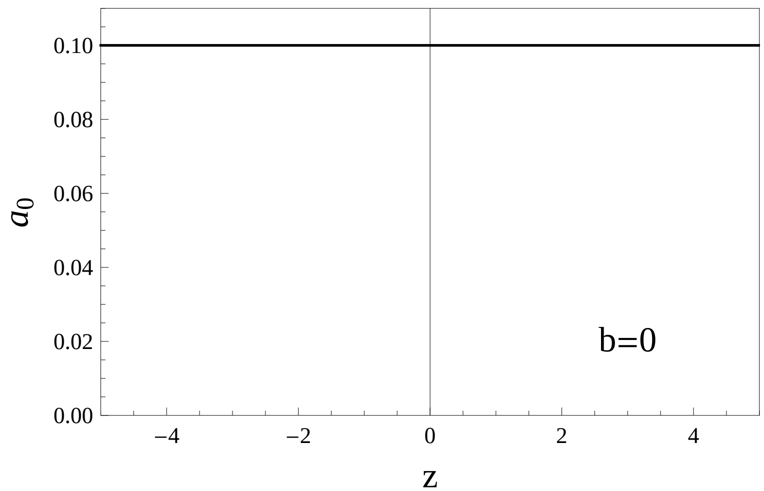
- to be solved for $a_0(u)$, $a_3(u)$, $x_4(u)$

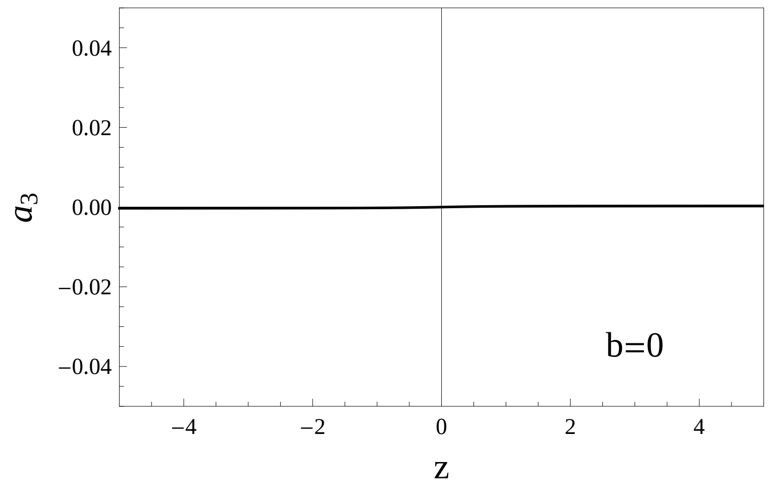
$$x_4(u) = \begin{cases} \text{const.} & \chi\text{S} \\ \text{nontrivial} & \chi\text{Sb} \end{cases}$$

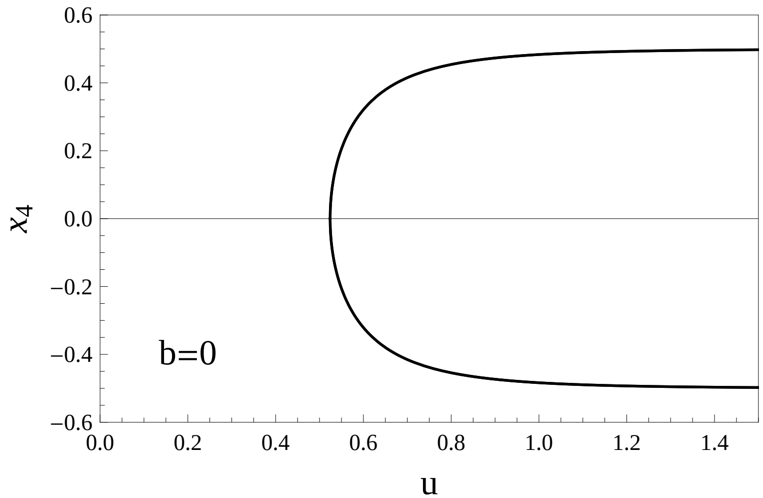


• Sketch of the holographic calculation (page 3/3)

- solutions of EoM; e.g., chirally broken phase, $u = (u_0^3 + u_0 z^2)^{1/3}$







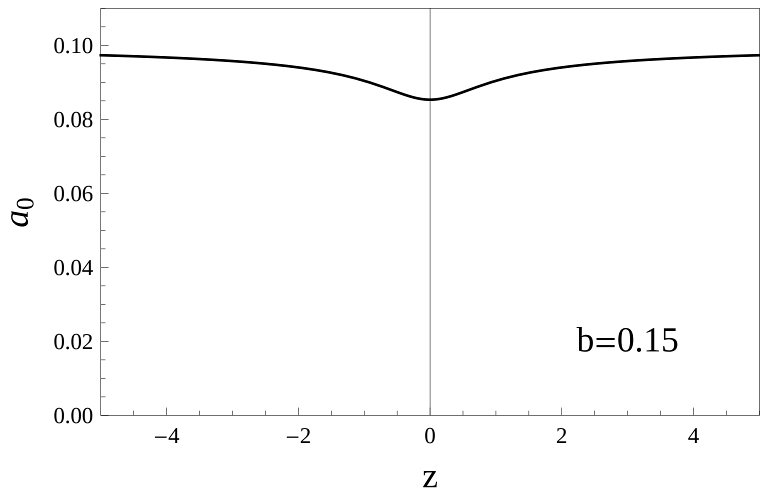
→ insert solutions back into

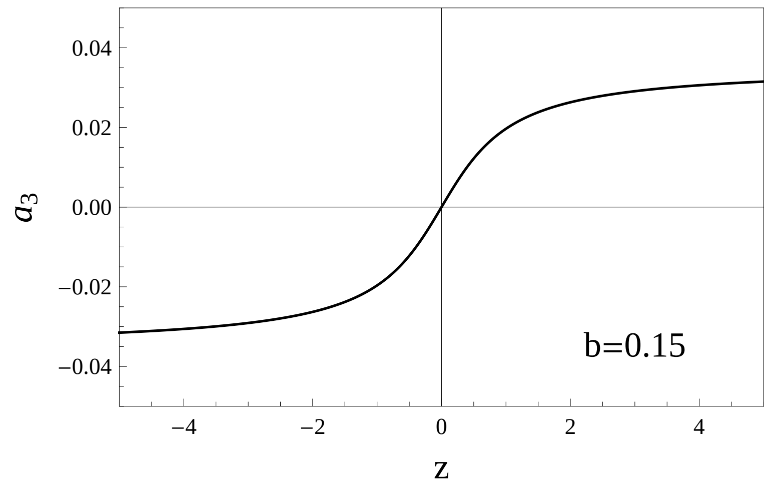
$$\Omega = \frac{T}{V} S_{\text{on-shell}}$$

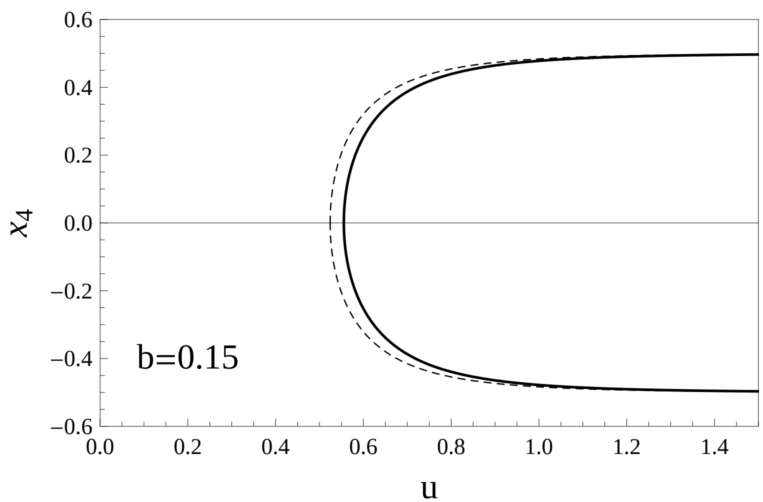
to compute
chiral phase transition

• Sketch of the holographic calculation (page 3/3)

- solutions of EoM; e.g., chirally broken phase, $u = (u_0^3 + u_0 z^2)^{1/3}$







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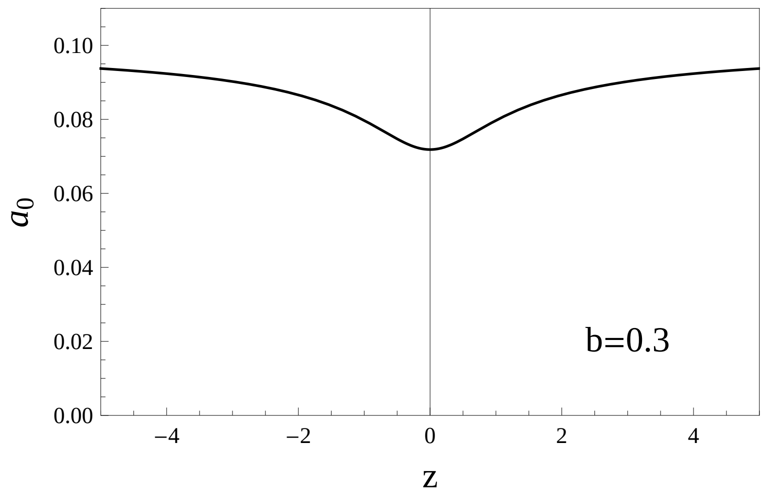
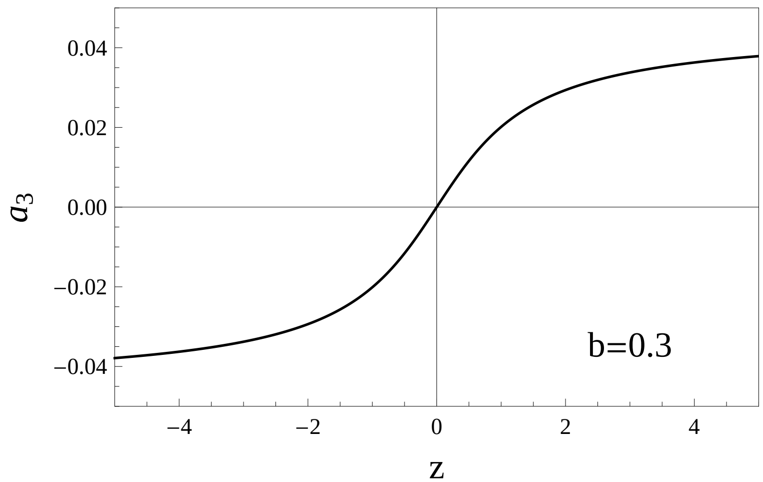
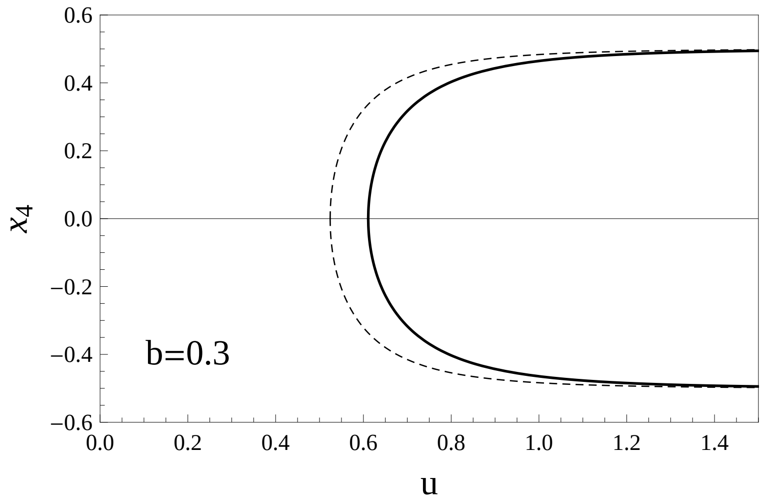




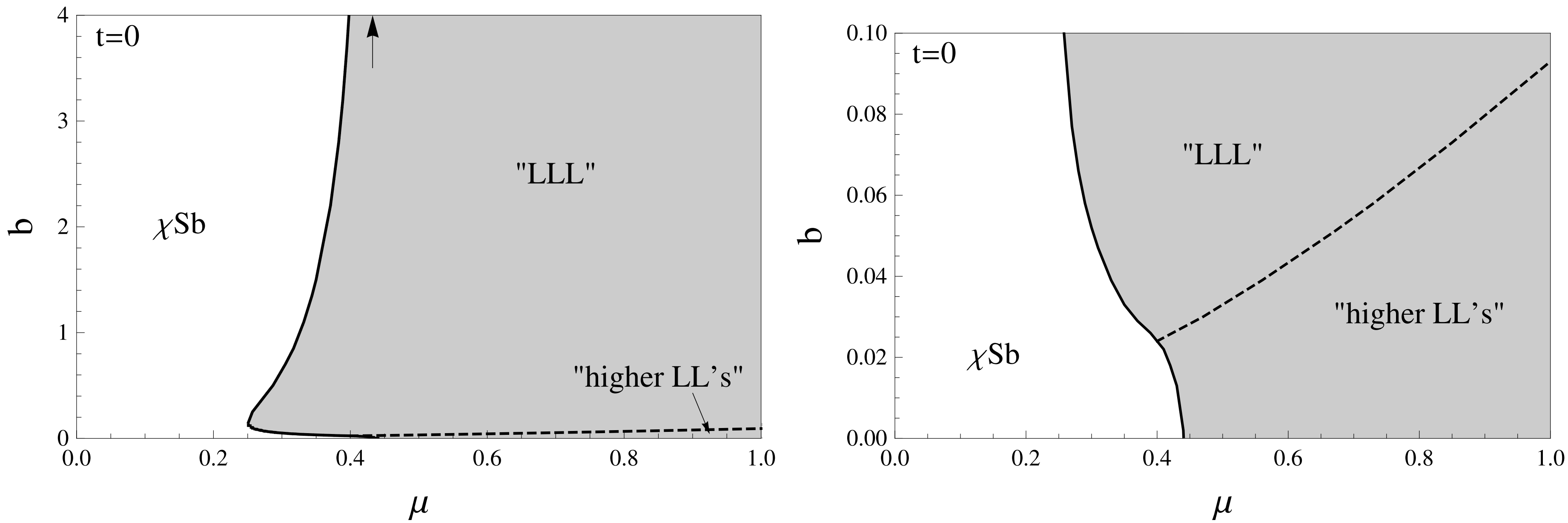


$\rightarrow$ insert solutions back into

$$\Omega = \frac{T}{V} S_{\text{on-shell}}$$

to compute
chiral phase transition

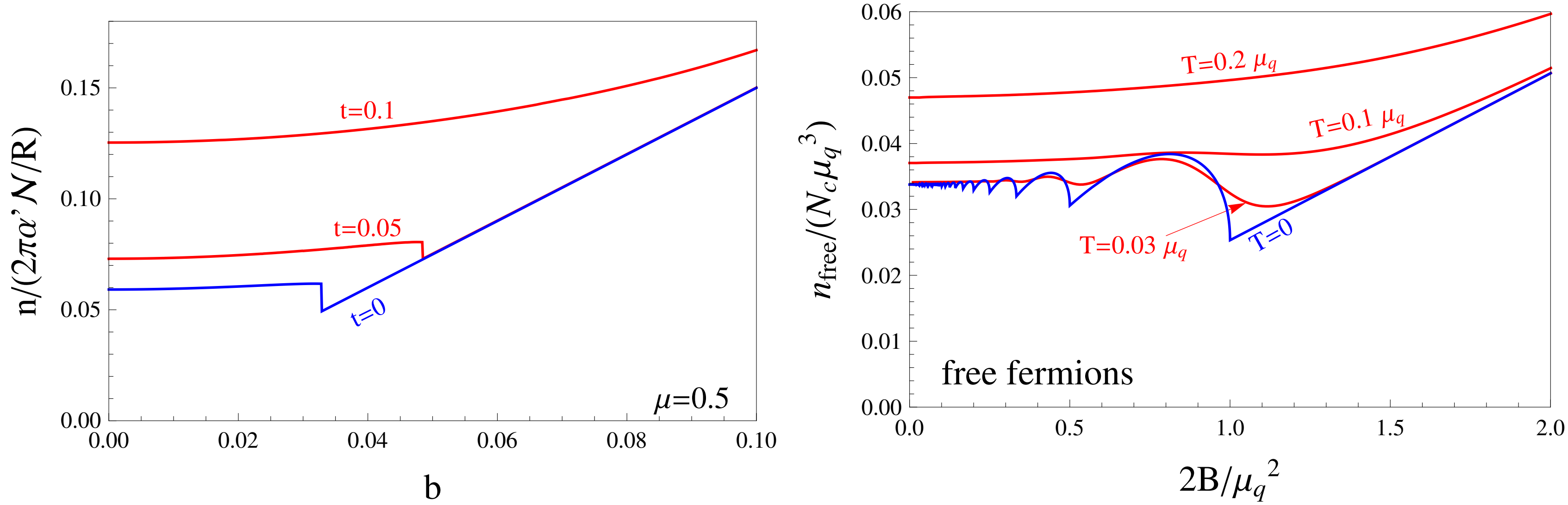
• $T = 0$ phase diagram



- Two main observations:
  - apparent Landau level transition
    G. Lifschytz, M. Lippert, PRD 80, 066007 (2009)
  - non-monotonic behavior of critical μ
    (doesn't magnetic catalysis suggest monotonic increase?)

• "LLL" in the Sakai-Sugimoto model

- compare density with free fermion system:



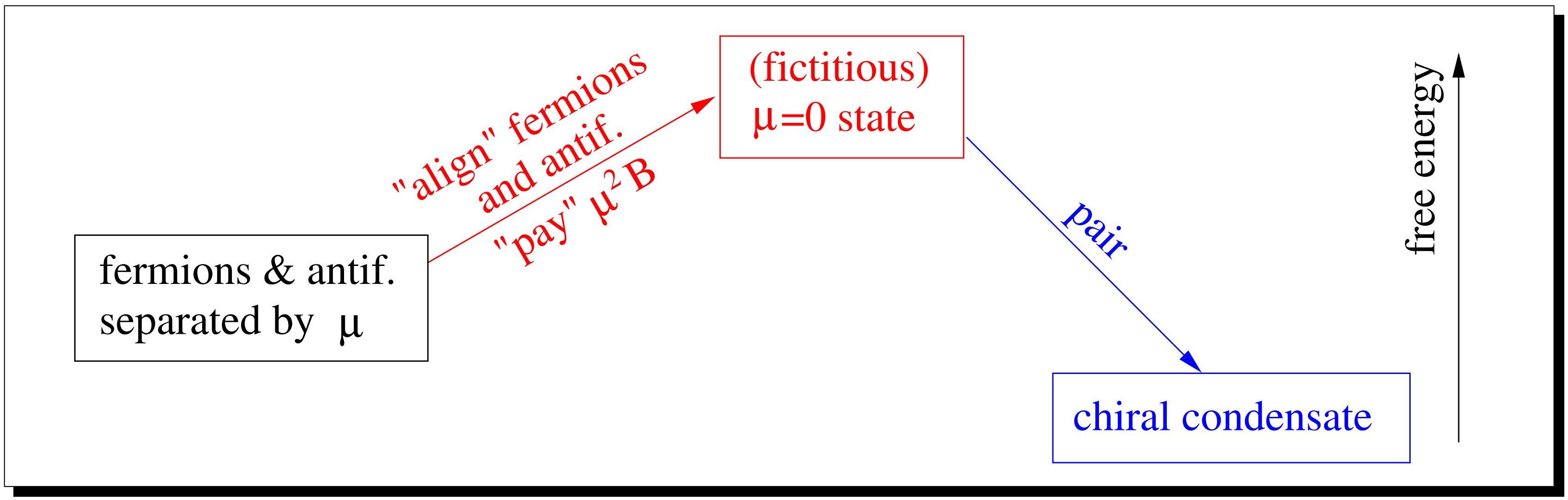
- no higher LL oscillations (expected due to strong coupling)
- linear behavior of n for large B exactly like for free fermions in LLL (all model parameters drop out!)

$$n = \frac{\mu B}{2\pi^2}$$

• Inverse magnetic catalysis (page 1/2)

Why does B *restore* chiral symmetry for certain μ?
("Inverse Magnetic Catalysis")

- chiral condensation (isotropic) at nonzero μ:



(analogous to Cooper pairing with mismatched Fermi surfaces)

- μ induces free energy *cost* for pairing; this cost depends on B!
- free energy *gain* from $\bar{\psi} - \psi$ pairing increases with B (magnetic catalysis)

• Inverse magnetic catalysis (page 2/2)

- this shows that inverse catalysis *can* happen
- whether it *does* happen, depends on details (and on coupling strength!)

weak coupling (NJL):

E. V. Gorbar *et al.*, PRC 80, 032801 (2009)

$$\Delta\Omega \propto B[\mu^2 - M(B)^2/2]$$

just like Clogston limit $\delta\mu = \frac{\Delta}{\sqrt{2}}$

in superconductivity

A. Clogston, PRL 9, 266 (1962)

B. Chandrasekhar, APL 1, 7 (1962)

Sakai-Sugimoto:

large B:

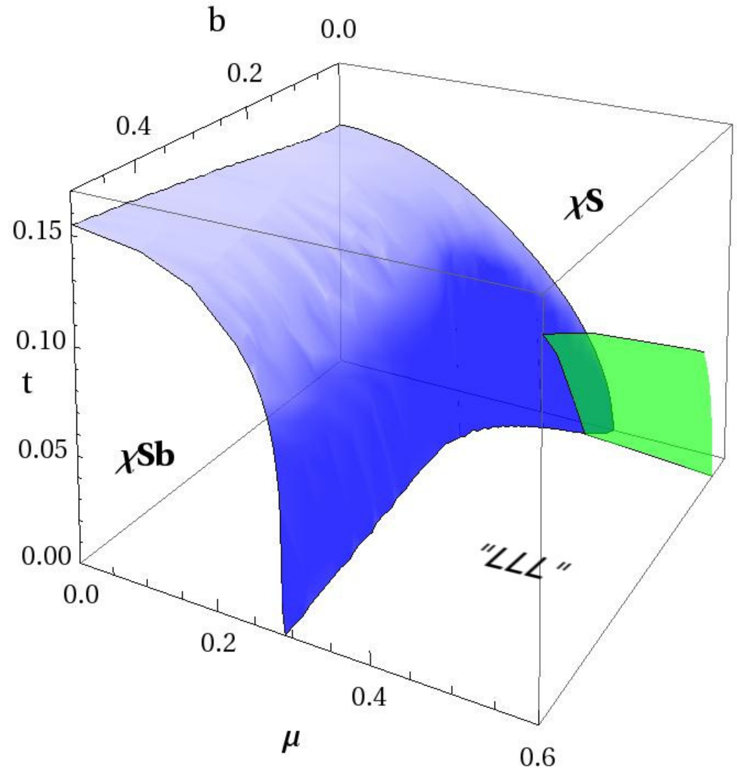
$$\Delta\Omega \propto B[\mu^2 - 0.12\, M(B)^2]$$

small B:

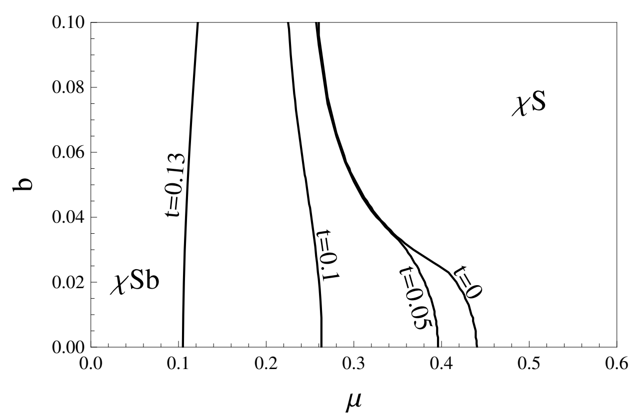
$$\Delta\Omega \propto \mu^2 B - \text{const} \times M(B)^{7/2}$$

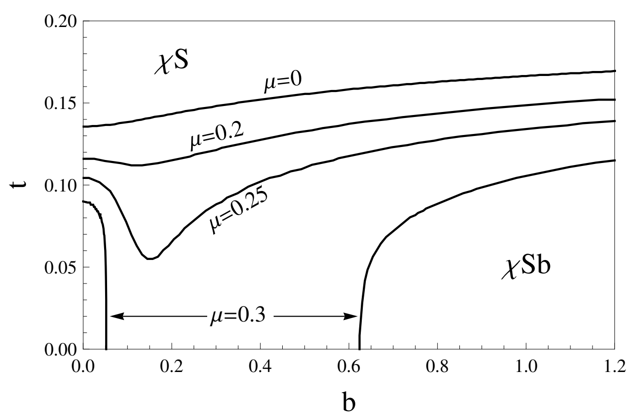
"Inverse magnetic catalysis"

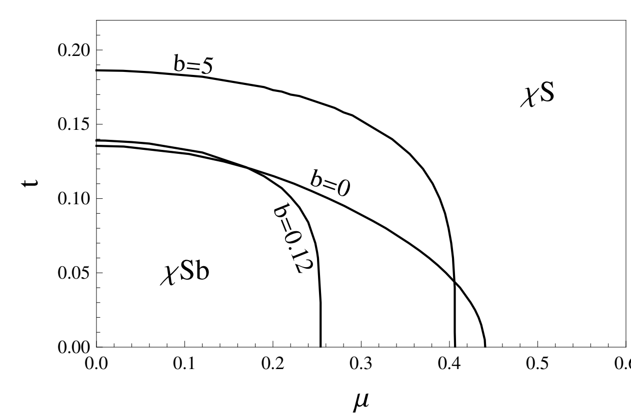
• Phase structure at nonzero temperature



blue: chiral phase transition
green: "LLL" transition



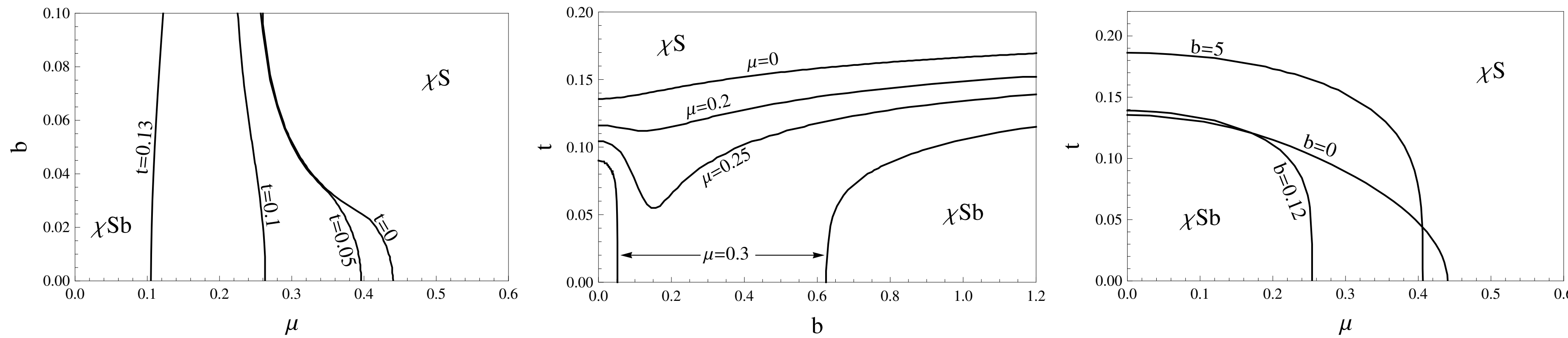




• Agreement with NJL calculation

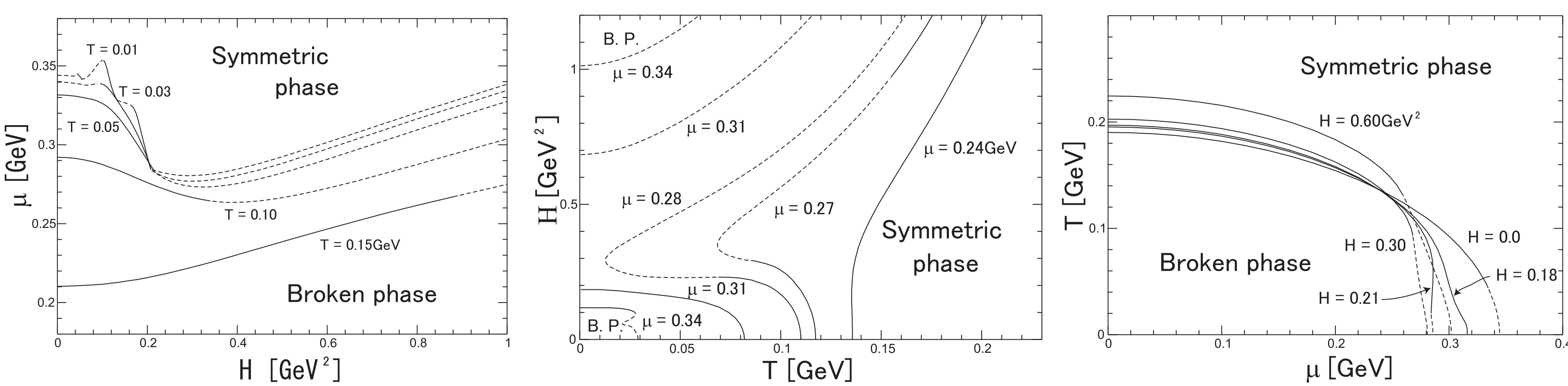
Sakai-Sugimoto:

F. Preis, A. Rebhan and A. Schmitt, JHEP 1103, 033 (2011)



NJL:

T. Inagaki, D. Kimura, T. Murata, Prog. Theor. Phys. 111, 371-386 (2004)



(IMC also in quark-meson model J. O. Andersen and A. Tranberg, arXiv:1204.3360 [hep-ph])

• Homogeneous baryonic matter in Sakai-Sugimoto

- baryons in AdS/CFT: wrapped D-branes with N_c strings

E. Witten, JHEP 9807, 006 (1998); D. J. Gross, H. Ooguri, PRD 58, 106002 (1998)

- baryons in Sakai-Sugimoto:
  - D4-branes wrapped on S^4
  - equivalently: instantons on D8-branes ($\to$ skyrmions)

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)

H. Hata, T. Sakai, S. Sugimoto, S. Yamato, Prog. Theor. Phys. 117, 1157 (2007)

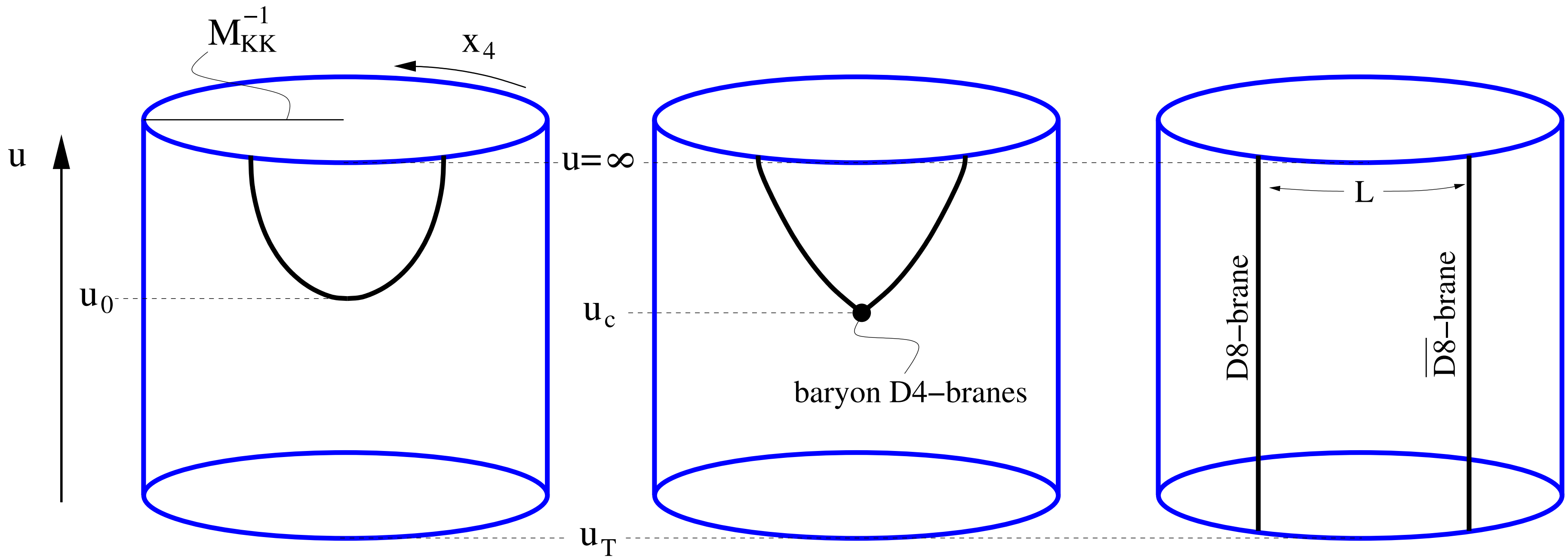
pointlike approximation for $N_f = 1$:

O. Bergman, G. Lifschytz, M. Lippert, JHEP 0711, 056 (2007)

$$S = S_{\text{from above}} + \underbrace{N_4 T_4 \int d\Omega_4 d\tau\, e^{-\Phi}\sqrt{\det g}}_{\propto\, n_4 N_c M_q} + \underbrace{\frac{N_c}{8\pi^2}\int_{\mathbb{R}^4\times\mathcal{U}} A_0 \,\mathrm{Tr}\, F^2}_{\propto\, n_4 \int A_0(u)\delta(u-u_c)}$$

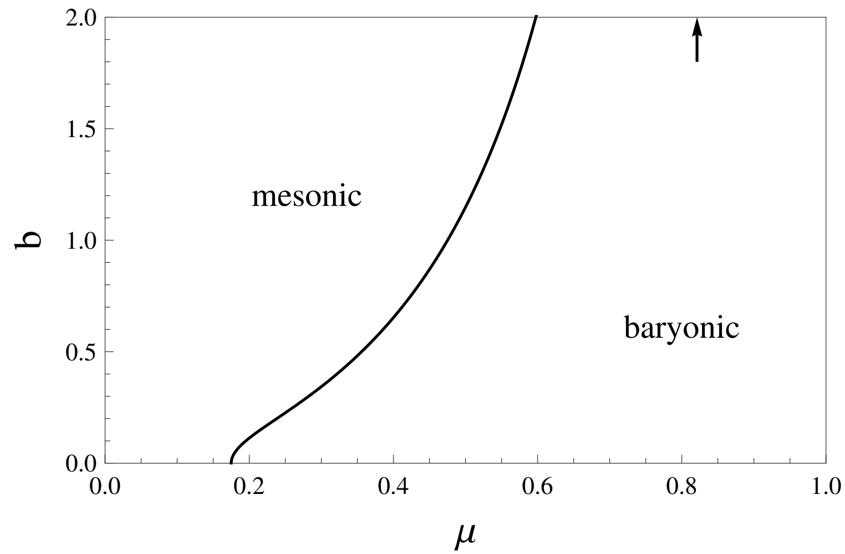
(n_4 baryon density, M_q constituent quark mass, u_c location of D4-branes)

- **Compare free energy of three phases**



mesonic	**baryonic**	**quark matter**
χS broken	χS broken	χS restored
$n_B \sim b\,\nabla\pi^0$	$n_B \sim n_4 + b\,\nabla\pi^0$	$n_B \sim N_c n_q$
$M_q \sim u_0$	$M_q \sim \frac{u_c}{3}$	$M_q = 0$

- **Onset of baryons (ignore quark matter for now)**



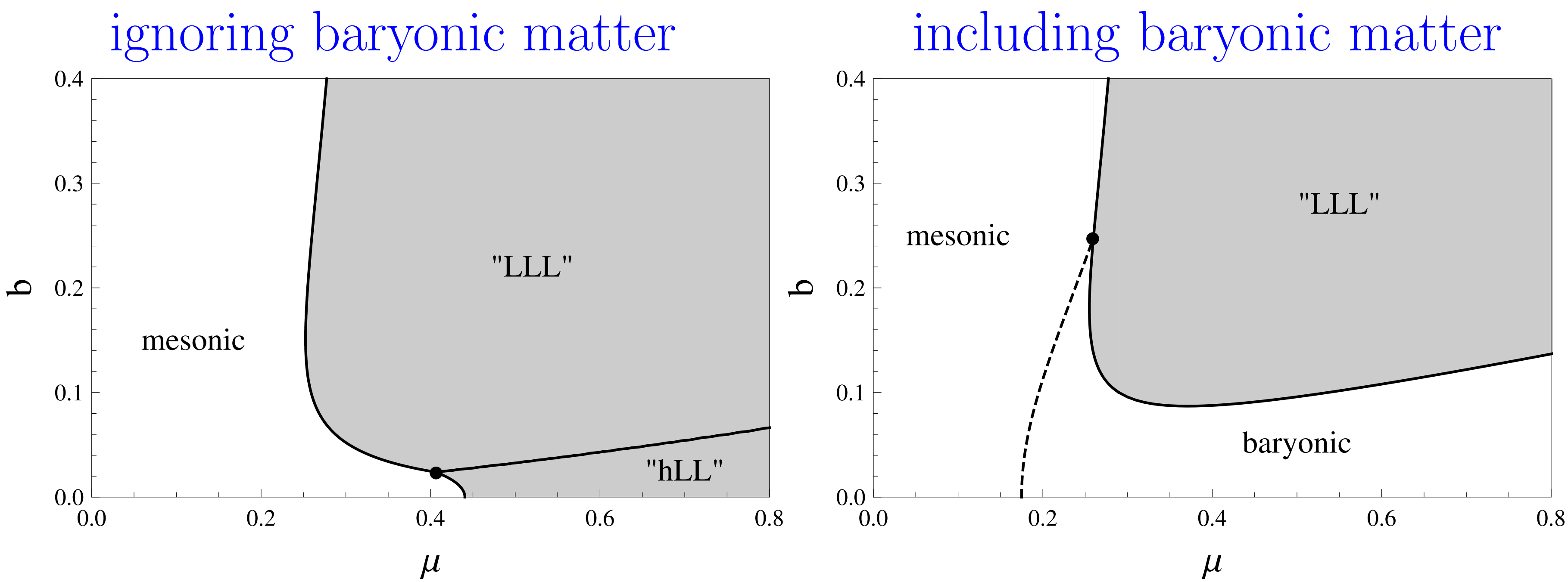
- **second-order** transition at $\mu_B = M_B$
- real-world: **first order** at $\mu_B = M_B - E_{\rm bind}$
- absence of $E_{\rm bind}$: large-N_c effect due to heaviness of σ ($m_\sigma \propto N_c$)?

  V. Kaplunovsky, J. Sonnenschein, JHEP 1105 (2011);

  L. Bonanno, F. Giacosa, NPA 859, 49-62 (2011)

• Effect of baryons on $T = 0$ phase diagram

F. Preis, A. Rebhan, A. Schmitt, JPG 39, 054006 (2012)



- small b: baryonic matter prevents the system from restoring chiral symmetry
- baryon onset line intersects chiral phase transition line $\rightarrow$ large b: mesonic matter superseded by quark matter
- with baryonic matter, IMC plays an even more prominent role in the phase diagram

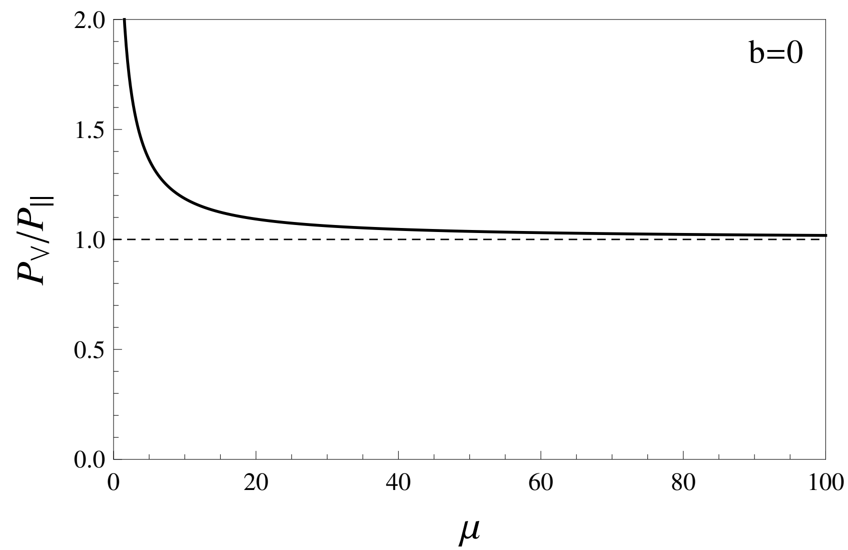
• Asymptotic baryonic matter

- For $\mu \to \infty$ baryonic and quark matter become indistinguishable:

$$P_{\vee}(b=0) \;=\; p\,\mu^{7/2} + \mathcal{O}(\mu^{5/2})$$

$$P_{||}(b=0) \;=\; p\,\mu^{7/2}$$

(where $p \equiv \frac{2}{7}\mathcal{N}\left[\frac{\Gamma\left(\frac{3}{10}\right)\Gamma\left(\frac{6}{5}\right)}{\sqrt{\pi}}\right]^{-5/2}$)



- is absence of chiral transition artifact of pointlike baryons?
  $\to$ overlap of baryons shifted to $\mu \to \infty$
- should redo analysis with finite-size baryons
  (here: instantons, $N_f > 1$)

- **Summary part 3**

	NJL	**Sakai-Sugimoto (small L)**
MC	$\surd$	$\surd$
IMC at finite μ	$\surd$	$\surd$
chiral trans. $(m = 0)$	1st & 2nd	1st
$m \neq 0$	easy	difficult
LL oscillations	$\surd$	–
LLL	$\surd$	$\surd$ (indirect)
baryons	difficult	$\surd$ (large N_c)

- **Conclusions: what can we learn from holography?**
  (in the given context of equilibrium phases of QCD)

- "Minimalistic" point of view:
  – consider Sakai-Sugimoto as just another model like NJL, PNJL, sigma model, ...
  – try to squeeze out model-independent physics
  (here: observe IMC, find physical picture which suggests model indep.)

- More "ambitious" point of view:
  – with AdS/CFT we have a "microscopic", reliable description of strongly coupled systems!
  – however, all systems considered so far are unrealistic
  (e.g., Sakai-Sugimoto dual to QCD at best for large-N_c and in inacc. limit)
  – try to learn about strongly coupled systems as such
  (absence of quasiparticles, viscosity bound, ...)
  – work hard to find gravity dual of QCD