Bulk viscosity in 2SC quark matter


• Color superconductivity: recent theoretical and astrophysical developments

• What is bulk viscosity?
  Why is it important for compact stars?

• Bulk viscosity in the 2SC phase
• **QCD phase diagram (1):**
  What is color superconductivity?

![Diagram of QCD phase transitions](image)

<table>
<thead>
<tr>
<th></th>
<th>Where?</th>
<th>What?</th>
<th>Attractive force</th>
<th>Cooper pairs</th>
<th>Broken gauge group</th>
</tr>
</thead>
<tbody>
<tr>
<td>“usual” superconductor</td>
<td>metals, alloys &amp; electrons</td>
<td>ion lattice &amp; electrons</td>
<td>phonons</td>
<td>electrons</td>
<td>$U(1)_{em}$</td>
</tr>
<tr>
<td>color superconductor</td>
<td>neutron stars &amp; gluons</td>
<td>quarks &amp; gluons</td>
<td>gluons</td>
<td>quarks</td>
<td>$SU(3)_{c}$</td>
</tr>
</tbody>
</table>
• **QCD phase diagram (2): Unknown territory**

![Diagram](image)

### Problems at moderate densities:

- perturbative QCD not valid
- strange mass not negligible
  - → neutrality requirements become nontrivial
Question:
What is the ground state of deconfined quark matter at moderate densities (in the interior of compact stars)?

1. Theoretical approach: start from CFL and ask “what is next phase down in density?” (if not hadronic matter)

2. Phenomenological approach: “guess” possible phase, compute its properties and compare with astrophysical observations

3. (Tabletop approach: learn from parallels to cold fermionic atoms in magnetic trap)
- On safe grounds: Asymptotically large density

\[ 0 \simeq m_s \simeq m_u \simeq m_d \ll \mu \quad \text{all quark masses negligible} \]

“color-flavor locked phase (CFL)”

\[ SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_{c+L+R} \]

- all quarks form Cooper pairs
- state is automatically color and electrically neutral
• **Large, but not asymptotically large densities**

  going down in density ⇔ “switching on” $m_s$ and maintaining neutrality

  → **mismatch in Fermi momenta of pairing quarks**
  ("stressed" pairing)

  BCS-pairing: 

  Neutral quark matter:

  → **stressed pairing is unavoidable**

  K. Rajagopal, A. Schmitt, PRD 73, 045003 (2006)
• **Less (and less symmetric) pairing**

For instance,

2SC phase ...

... and many others

• **One-flavor pairing: Color-Spin-Locking, A-phase, ...**


• **Gapless superconductors: g2SC, gCFL**

  I. Shovkovy, M. Huang, PLB 564, 205 (2003)

• **Counter-propagating currents: LOFF, meson current**

  M. Alford, J. Bowers, K. Rajagopal, PRD 63, 074016 (2001)
• **Astrophysical approach (page 1/2)**

• **neutrino emissivity/cooling of the star**
  
  – **normal quark matter** N. Iwamoto, PRL 44, 1637 (1980)
  – **gCFL** M. Alford, P. Jotwani, C. Kouvaris, J. Kundu, K. Rajagopal, PRD, 114011 (2005)

**direct Urca processes**

\[ u + e \rightarrow d + \nu, \quad d \rightarrow u + e + \bar{\nu} \]

\(\nu\)-emissivity sensitive to **magnitude of gap** and number and dimensionality of **ungapped modes**

**tendency:** need **fully gapped** phase with **not-too-large gap** to explain observed cooling (\(\rightarrow\) **CSL**?)
Astrophysical approach (page 2/2)

magnetic fields

- **spin-0, e.g. CFL** “rotated electromagnetism”
  \[ \rightarrow \text{no Meissner effect} \]
  M.G. Alford, J. Berges, K. Rajagopal, NPB 571, 269 (2000)

- **spin-1** Meissner effect

- **precession of the star** B. Link, PRL 91, 101101 (2003)

- **crust confinement of magnetic field**

viscosity

- **shear, bulk viscosity in CFL**

- **bulk viscosity in spin-1 phases**
  B.A. Sa’d, I.A. Shovkovy, D.H. Rischke, astro-ph/0607643

- **bulk viscosity in 2SC phase**
• **Why compute bulk viscosity?**

• **r-modes:** non-radial pulsation modes

• **grow unstable**
  in a **perfect-fluid** rotating star
  → emission of gravitational waves

• **spin down** the star drastically and quickly (within days)

• fast rotating stars are observed!
  \( \omega \sim 1\text{ms}^{-1} \)

• must be some damping mechanism
  → **bulk/shear viscosity**

• deduce upper limit for \( \omega \) from **viscosity**
• What is bulk viscosity?

- compression/expansion changes chemical composition
- system out of chemical equilibrium

\[ \delta \mu \equiv \mu_s - \mu_d \neq 0 \]

re-equilibration via

\[ u + d \leftrightarrow u + s \]

• 2 time scales:
  external oscillation \( \omega \) vs. microscopic rate \( \gamma \)
• **Bulk viscosity is a resonance phenomenon**

Just like an electric circuit!

\[
\begin{align*}
I(t) & \sim \sin(\omega t + \phi) \\
U(t) & \sim \sin \omega t \\
\text{dissipation} & \sim \langle I(t) \ U(t) \rangle
\end{align*}
\]

\[
\begin{align*}
\delta V(t) & \sim \sin \omega t \\
\delta \mu(t) & \sim \sin(\omega t + \phi) \\
\text{dissipation} & \sim \langle \delta V(t) \ \delta \mu(t) \rangle
\end{align*}
\]

"**capacitance**"  
\( C \leftrightarrow \text{inverse microscopic rate } \gamma^{-1} \)  
(slow process → store large chemical energy)

"**resistance**"  
\( R \leftrightarrow \left( n_u \frac{\partial \mu_d}{\partial n_u} + n_d \frac{\partial \mu_d}{\partial n_d} - n_s \frac{\partial \mu_s}{\partial n_s} \right)^{-1} \)  
(same dispersion for \( d \) and \( s \) → infinite "resistance" → no dissipation)

**Bulk viscosity**

\[
\begin{align*}
\zeta & = \alpha \frac{\gamma}{\gamma^2 + \omega^2} \\
\alpha & \equiv \frac{n_u \frac{\partial \mu_d}{\partial n_u} + n_d \frac{\partial \mu_d}{\partial n_d} - n_s \frac{\partial \mu_s}{\partial n_s}}{\frac{\partial \mu_d}{\partial n_d} + \frac{\partial \mu_s}{\partial n_s}}
\end{align*}
\]
• **Compute rate for** $u + d \leftrightarrow u + s$ **in 2SC**

$$\Gamma_{2SC} = 4 \, \Gamma_{2SC} = 1$$

**paired:**

$$
\begin{align*}
\begin{array}{c}
\text{d} \quad \text{u} \\
\text{d} \quad \text{u}
\end{array}
\end{align*}
$$

**unpaired:**

$$
\begin{align*}
\begin{array}{c}
\text{s} \quad \text{d} \quad \text{u} \\
\text{s} \quad \text{s}
\end{array}
\end{align*}
$$

**small temperatures,**

$$T \ll T_c \approx 30\text{MeV}$$

$$\Gamma_{2SC} = \frac{1}{9}\Gamma_{\text{unpaired}}$$

due to **exponential suppression**

$$\exp(-\Delta/T)$$

d of gapped modes
• **Results for all temperatures** \( T < T_c \)

(i) fixed \( \delta \mu = \mu_s - \mu_d > 0 \) → net production of \( d \) quarks, \( \Gamma_d > 0 \)

• contributions of subprocesses

• total rate compared to unpaired quark matter

(ii) for bulk viscosity, consider \( \gamma = \frac{\partial \Gamma_d}{\partial \delta \mu} \)
• Results for bulk viscosity

\[ \zeta = \alpha \frac{\gamma}{\gamma^2 + \omega^2} \]
• **Astrophysical implications**
• bulk viscosity in superconductor can be **larger** than in normal phase
• results important for young neutron stars, $T > 1$ MeV
• → first days of neutron star’s life (potentially enough for r-mode instabilities to grow)

J. Madsen, PRL 85, 10 (2000)
• ms = millisecond pulsars
• LMXB = low-mass x-ray binaries
• Conclusions

• Bulk viscosity
  – shear and bulk viscosities damp \textit{r-mode instabilities}
  – \textit{bulk viscosity} of quark matter in a neutron star dominated by \textit{weak processes} (unlike heavy-ion collisions; different \textit{external time scale})
  – \textit{2SC quark matter} has larger bulk viscosity than unpaired quark matter in \textit{very young neutron stars}

• More general
  – phase(s) between \textit{CFL} and \textit{hadronic matter} are unknown
  – use astrophysical observations to learn about these phases (\textit{cooling curves, rotation frequencies, magnetic fields ...})
  – how does quark matter deal with \textit{mismatched Fermi surfaces? exotic superconductors?}