Stressed Cooper pairing in dense quark matter

For a review, see


• Color-flavor locking (CFL) – highest densities
• Stressed Cooper pairing – below CFL densities
Superconductivity: Cooper pairing of fermions

- free energy $\Omega = E - \mu N$
- no interactions: add fermion at $E = \mu$ without cost
- attractive interaction: add pair with gain
- pairs condense → superconductivity

This Bardeen-Cooper-Schrieffer (BCS) argument holds for electrons in metal, $^3$He atoms, ..., and quarks in quark matter
• **Order parameter in color-flavor space**

• one-gluon exchange

\[ SU(3)_c : \quad [3]_c \otimes [3]_c = \bar{3}^a_c \oplus [6]^s_c \]

attractive in antisymmetric antitriplet channel \( [\bar{3}]^a_c \)

• **flavor space**

\[ SU(3)_f : \quad [3]_f \otimes [3]_f = \bar{3}^a_f \oplus [6]^s_f \]

• **order parameter** (for spin-0 pairing):

\[ \langle \psi_i^\alpha C \gamma_5 \psi_j^\beta \rangle \propto \epsilon^{\alpha\beta A} \epsilon_{ij} B \phi_A^B \in [\bar{3}]^a_c \otimes [\bar{3}]^a_f \]
QCD phase diagram (page 1/2): Known and unknown territories

High densities:
- rigorous theoretical control
- color-flavor locking (CFL)

Moderate densities:
- perturbative QCD not valid
- strange mass & neutrality: stress on Cooper pairing
• QCD phase diagram (page 2/2): 2 Possible scenarios

CFL superseded by nuclear matter:
- effective theory for CFL in strongly-coupled regime
- CFL matter in the core of compact stars?

CFL superseded by “non-CFL” matter:
- complicated phase structure?
- rely on Nambu-Jona-Lasinio-type models
Question:
What is the ground state of deconfined quark matter at moderate densities (in the interior of compact stars)?

1. Theoretical approach: start from CFL and ask “what is next phase down in density?” (if not hadronic matter)

2. Phenomenological approach: “guess” possible phase, compute its properties and compare with astrophysical observations
• On safe grounds: Asymptotically large density

$$0 \simeq m_s \simeq m_u \simeq m_d \ll \mu \quad \text{all quark masses negligible}$$

"color-flavor locked phase (CFL)"

M. Alford, K. Rajagopal, F. Wilczek, NPB 537, 443 (1999)

$$\phi^A_B = \delta^A_B \quad \Rightarrow \quad \langle \psi^\alpha_i C \gamma_5 \psi^\beta_j \rangle \propto \epsilon^{\alpha\beta A} \epsilon_{ijA}$$

$$\Rightarrow \quad SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times U(1)_Q \times \mathbb{Z}_2$$
• **Properties of CFL (page 1/2)**

**1) chiral symmetry breaking**

• usual chiral symmetry breaking: LR pairing \( \langle \bar{\psi}_R \psi_L \rangle \)

• **CFL**: LL, RR pairing \( \langle \psi_R \psi_R \rangle, \langle \psi_L \psi_L \rangle \), however

\[
SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times \mathbb{Z}_2
\]

• chiral symmetry broken through “locking” to color

• octet of pseudo-Goldstone modes \( K^0, K^\pm, \pi^0, \ldots \)
• Properties of CFL (page 2/2)

(2) superfluidity

\[ SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times \mathbb{Z}_2 \]

- exactly massless Goldstone mode \( \phi \)

(3) rotated electromagnetism

\[ SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \]

- Cooper pairs neutral under \( \tilde{Q} = Q + \frac{2}{\sqrt{3}}T_8 \)
- photon-gluon mixing with (small) mixing angle

\[
\cos^2 \theta = \frac{3g^2}{3g^2 + 4e^2} \approx 1
\]
• **Going down in density:**
  Large, but not asymptotically large, densities

• strange mass $m_s \simeq 120$ MeV no longer $\ll \mu \simeq 400$ MeV

• fixed $\mu$, $\mu_e$: $m_s \neq 0$ reduces $p_F^s$

• electric neutrality: increase in $p_F^d$ to compensate

• neutral, unpaired quark matter in $\beta$-equilibrium ($\mu_u + \mu_e = \mu_d$): Fermi momenta split apart
• **Stressed Cooper pairing: a general phenomenon (page 1/2)**

BCS-pairing:  

\[ k \rightarrow -k \]  

\[ k_F \]

split Fermi momenta:  

\[ k_F^d \]

\[ k_F^s \]

• for instance: two species (spin states) of cold fermionic atoms  

– Experiments:


– Theory (review):

- **Stressed Cooper pairing:**
  a general phenomenon (page 2/2)


- **phase separation** of superfluid and normal components
- **phase separation** unlikely in quark matter (local color neutrality!)
• CFL pairing with small stress

- create common Fermi surface:
  cost in free energy
  \[ \sim \delta p_F^2 \mu^2 \sim m_s^4 \]
- form pairs:
  gain in free energy \( \sim \Delta^2 \mu^2 \)
- CFL survives for \( \Delta \gtrsim \frac{m_s^2}{\mu} \)

\[ \epsilon_k = \pm \sqrt{(k - \mu)^2 + \Delta^2} \]

\[ \epsilon_k = \pm \left( \sqrt{(k - \bar{\mu})^2 + \Delta^2} \pm \delta \mu \right) \]
• **Pairing patterns with stress**

**unpaired**

\[ \frac{M_s^2}{4\mu} \]

\[ \frac{M_s^2}{4\mu} \]

\[ s \]

\[ d \]

\[ u \]

\[ p_F \]

**2SC pairing**

\[ d \]

\[ u \]

\[ s \]

\[ p_F \]

**CFL pairing**

\[ d \]

\[ u \]

\[ s \]

\[ p_F \]

- any pairing pattern most “comfortable” with \( m_s \) and neutrality?
- stressed pairing is **unavoidable**!

K. Rajagopal, A. Schmitt, PRD 73, 045003 (2006)
• Less (and less symmetric) pairing (page 1/4)

**Kaon condensation “CFL-\(K^0\)”**


- chiral field
  \[ \Sigma = \phi_L^\dagger \phi_R \]

- pure CFL: \(\Sigma = 1\)

- kaon condensation \(\Rightarrow\) \(\Sigma = e^{i\varphi T_6}\)
  (relative L/R rotation)

• in other words:

  create kaon with mass
  \[ m_{K^0}^2 = a m_d (m_s + m_u) \ll \Delta^2 \]
  from \(0 \rightarrow \bar{s} + u + \bar{u} + d\)
  \((a \sim \Delta^2/\mu^2)\)
• Less (and less symmetric) pairing (page 2/4)

(Super)currents in CFL: “curCFL-$K^0$”


A. Schmitt, NPA 820, 49C (2009)

\[
\phi_L(x) = \Delta e^{iJ_K \cdot x T_8} e^{i(\varphi/2)T_6} \\
\phi_R(x) = \Delta e^{iJ_K \cdot x T_8} e^{-i(\varphi/2)T_6}
\]

• “anisotropic breach”

• stable and unstable Fermi surface topologies:

"breach" (unstable)  \hspace{2cm} curCFL–$K^0$ (stable)
• Less (and less symmetric) pairing (page 3/4)

More currents in CFL: crystalline structures (LOFF)

M. Alford, J. Bowers, K. Rajagopal, PRD 63, 074016 (2001)
M. Mannarelli, K. Rajagopal and R. Sharma, PRD 73, 114012 (2006)

\[
\langle ud \rangle \sim \Delta_3 \sum_a \exp(2i\mathbf{q}_3^a \cdot \mathbf{x})
\]
\[
\langle us \rangle \sim \Delta_2 \sum_a \exp(2i\mathbf{q}_2^a \cdot \mathbf{x})
\]
\[
\langle ds \rangle \sim 0
\]

• here: “CubeX”

• \{\mathbf{q}_3\}, \{\mathbf{q}_2\} each contain 4 vectors, together pointing to the corners of a cube

\(\Delta_3(\mathbf{x}), \Delta_2(\mathbf{x})\)
• **Less (and less symmetric) pairing (page 4/4)**

**Last resort: single flavor pairing**

• need $J = 1$ Cooper pairs
  \[
  \phi \in [\bar{3}]_c^a \otimes [3]_J^s
  \]

• different possible phases:
  Color-spin locking (CSL), $A$-phase, polar phase . . .

  A. Schmitt, PRD 71, 054016 (2005)

• preferred phase at high densities: CSL

• gap much smaller than in spin-0 phases:
  \[
  \Delta J=1 \lesssim 10^{-2} \Delta J=0
  \]
- **Stressed pairing: free energy comparison**

here: $\Delta_{CFL} = 25$ MeV

(pert. QCD: $\Delta_{CFL} \simeq 20$ MeV, NJL: $\Delta_{CFL} \simeq (20 - 100)$ MeV).
• **Color-superconducting quark matter in compact stars**


• relate astrophysical observations to (transport) properties

  cooling curve – neutrino emissivity

  \( r \)-mode instability – viscosity

  mass/radius – equation of state

  glitches – shear modulus/vortices in LOFF

  
  
  \[
  \ldots
  \]

  

  \[\rightarrow\] M. Alford’s talk, tomorrow 15:30
**Summary**

- 3-flavor quark matter at asymptotically high densities is in the **CFL** state
- at lower densities CFL is “stressed”
- phase(s) between **CFL** and **hadronic matter** (if there is/are any) is/are uncertain

**Open questions**

- BCS – BEC crossover? (but: quark bound states contain 3 quarks, not 2)
- insight from large-$N_c$ QCD: competition/coexistence with quarkyonic phase?
  L. McLerran, R. D. Pisarski, NPA 796, 83 (2007)

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