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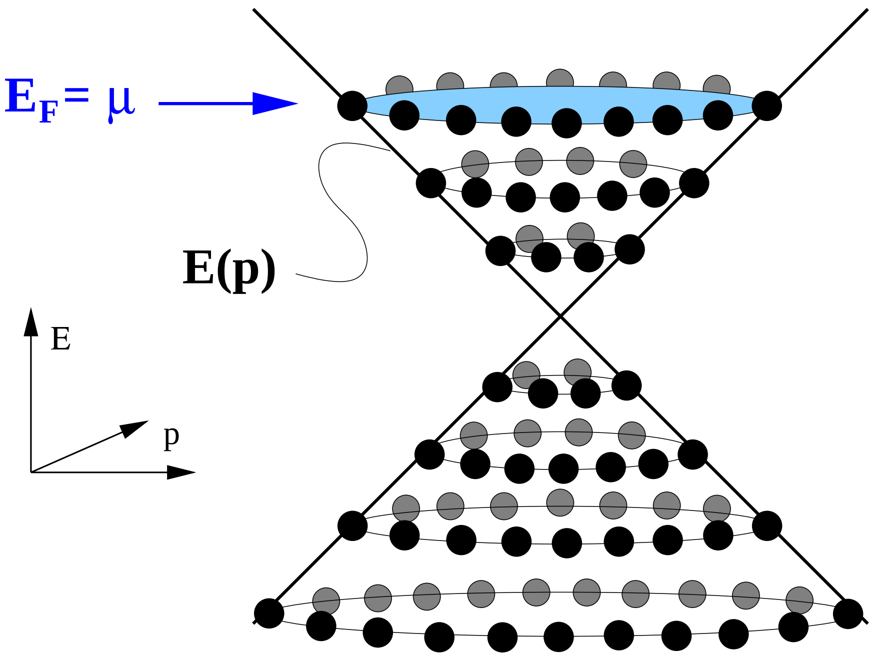
Stressed Cooper pairing in dense quark matter

For a review, see

M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, Rev. Mod. Phys. 80, 1455 (2008)

- **Color-flavor locking (CFL) – highest densities**
- **Stressed Cooper pairing – below CFL densities**

• Superconductivity: Cooper pairing of fermions



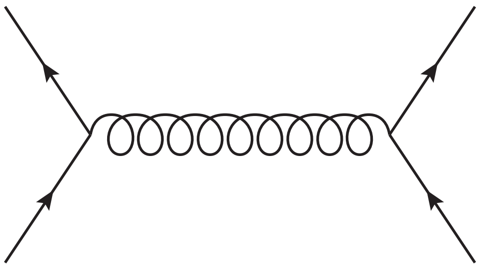
free energy $\Omega = E - \mu N$

- no interactions: add fermion at $E = \mu$ without cost
- attractive interaction: add pair with gain
- pairs condense $\rightarrow$ superconductivity

This Bardeen-Cooper-Schrieffer (BCS) argument holds for electrons in metal, ^{3}He atoms, ..., and quarks in quark matter

• Order parameter in color-flavor space

- one-gluon exchange



attractive in antisymmetric antitriplet channel $[\bar{\mathbf{3}}]_c^a$

$$SU(3)_c : \qquad [\mathbf{3}]_c \otimes [\mathbf{3}]_c = [\bar{\mathbf{3}}]_c^a \oplus [\mathbf{6}]_c^s$$

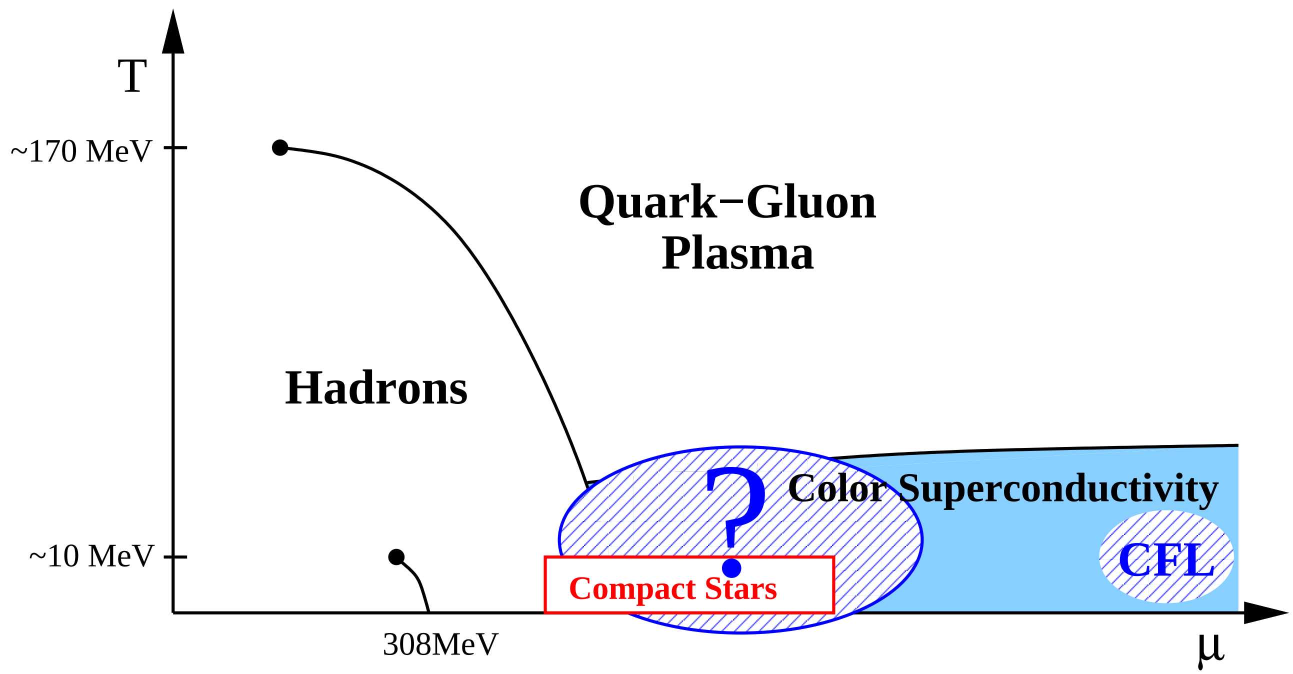
- flavor space

$$SU(3)_f : \qquad [\mathbf{3}]_f \otimes [\mathbf{3}]_f = [\bar{\mathbf{3}}]_f^a \oplus [\mathbf{6}]_f^s$$

- order parameter (for spin-0 pairing):

$$\langle \psi_i^\alpha C\gamma_5 \psi_j^\beta \rangle \;\propto\; \epsilon^{\alpha\beta A}\,\epsilon_{ijB}\,\phi_B^A \;\in\; [\bar{\mathbf{3}}]_c^a \otimes [\bar{\mathbf{3}}]_f^a$$

- **QCD phase diagram (page 1/2): Known and unknown territories**



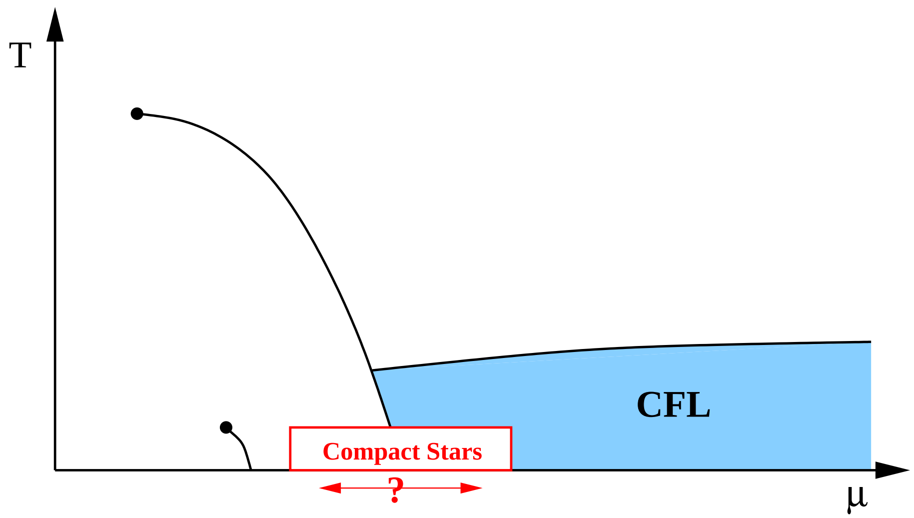
High densities:

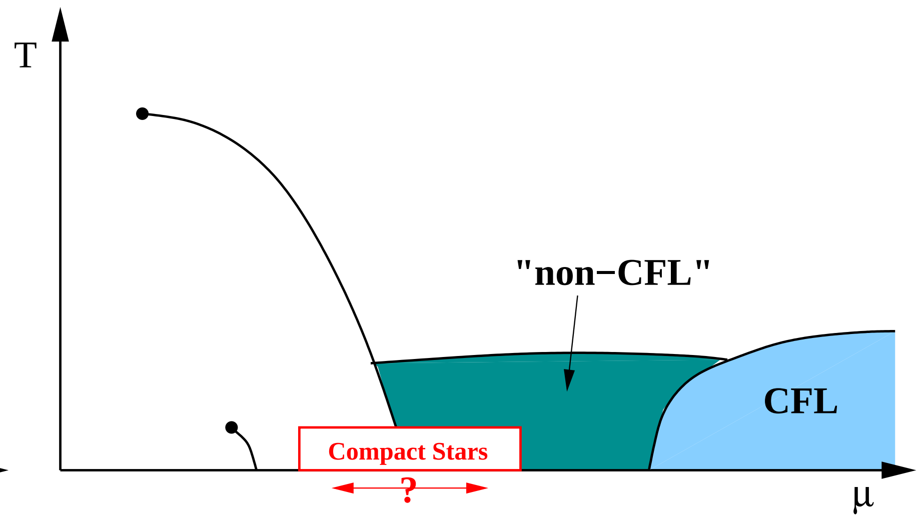
- rigorous theoretical control
- color-flavor locking (CFL)

Moderate densities:

- perturbative QCD not valid
- strange mass & neutrality: stress on Cooper pairing

- **QCD phase diagram (page 2/2): 2 Possible scenarios**





CFL superseded by nuclear matter:

- effective theory for CFL in strongly-coupled regime
- CFL matter in the core of compact stars?

CFL superseded by “non-CFL” matter:

- complicated phase structure?
- rely on Nambu-Jona-Lasinio-type models

Question:

What is the ground state of deconfined quark matter at moderate densities (in the interior of compact stars)?

1. **Theoretical approach: start from CFL and ask "what is next phase down in density?" (if not hadronic matter)**
2. **Phenomenological approach: "guess" possible phase, compute its properties and compare with astrophysical observations**

- **On safe grounds: Asymptotically large density**

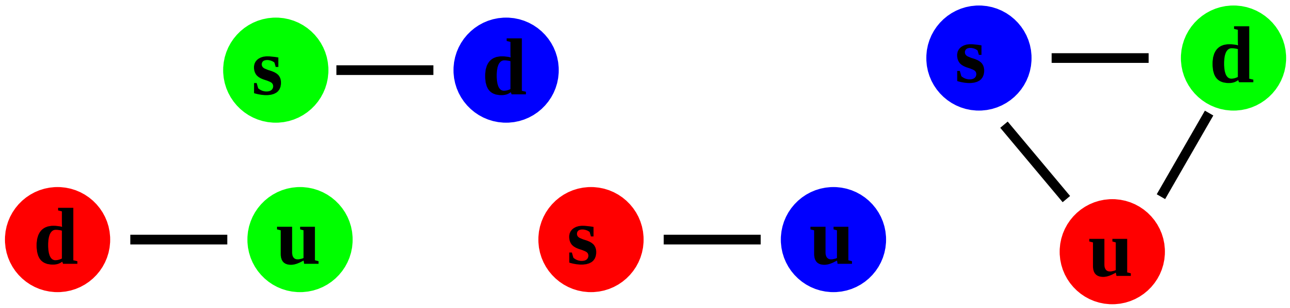
$$0 \simeq m_s \simeq m_u \simeq m_d \ll \mu \qquad \text{all quark masses negligible}$$

"color-flavor locked phase (CFL)"

M. Alford, K. Rajagopal, F. Wilczek, NPB 537, 443 (1999)

$$\phi^A_B = \delta^A_B \quad \Rightarrow \quad \langle \psi^\alpha_i C\gamma_5 \psi^\beta_j \rangle \ \propto \ \epsilon^{\alpha\beta A}\,\epsilon_{ijA}$$

$$\Rightarrow \quad SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$



• Properties of CFL (page 1/2)

(1) chiral symmetry breaking

- usual chiral symmetry breaking: LR pairing $\langle \bar{\psi}_R \psi_L \rangle$
- CFL: LL, RR pairing $\langle \psi_R \psi_R \rangle$, $\langle \psi_L \psi_L \rangle$, however

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \rightarrow \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

- chiral symmetry broken through "locking" to color
- octet of pseudo-Goldstone modes K^0, $K^{\pm}$, π^0, ...

• Properties of CFL (page 2/2)

(2) superfluidity

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \rightarrow \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

- *exactly massless* Goldstone mode ϕ
- vortices in rotating CFL M. M. Forbes, A. R. Zhitnitsky, PRD 65, 085009 (2002)

(3) rotated electromagnetism

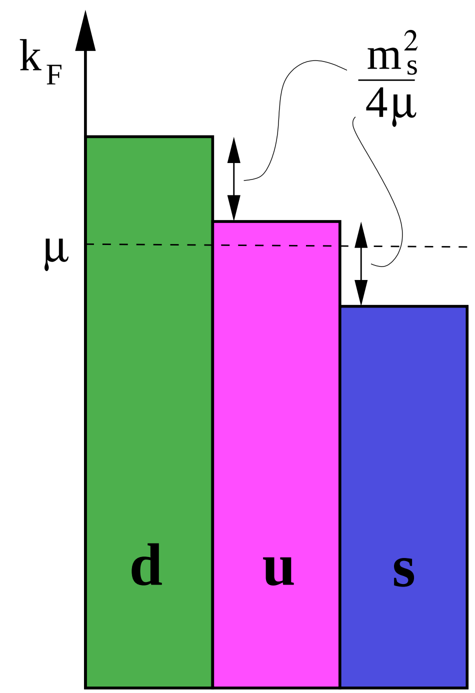
$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \rightarrow \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}}$$

- Cooper pairs neutral under $\tilde{Q} = Q + \frac{2}{\sqrt{3}} T_8$
- photon-gluon mixing with (small) mixing angle

$$\cos^2\theta = \frac{3g^2}{3g^2 + 4e^2} \simeq 1$$

- **Going down in density:**
  Large, but not asymptotically large, densities

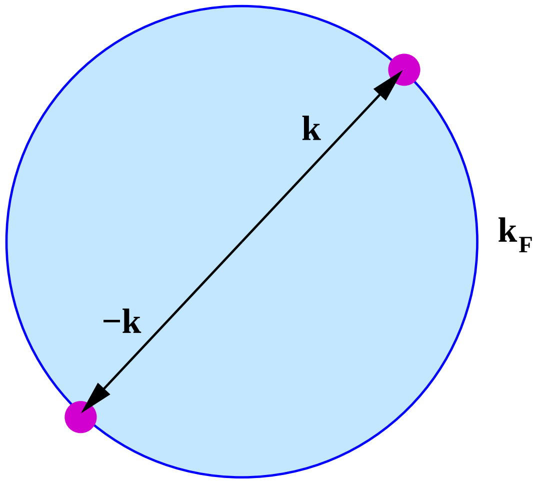
- strange mass $m_s \simeq 120\,\mathrm{MeV}$ no longer $\ll \mu \simeq 400\,\mathrm{MeV}$
- fixed μ, μ_e: $m_s \neq 0$ reduces p_F^s
- electric neutrality: increase in p_F^d to compensate



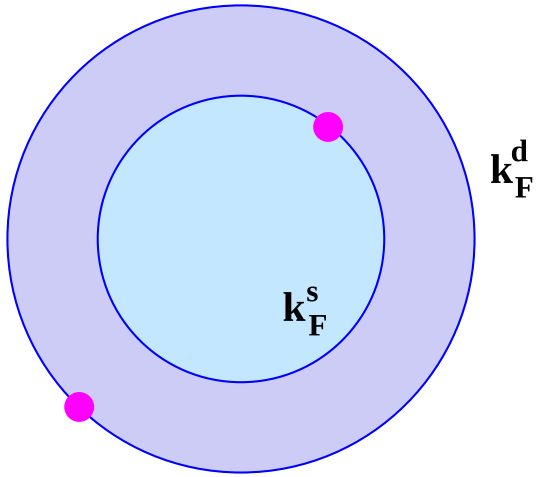
- neutral, unpaired quark matter
  in β-equilibrium ($\mu_u + \mu_e = \mu_d$):
  Fermi momenta split apart

• Stressed Cooper pairing: a general phenomenon (page 1/2)

BCS-pairing:



split Fermi momenta:



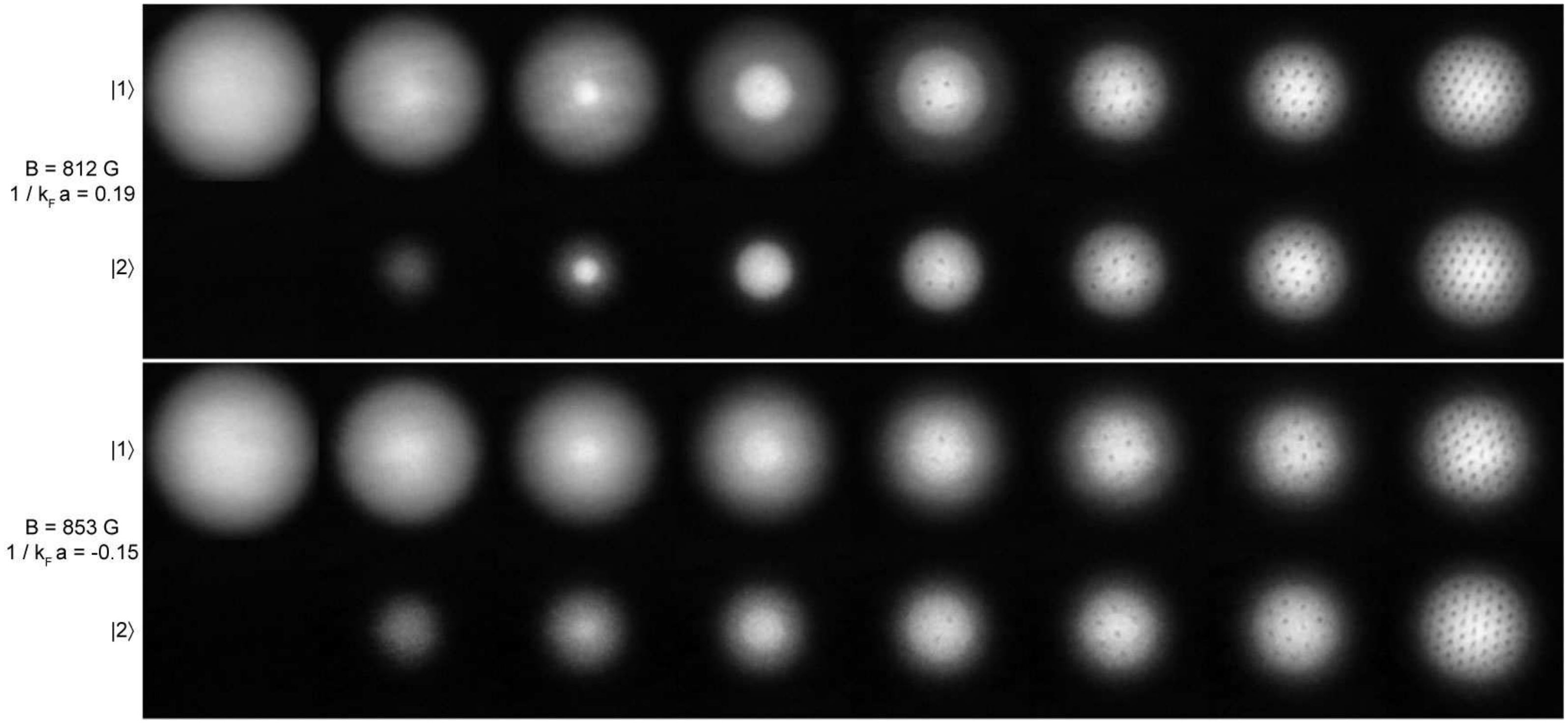
- for instance: two species (spin states) of cold fermionic atoms
  - Experiments:

    Y. Chin, M.W. Zwierlein, C.H. Schunck, A. Schirotzek, W. Ketterle, PRL 97, 030401 (2006)

    G.B. Partridge, W. Li, R.I. Kamar, Y. Liao, R.G. Hulet, Science 311, 503 (2006)
  - Theory (review):

    D.E. Sheehy, L. Radzihovsky, Ann. Phys. 322, 1790 (2007)

• Stressed Cooper pairing: a general phenomenon (page 2/2)

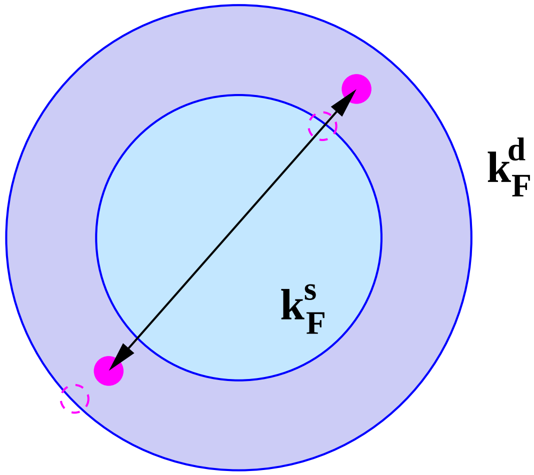


M.W. Zwierlein, A. Schirotzek, C.H. Schunck, W. Ketterle, Science 311, 492 (2006)

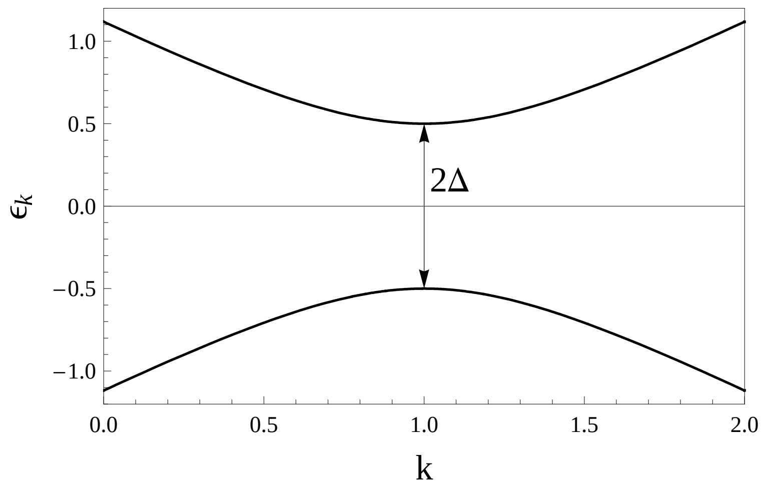
- **phase separation** of superfluid and normal components
- phase separation unlikely in quark matter (local color neutrality!)

M. Alford, C. Kouvaris, K. Rajagopal, PRL 92, 222001 (2004)

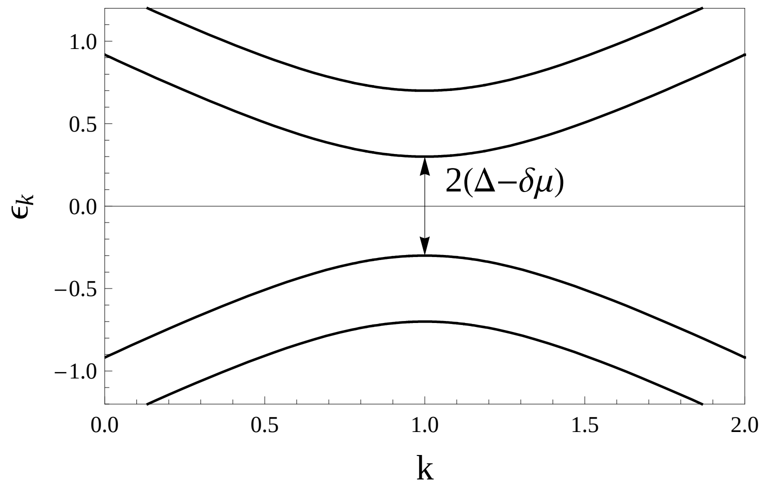
• CFL pairing with small stress



- create common Fermi surface:
  cost in free energy
  $\sim \delta p_F^2 \, \mu^2 \sim m_s^4$
- form pairs:
  gain in free energy $\sim \Delta^2 \mu^2$
- CFL survives for $\Delta \gtrsim \frac{m_s^2}{\mu}$

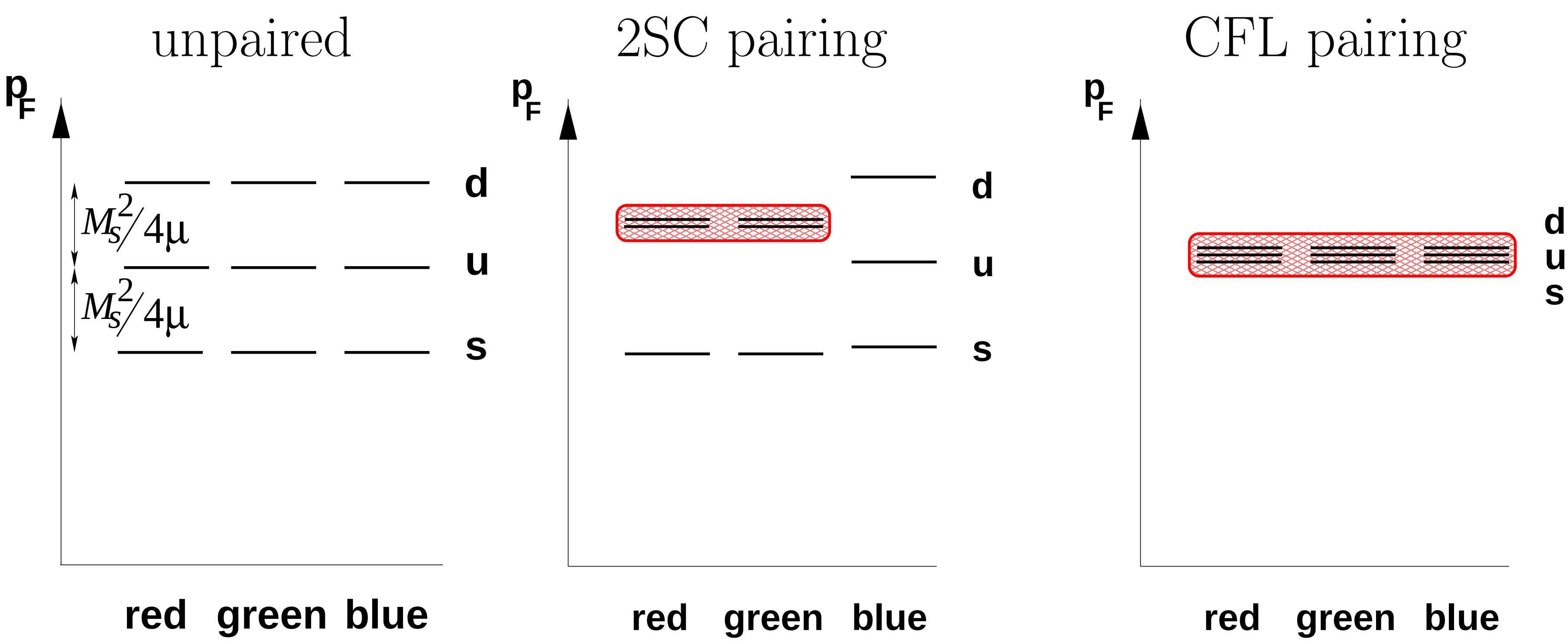


$$\epsilon_k = \pm\sqrt{(k-\mu)^2 + \Delta^2}$$



$$\epsilon_k = \pm\left(\sqrt{(k-\bar{\mu})^2 + \Delta^2} \pm \delta\mu\right)$$

• Pairing patterns with stress



- any pairing pattern most "comfortable" with m_s and neutrality?
- stressed pairing is **unavoidable**!

K. Rajagopal, A. Schmitt, PRD 73, 045003 (2006)

• Less (and less symmetric) pairing (page 1/4)

Kaon condensation "CFL-K^0"

P. F. Bedaque and T. Schäfer, NPA 697, 802 (2002)

- chiral field

$$\Sigma = \phi_L^\dagger \phi_R$$

- pure CFL: $\Sigma = \mathbf{1}$
- kaon condensation $\Rightarrow \Sigma = e^{i\varphi T_6}$ (relative L/R rotation)

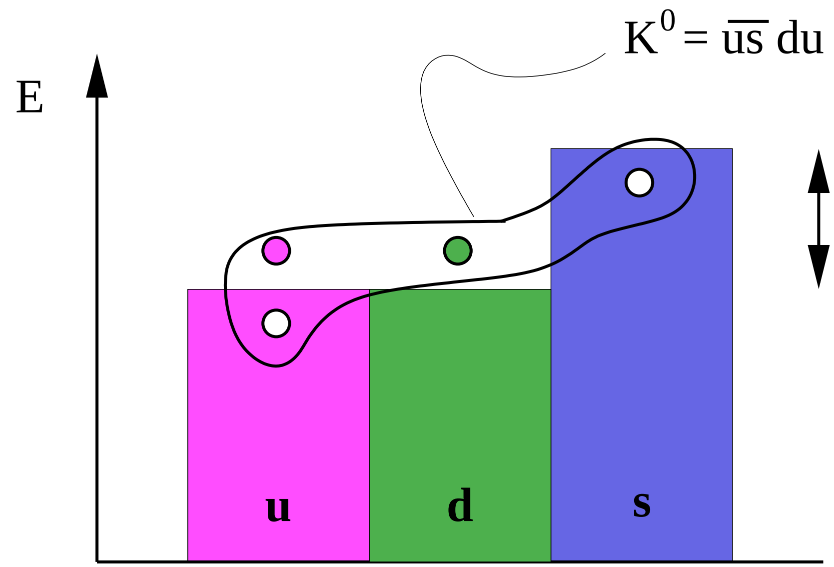
- in other words:

create kaon with mass

$m^2_{K^0} = a\, m_d(m_s{+}m_u) \ll \Delta^2$

from $0 \to \bar{s} + u + \bar{u} + d$

$(a \sim \Delta^2/\mu^2)$



• Less (and less symmetric) pairing (page 2/4)

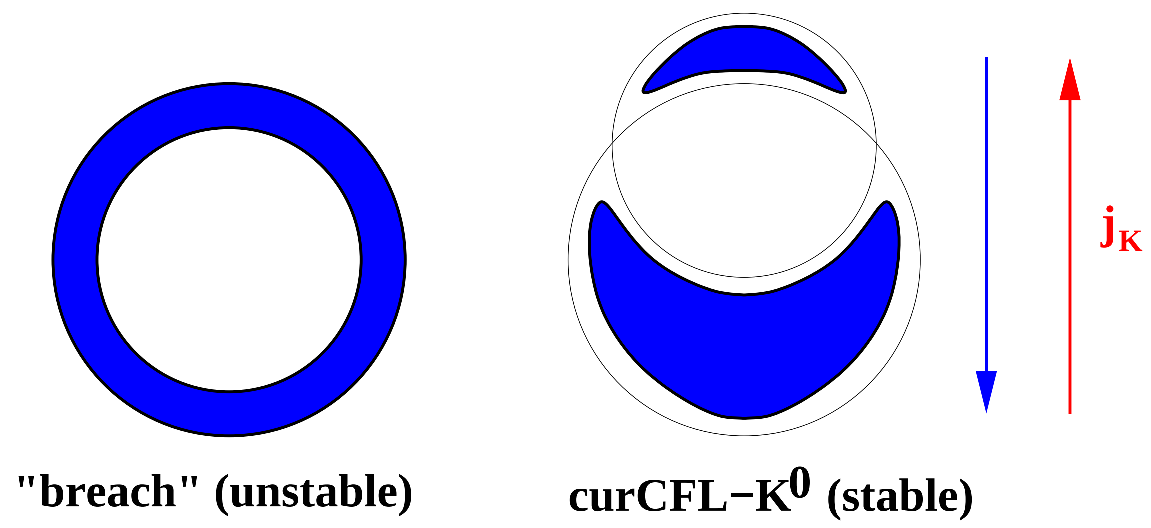
(Super)currents in CFL: "curCFL-K^0"

T. Schäfer, PRL 96, 012305 (2006); A. Kryjevski, PRD 77, 014018 (2008)

A. Schmitt, NPA 820, 49C (2009)

$$\phi_L(\mathbf{x}) = \Delta\, e^{i\boldsymbol{j}_K\cdot\mathbf{x}\,T_8}\; e^{i(\varphi/2)T_6}$$
$$\phi_R(\mathbf{x}) = \Delta\, e^{i\boldsymbol{j}_K\cdot\mathbf{x}\,T_8}\; e^{-i(\varphi/2)T_6}$$

- "anisotropic breach"
- stable and unstable Fermi surface topologies:



- **Less (and less symmetric) pairing (page 3/4)**

More currents in CFL: crystalline structures (LOFF)

M. Alford, J. Bowers, K. Rajagopal, PRD 63, 074016 (2001)

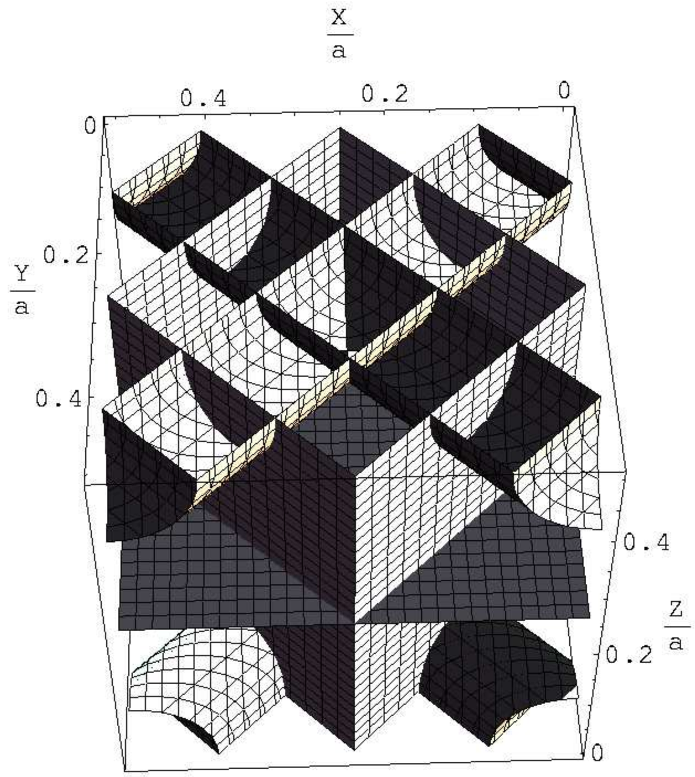
M. Mannarelli, K. Rajagopal and R. Sharma, PRD 73, 114012 (2006)

$$\langle ud \rangle \sim \Delta_3 \sum_a \exp(2i\mathbf{q}_3^a \cdot \mathbf{x})$$

$$\langle us \rangle \sim \Delta_2 \sum_a \exp(2i\mathbf{q}_2^a \cdot \mathbf{x})$$

$$\langle ds \rangle \simeq 0$$

- here: “CubeX”
- $\{\mathbf{q}_3\}$, $\{\mathbf{q}_2\}$ each contain 4 vectors, together pointing to the corners of a cube



$\Delta_3(\mathbf{x}), \Delta_2(\mathbf{x})$

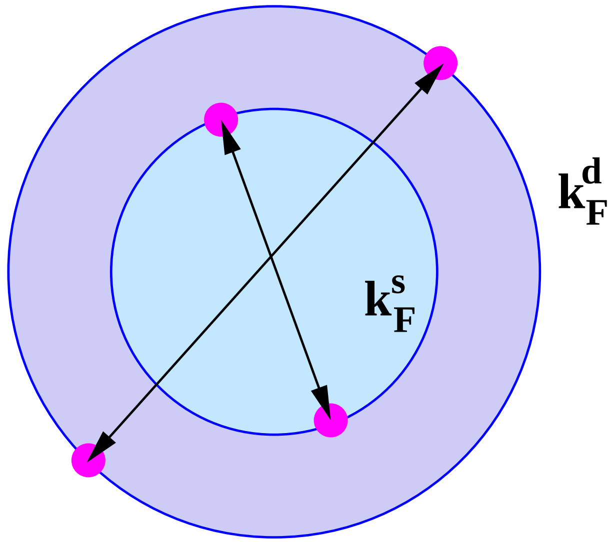
• Less (and less symmetric) pairing (page 4/4)

Last resort: single flavor pairing

- need $J = 1$ Cooper pairs

$$\phi \in [\bar{\mathbf{3}}]^a_c \otimes [\mathbf{3}]^s_J$$

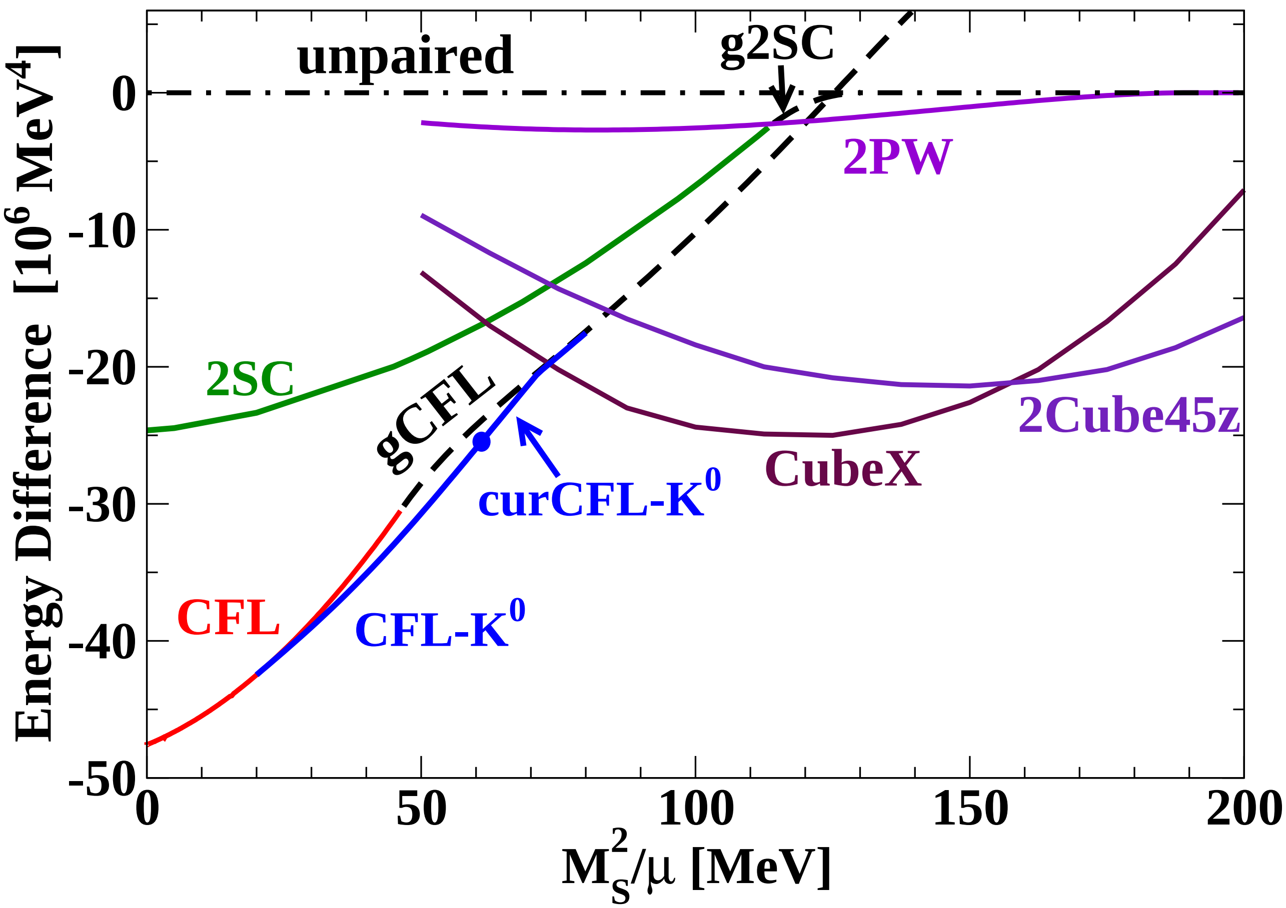
- different possible phases:
  Color-spin locking (CSL),
  A-phase, polar phase ...
  T. Schäfer, PRD 62, 094007 (2000)
  A. Schmitt, PRD 71, 054016 (2005)



- preferred phase at high densities: CSL
- gap much smaller than in spin-0 phases:

$$\Delta_{J=1} \lesssim 10^{-2} \Delta_{J=0}$$

- **Stressed pairing: free energy comparison**



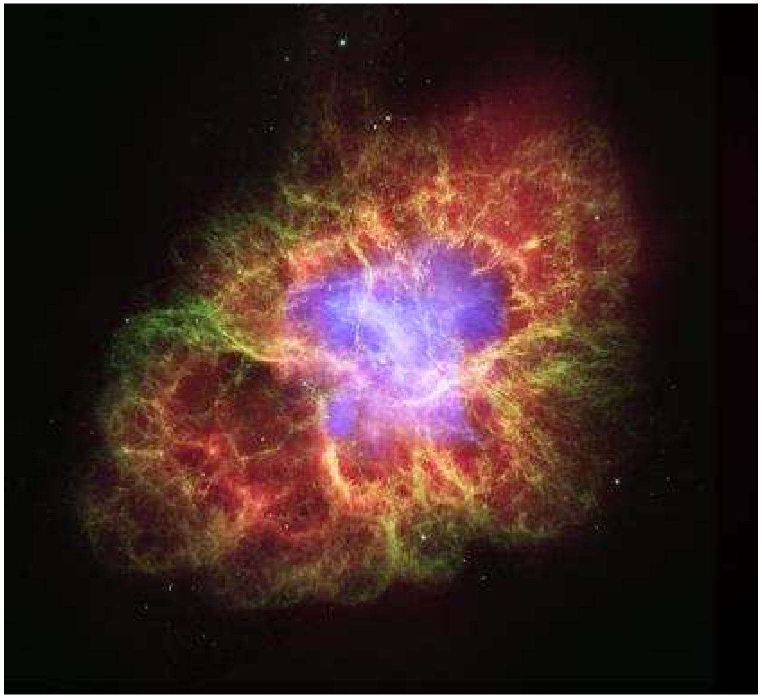
here: $\Delta_{\text{CFL}} = 25$ MeV

(pert. QCD: $\Delta_{\text{CFL}} \simeq 20$ MeV, NJL: $\Delta_{\text{CFL}} \simeq (20 - 100)$ MeV).

• Color-superconducting quark matter in compact stars

see for instance: A. Schmitt, arXiv:1001.3294, to appear in Lect. Notes Phys. (Springer)

- relate astrophysical observations to (transport) properties



cooling curve – neutrino emissivity

r-mode instability – viscosity

mass/radius – equation of state

glitches – shear modulus/vortices in LOFF

...

⟶ M. Alford's talk, tomorrow 15:30

- **Summary**

  - 3-flavor quark matter at asymptotically high densities is in the **CFL** state
  - at lower densities CFL is “stressed”
  - phase(s) between **CFL** and **hadronic matter** (if there is/are any) is/are uncertain

- **Open questions**

  - BCS – BEC crossover? (but: quark bound states contain 3 quarks, not 2)
  - insight from large-N_c QCD: competition/coexistence with quarkyonic phase?

    L. McLerran, R. D. Pisarski, NPA 796, 83 (2007)

  ...