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The Chiral Magnetic Effect in "Holographic QCD"

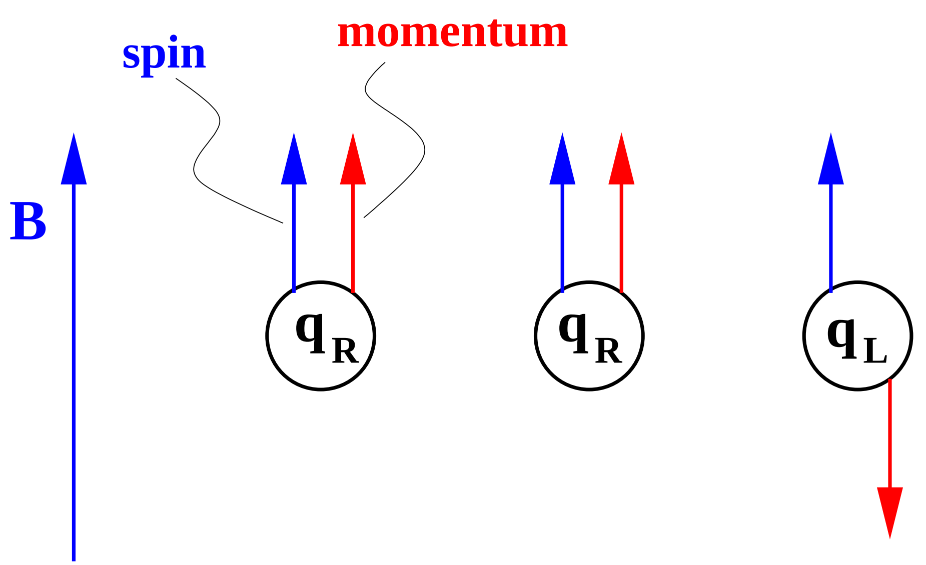
A. Rebhan, A. Schmitt, S.A. Stricker, JHEP 1001, 026 (2010)

- **The Chiral Magnetic Effect (CME)**
- **The Sakai-Sugimoto model**
- **Absence of the CME in the Sakai-Sugimoto model**
  - correct definition of currents
  - correct implementation of axial anomaly

• The Chiral Magnetic Effect (CME) (page 1/2)

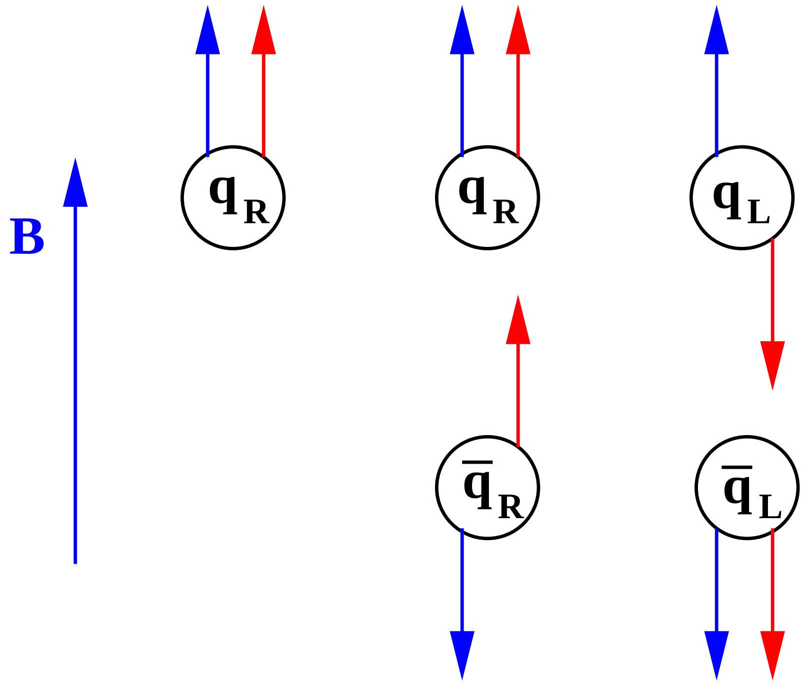
D.E. Kharzeev, L.D. McLerran, H.J. Warringa, NPA 803, 227 (2008)
K.Fukushima, D.E. Kharzeev, H.J. Warringa, PRD 78, 074033 (2008)

- massless chiral quarks q_R, q_L
- external magnetic field **B**
- nonzero axial charge $N_5 \equiv \langle q^\dagger \gamma^5 q \rangle = N_R - N_L > 0$



⇒ electric current parallel to B

• The Chiral Magnetic Effect (CME) (page 2/2)



- add antiquarks $\bar{q}_R$, $\bar{q}_L$
  (N_5 unchanged, but N changed)
- remember:
  $\bar{q}_R \longrightarrow$ negative helicity
  $\bar{q}_L \longrightarrow$ positive helicity
- current does not depend on μ
- lowest-Landau-level phenomenon

$$J = \frac{e^2 N_c}{2\pi^2} \mu_5 B$$

A.Y. Alekseev, V.V. Cheianov, J. Fröhlich, PRL 81, 3503 (1998)

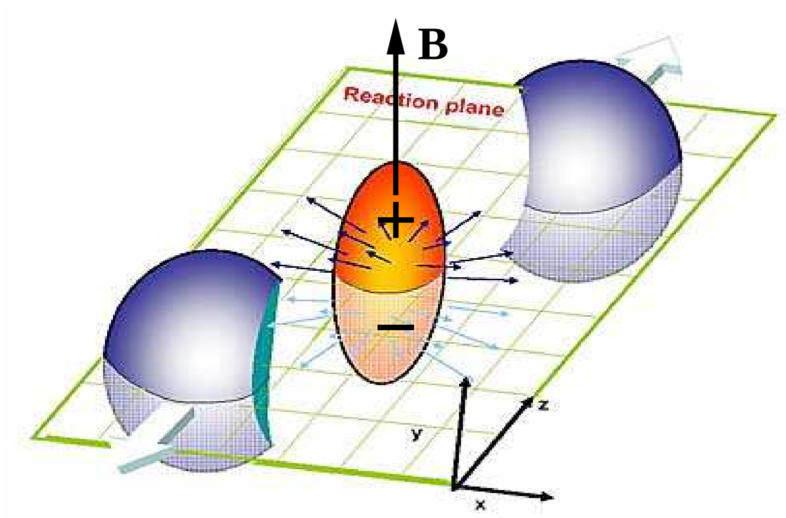
K.Fukushima, D.E.Kharzeev, H.J.Warringa, PRD 78, 074033 (2008)

- analogously for axial current:

  M.A. Metlitski, A.R. Zhitnitsky, PRD 72, 045011 (2005)

$$J_5 = \frac{e^2 N_c}{2\pi^2} \mu B$$

• CME in heavy-ion collisions (page 1/2)



- non-central collisions: $eB \sim 10^{17}\,\mathrm{G}$

  V. Skokov *et. al.*, Int. J. Mod. Phys. A24, 5925 (2009)

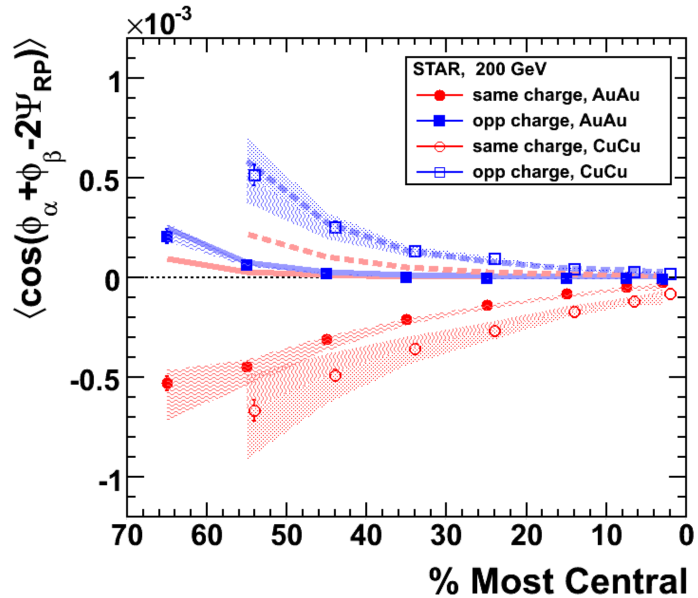
- N_5 from QCD axial anomaly

$$\frac{dN_5}{dt} = -\frac{g^2}{16\pi^2}\int d^3x\, F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

(large T: sphalerons)

- simplified description with μ_5: take "snapshot" with conserved N_5
- "event-by-event" charge separation

  B. I. Abelev *et al.* [STAR Collaboration], PRL 103, 251601 (2009)



- **CME in heavy-ion collisions (page 2/2)**

- STAR result due to CME?
  Not clear yet. F. Wang, arXiv:0911.1482 [nucl-ex]

- how about **chirally broken phase**? suppression of the effect?
- quark-gluon plasma at RHIC **strongly coupled**
  $\longrightarrow$ small viscosity P. Romatschke, U. Romatschke, PRL 99, 172301 (2007)
- modification of current J at strong coupling?

$\longrightarrow$ study CME in the **Sakai-Sugimoto model**

• The gauge/gravity duality: basic idea

J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998)

For a pedagogical review, see S. S. Gubser, A. Karch, Ann. Rev. Nucl. Part. Sci. 59, 145 (2009)

string theory (in higher dimensions)	$\Leftrightarrow$	gauge theory (on boundary)

original "AdS/CFT correspondence":

string theory on $AdS_5 \times S^5 \Leftrightarrow \mathcal{N} = 4\ SU(N_c)$ SYM theory on $\mathbb{R}^{3,1}$

$$\frac{R^4}{\ell_s^4} = g_{\rm YM}^2 N_c \equiv \lambda$$

R curvature radius; ℓ_s string length

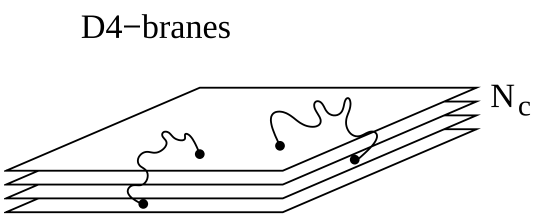
$\ell_s \ll R$		$\lambda \gg 1$
supergravity limit (*easy!*)	$\Leftrightarrow$	strong coupling limit (*difficult!*)

• The Sakai-Sugimoto model: background geometry (page 1/3)

E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998)

M. Kruczenski, D. Mateos, R. C. Myers, D. J. Winters, JHEP 0405, 041 (2004)

N_c D4-branes compactified on circle $x_4 \equiv x_4 + 2\pi/M_{\rm KK}$



- 4-4 strings $\longrightarrow$ adjoint scalars & fermions, gauge fields
- periodic $x_4 \longrightarrow$ break SUSY by giving mass $\sim M_{\rm KK}$ to scalars & fermions

$\Rightarrow U(N_c)$ gauge theory

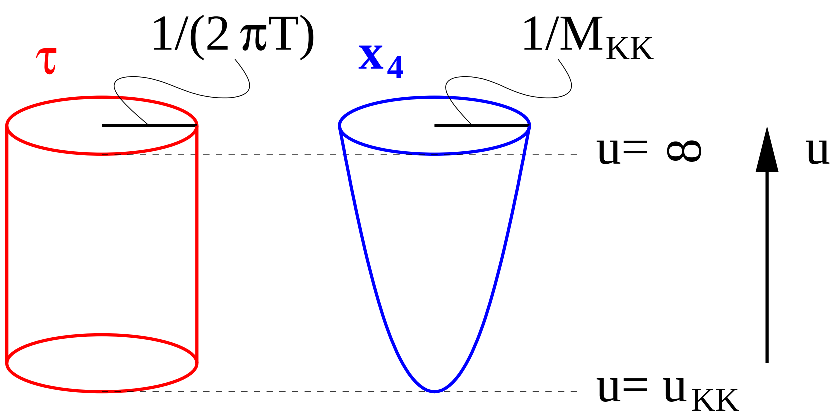
$$\lambda = \frac{g_5^2 N_c}{2\pi/M_{\rm KK}}$$

	$\lambda \ll 1$	$\lambda \gg 1$
dual to large-N_c QCD (at energies $\ll M_{\rm KK}$)	√	x
supergravity works	x	√

• Background geometry (page 2/3): two solutions

Confined phase

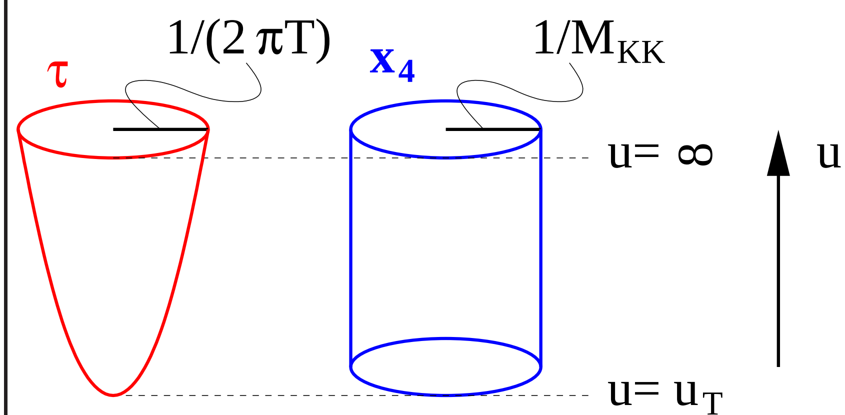
$$ds^2_{\text{conf}} = \left(\frac{u}{R}\right)^{3/2} [d\tau^2 + d\mathbf{x}^2 + f(u) dx_4^2] + \left(\frac{R}{u}\right)^{3/2} \left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$



$$M_{\text{KK}} = \frac{3}{2}\frac{u_{\text{KK}}^{1/2}}{R^{3/2}} \qquad f(u) \equiv 1 - \frac{u_{\text{KK}}^3}{u^3}$$

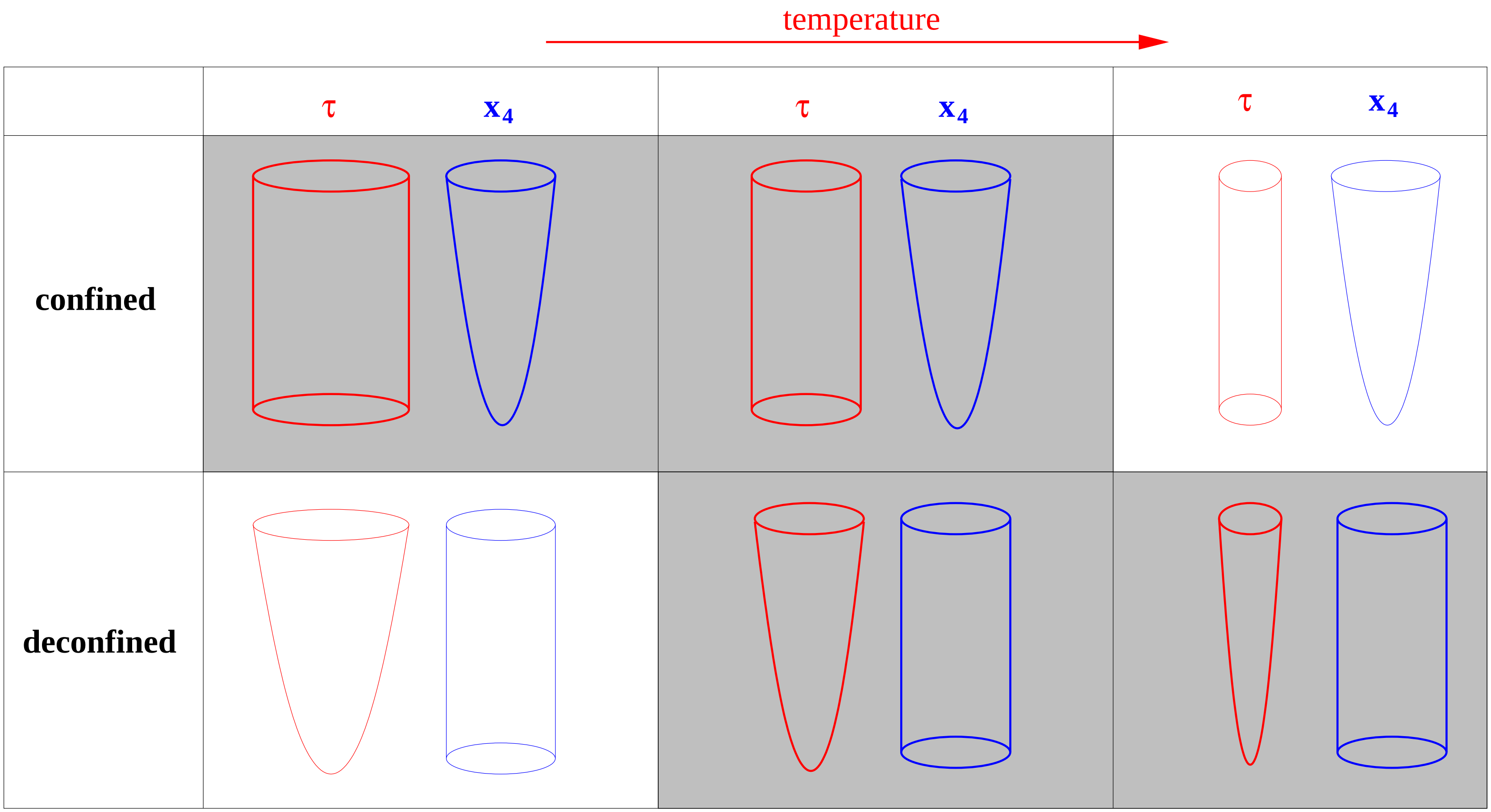
Deconfined phase

$$ds^2_{\text{deconf}} = \left(\frac{u}{R}\right)^{3/2} [\tilde{f}(u) d\tau^2 + \delta_{ij} d\mathbf{x}^2 + dx_4^2] + \left(\frac{R}{u}\right)^{3/2} \left[\frac{du^2}{\tilde{f}(u)} + u^2 d\Omega_4^2\right]$$



$$T = \frac{3}{4\pi}\frac{u_T^{1/2}}{R^{3/2}} \qquad \tilde{f}(u) \equiv 1 - \frac{u_T^3}{u^3}$$

- **Background geometry (page 3/3): deconfinement phase transition**



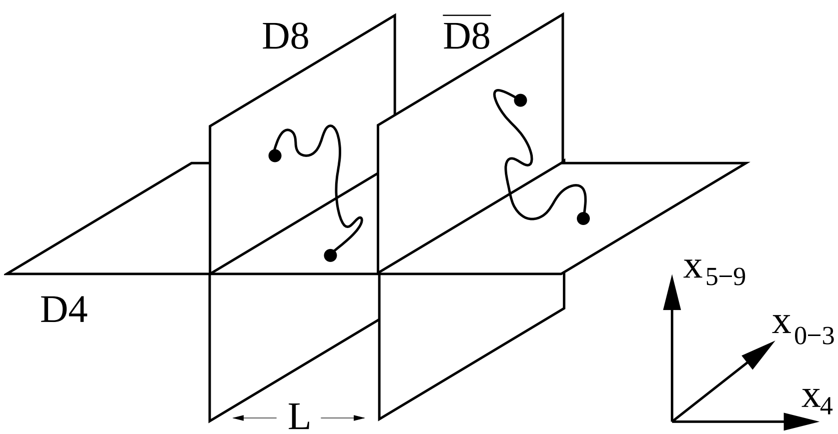
$$T_c = \frac{M_{\mathrm{KK}}}{2\pi}$$

• Add flavor (page 1/2)

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

- add N_f D8- and $\overline{\text{D8}}$-branes, separated in x_4

	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8/$\overline{\text{D8}}$	x	x	x	x		x	x	x	x	x



- 4-8, 4-$\bar{8}$ strings

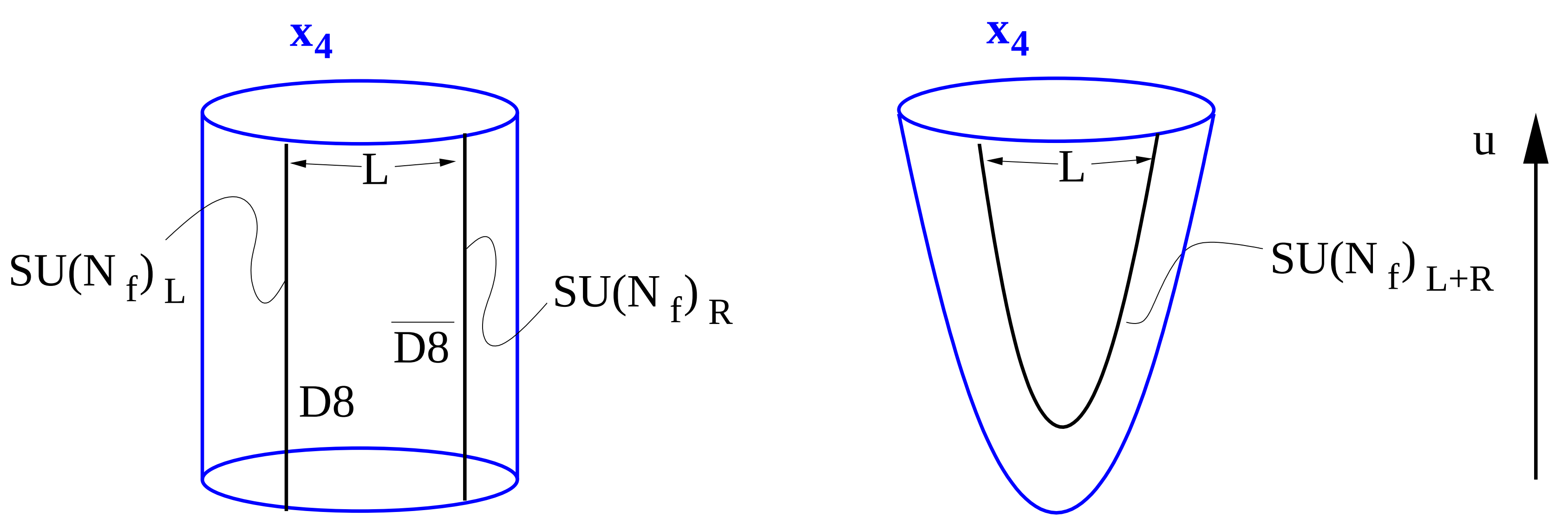
  → fundamental, massless chiral fermions

  under $U(N_f)_L \times U(N_f)_R$

  ⇒ quarks & gluons

• Add flavor (page 2/2): Chiral symmetry breaking

- background geometry unchanged if $N_f \ll N_c$ ("probe branes") $\longrightarrow$ "quenched" approximation
- gauge symmetry on the branes $\longrightarrow$ global symmetry at $u = \infty$



- chiral symmetry breaking

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_{L+R}$$

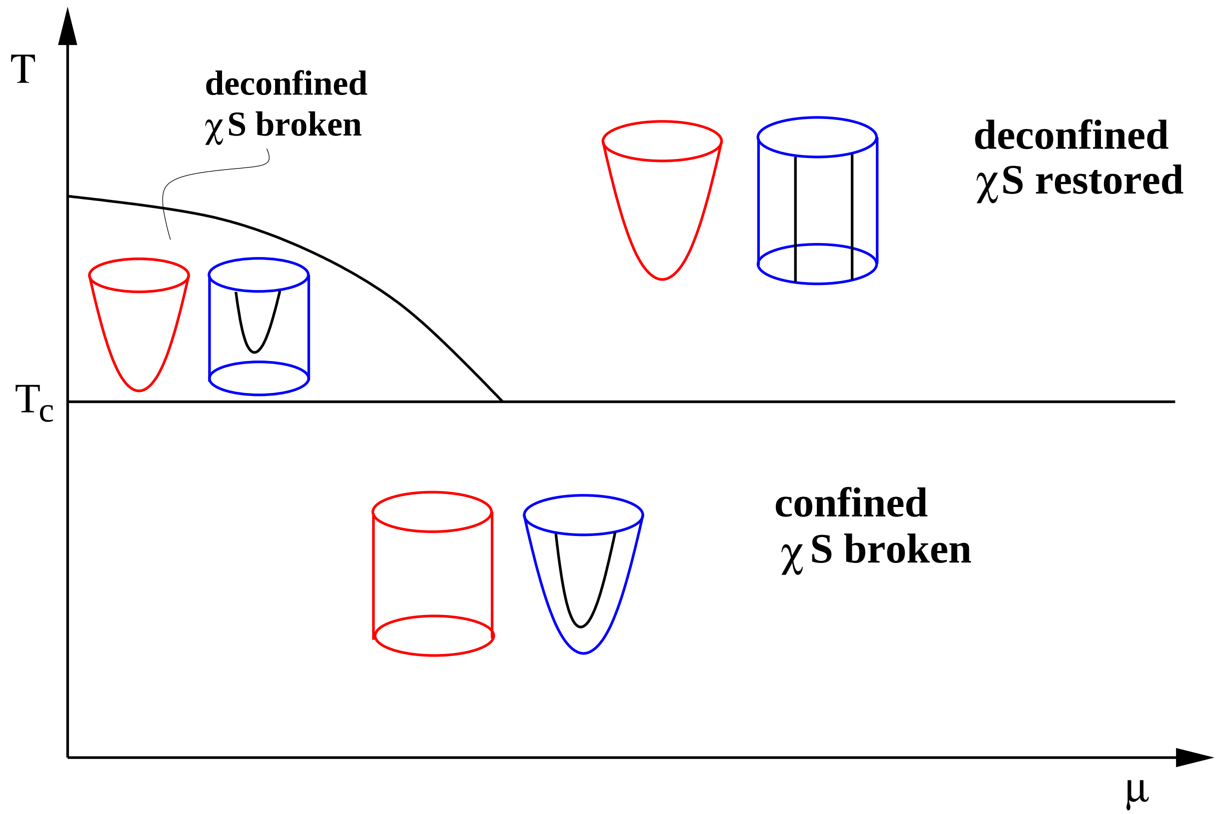
• T-μ phase diagram

N. Horigome, Y. Tanii, JHEP 0701, 072 (2007)

$M_{\rm KK} = 949\,{\rm MeV}$

(fit to ρ mass)

$\Rightarrow \quad T_c \simeq 150\,{\rm MeV}$



- deconfined, chirally broken phase only for $L < 0.3\,\pi/M_{\rm KK}$

  O. Aharony, J. Sonnenschein, S. Yankielowicz, Annals Phys. 322, 1420 (2007)

- similar diagram from large-N_c QCD

  L. McLerran, R. D. Pisarski, Nucl. Phys. A 796, 83 (2007)

• Gauge field action on D8-branes

- maximal separation $L = \pi/M_{\rm KK}$
- one flavor $N_f = 1$
- holographic coordinate $u \to z \in [-\infty, \infty]$ (broken) $z \in [0, \infty]$ (symmetric)
- gauge choice $A_z = 0$

$$S = S_{\rm YM} + S_{\rm CS}$$

$$S_{\rm YM} = \kappa M_{\rm KK}^2 \int d^4x \int_{-\infty}^{\infty} dz \left[k(z) F_{z\mu} F^{z\mu} + \frac{h(z)}{2M_{\rm KK}^2} F_{\mu\nu} F^{\mu\nu} \right]$$

$$k(z) \equiv 1 + z^2$$

$$S_{\rm CS} = \frac{N_c}{24\pi^2} \int d^4x \int_{-\infty}^{\infty} dz\, A_\mu F_{z\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$$

$$\kappa \equiv \frac{\lambda N_c}{216\pi^3}$$

equations of motion

$$\frac{\delta(\mathcal{L}_{\rm YM} + \mathcal{L}_{\rm CS})}{\delta A_\mu} = 0$$

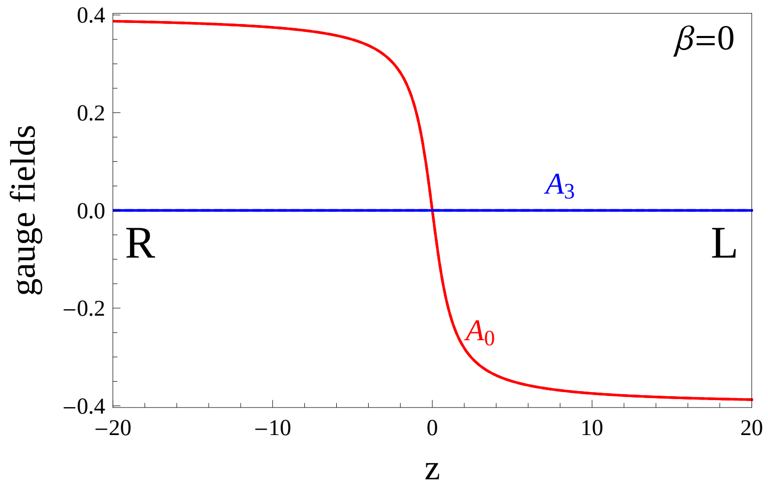
boundary conditions

$$A_0(z = \pm\infty) = \mu_{L/R}$$

$$A_1(x_2, z = \pm\infty) = -x_2 B$$

• Gauge fields in the bulk

Chirally broken phase

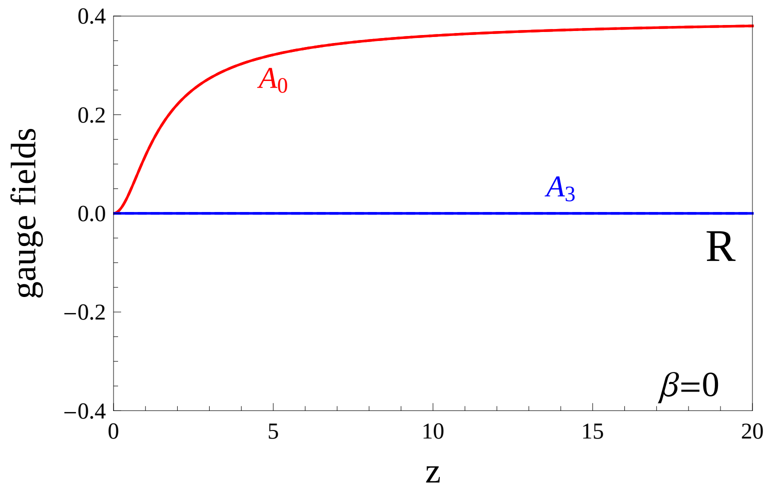


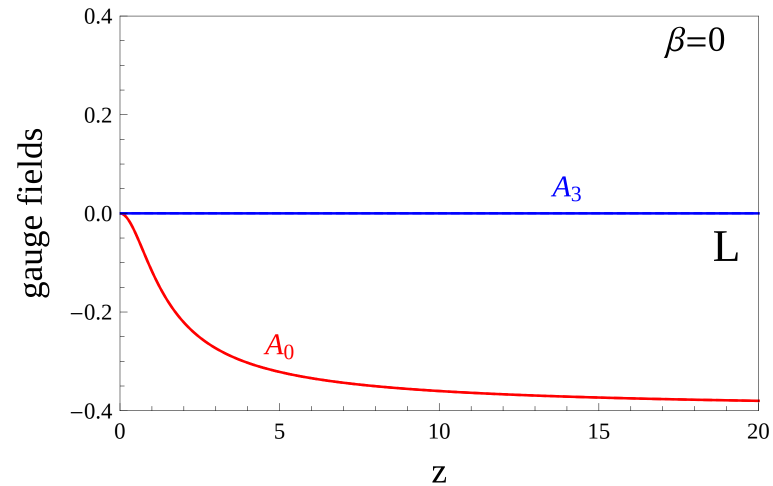
$$\mu_5 = \frac{\mu_R - \mu_L}{2}$$

$$(\text{here: } \mu = 0)$$

$$\beta \equiv \frac{27\pi B}{2\lambda M_{\text{KK}}^2} \simeq \frac{B}{2 \cdot 10^{19}\,\text{G}}$$

Chirally restored phase





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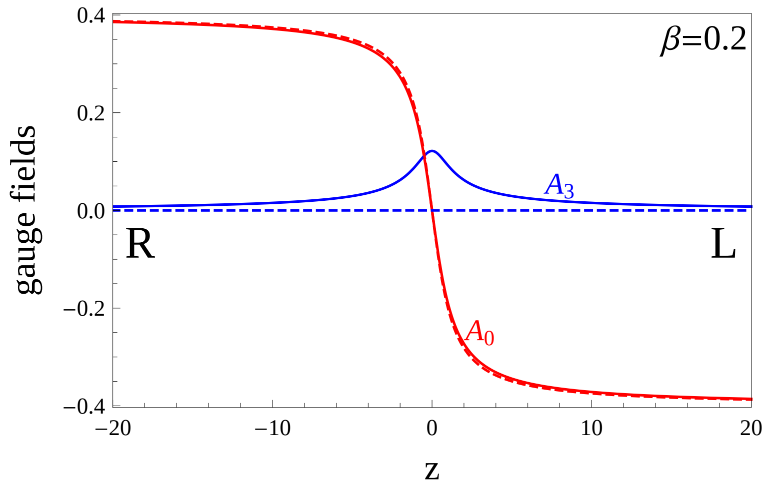

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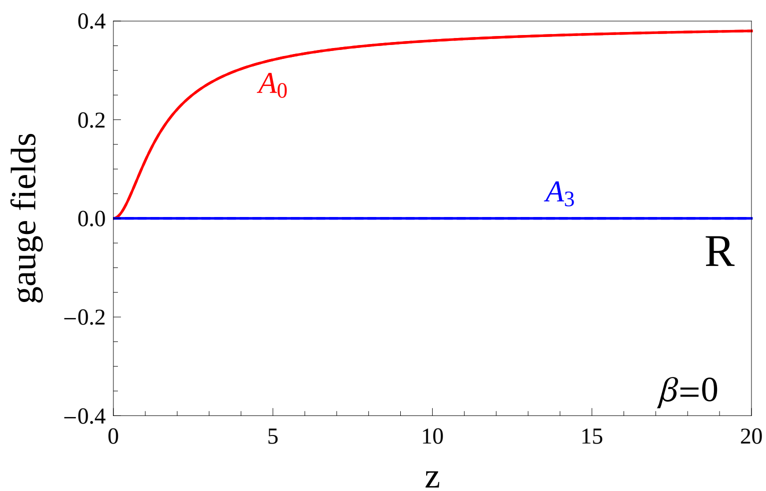

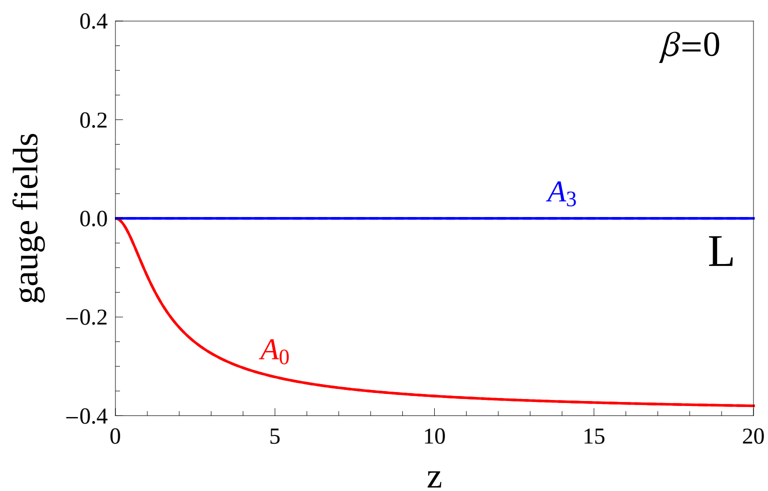

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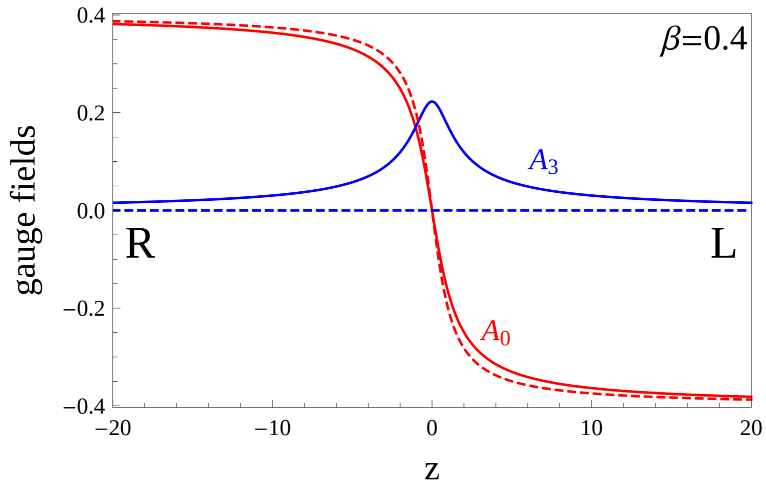

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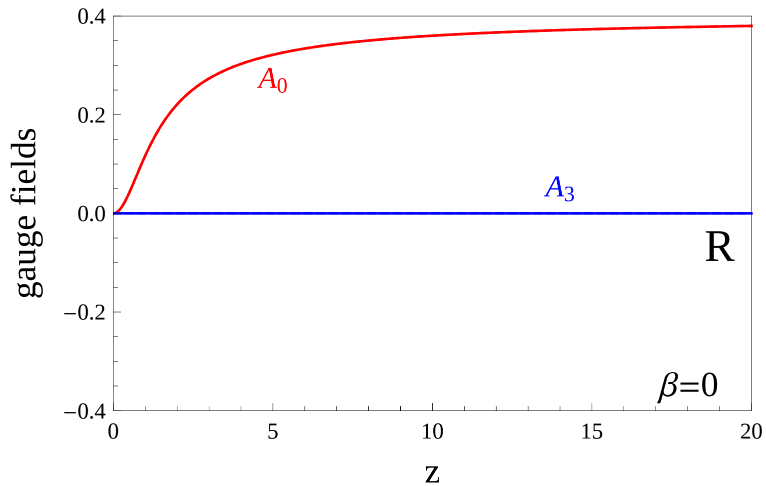

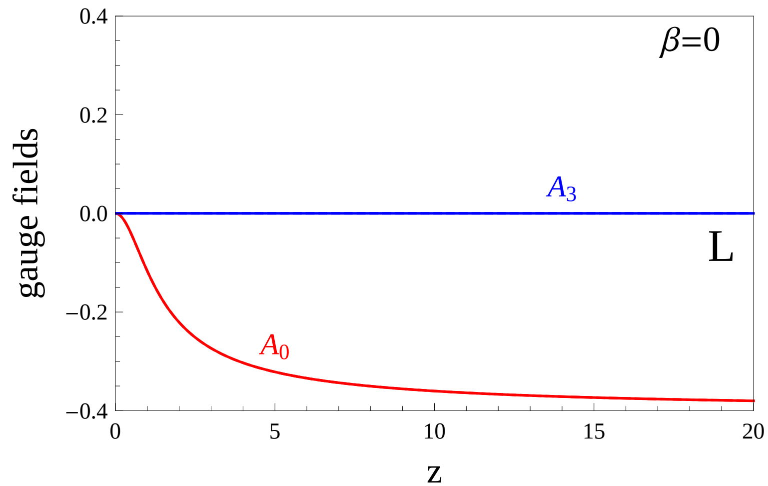

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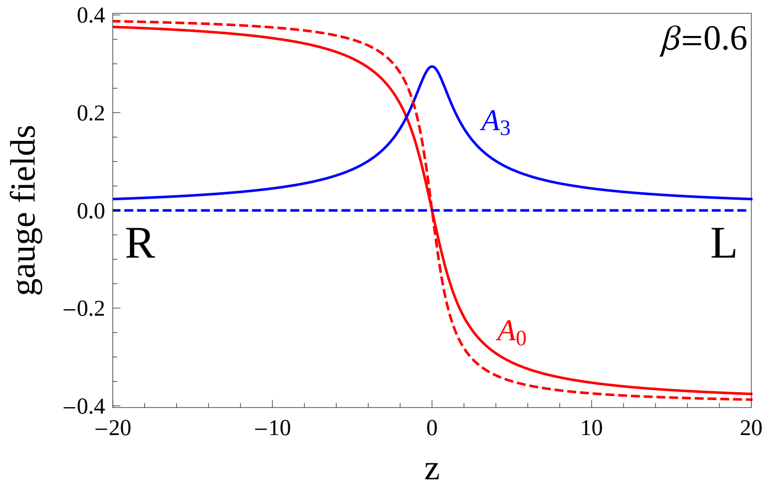

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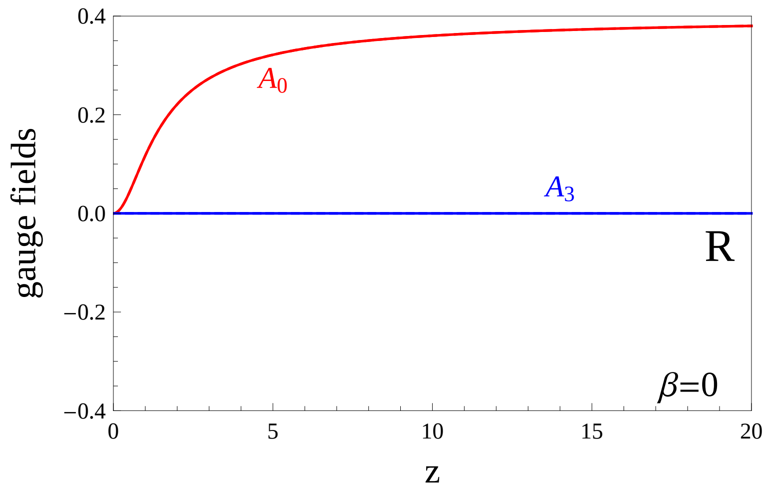

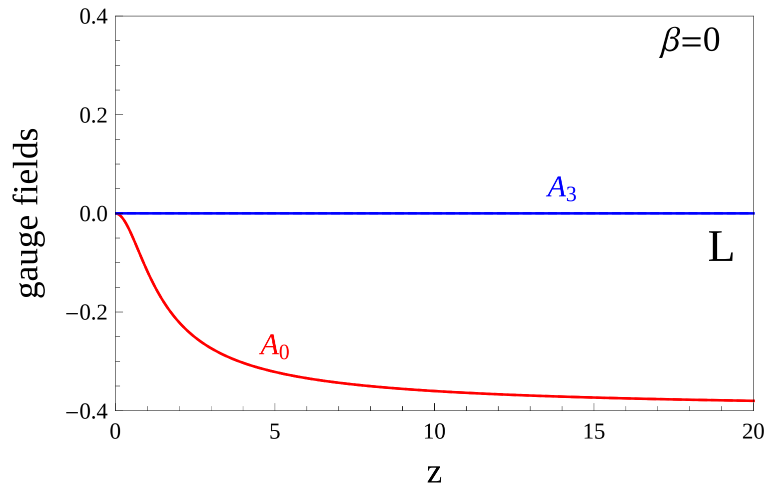

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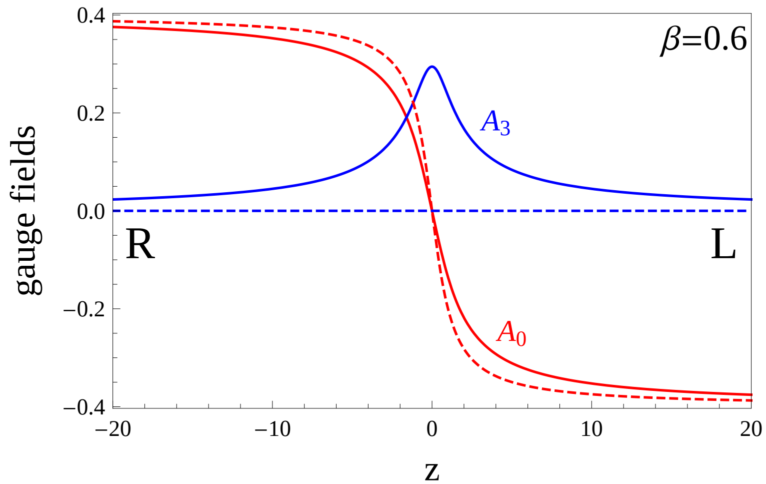

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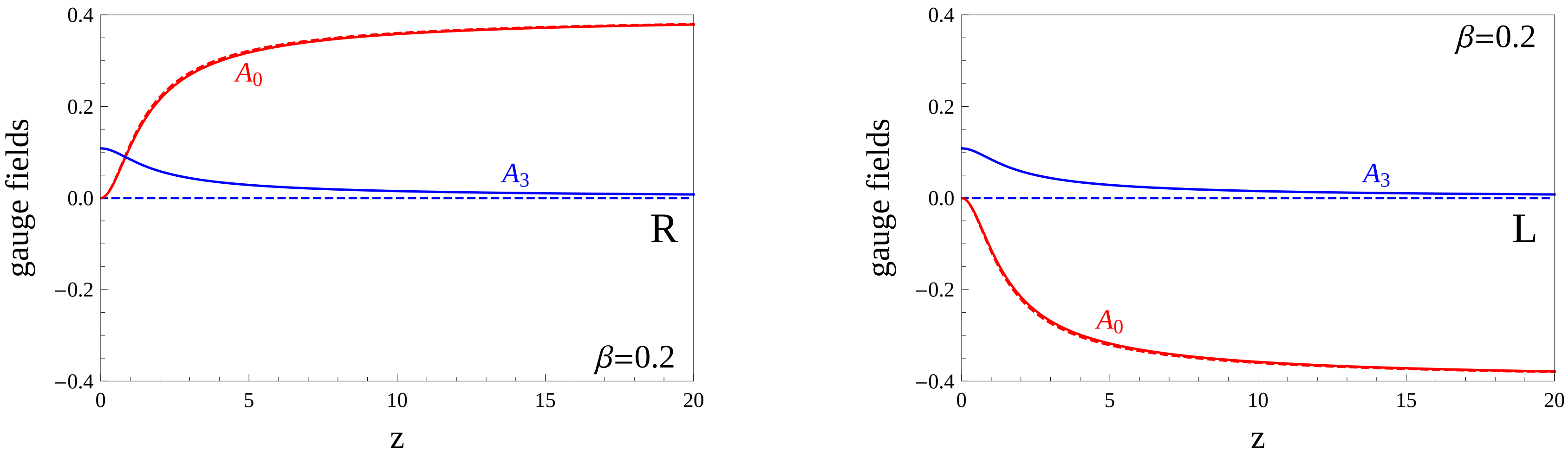
Chirally restored phase



• Gauge fields in the bulk

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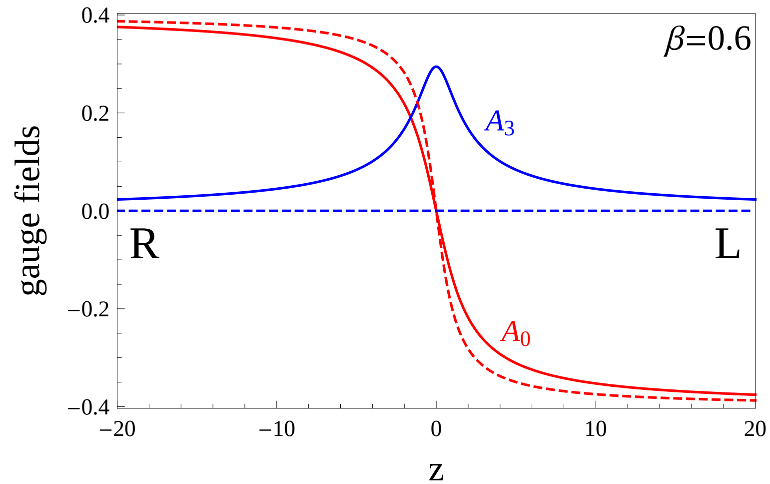

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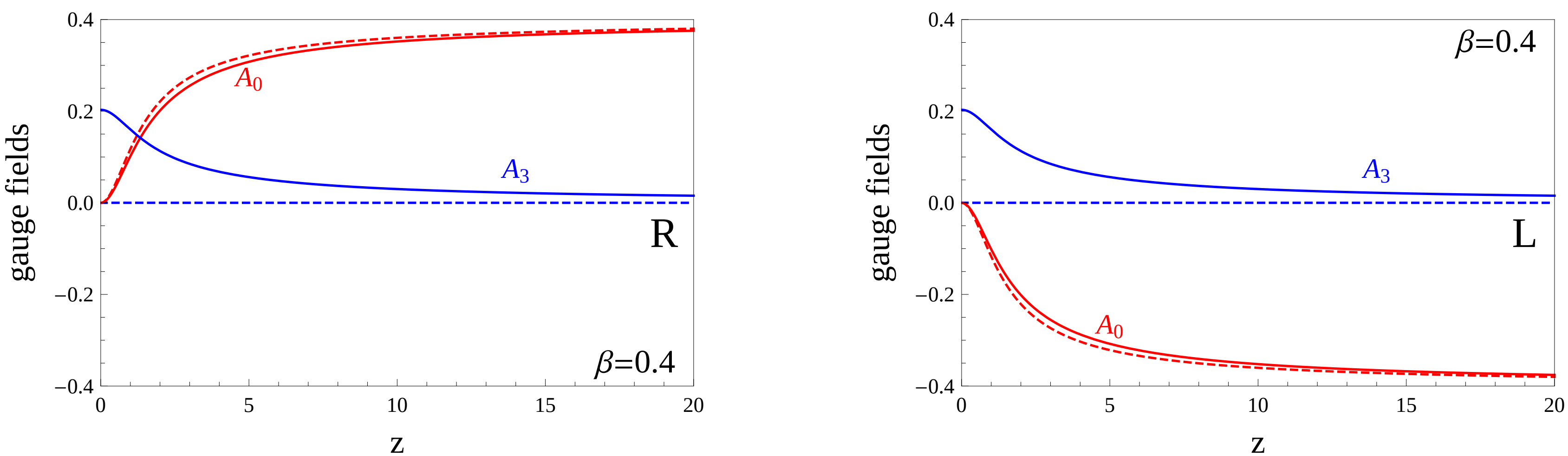
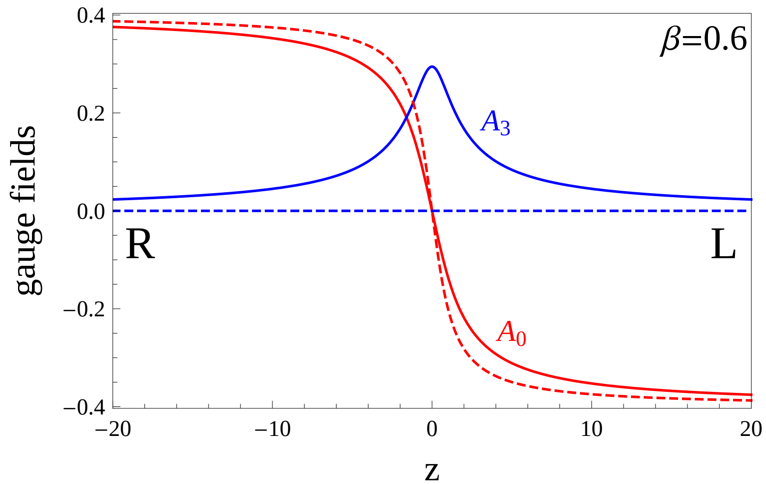
Chirally restored phase



• Gauge fields in the bulk

Chirally broken phase



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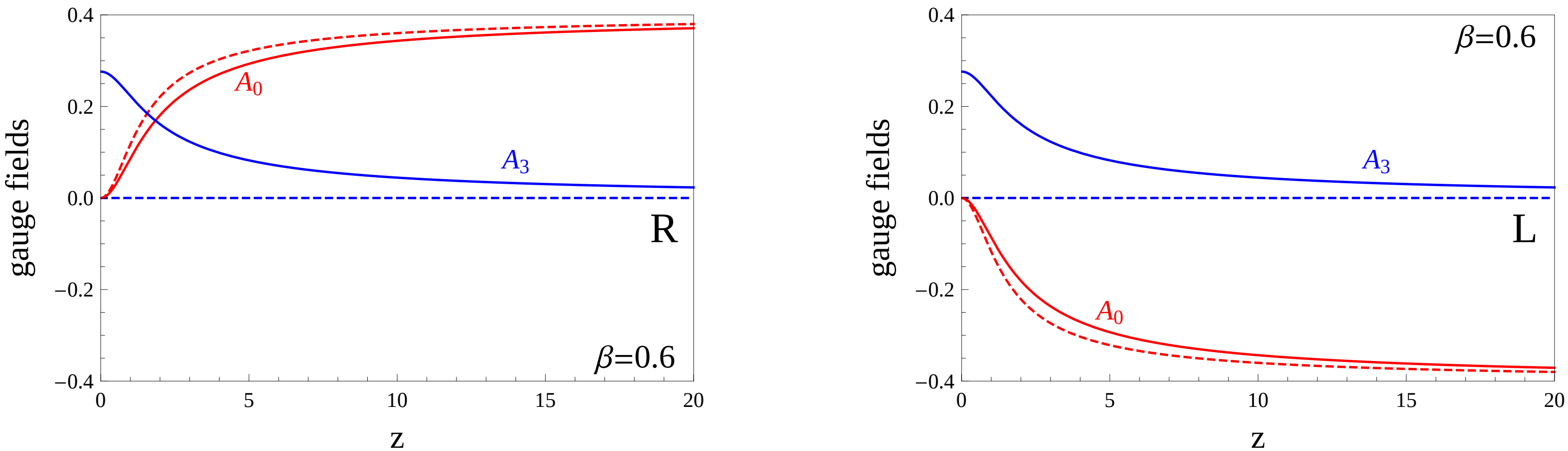
Chirally restored phase



• Chiral currents

- YM and CS contributions to **chiral currents**

H. Hata, M. Murata, S. Yamato, PRD 78, 086006 (2008)

K. Hashimoto, T. Sakai, S. Sugimoto, Prog. Theor. Phys. 120, 1093 (2008)

$$S = S_{\rm YM} + S_{\rm CS}$$

$$\Rightarrow \qquad \mathcal{J}^\mu_{L/R} \equiv -\frac{\delta S}{\delta A_\mu(x, z=\pm\infty)} = \mathcal{J}^\mu_{L/R,{\rm YM}} + \mathcal{J}^\mu_{L/R,{\rm CS}}$$

- sometimes CS contribution is ignored

H. U. Yee, JHEP 0911, 085 (2009)

D. T. Son, P. Surowka, PRL 103, 191601 (2009)

only YM part in asymptotics of A_μ:

$$A^\mu(x,z) = A^\mu(x, z=\pm\infty) \pm \frac{\mathcal{J}^\mu_{L/R,{\rm YM}}}{2\kappa M^2_{\rm KK}}\frac{1}{z} + \mathcal{O}\left(\frac{1}{z^2}\right) .$$

- **Anomalies**
  - axial and vector anomalies $\left[F_{\mu\nu}^{L/R}(x) \equiv F_{\mu\nu}(x, z = \pm\infty)\right]$

$$\partial_\mu \mathcal{J}_5^\mu = \frac{N_c}{24\pi^2}\left(F_{\mu\nu}^V \widetilde{F}_V^{\mu\nu} + F_{\mu\nu}^A \widetilde{F}_A^{\mu\nu}\right)$$

$$\partial_\mu \mathcal{J}^\mu = \frac{N_c}{12\pi^2} F_{\mu\nu}^V \widetilde{F}_A^{\mu\nu}$$

"consistent" anomaly

W.A. Bardeen, Phys. Rev. 184, 1848 (1969); C.T. Hill, PRD 73, 085001 (2006)

- need **Bardeen's counterterm**

$$\Delta S = c \int d^4x (A_\mu^L A_\nu^R F_{\rho\sigma}^L + A_\mu^L A_\nu^R F_{\rho\sigma}^R)\epsilon^{\mu\nu\rho\sigma}$$

(here interpreted as **holographic renormalization**)

$\longrightarrow$ determine c to get **QED ("covariant") anomaly**

• Correct anomaly with Bardeen's counterterm

- Bardeen's counterterm $\Delta S \longrightarrow$ new chiral currents

$$\bar{\mathcal{J}}^{\mu}_{L/R} \equiv \mathcal{J}^{\mu}_{L/R,\mathrm{YM}} + \mathcal{J}^{\mu}_{L/R,\mathrm{CS}} + \Delta\mathcal{J}^{\mu}_{L/R}$$

$$\Rightarrow \quad \begin{aligned} \partial_\mu \bar{\mathcal{J}}^{\mu}_{5} &= \frac{N_c}{8\pi^2} F^{V}_{\mu\nu} \widetilde{F}^{\mu\nu}_{V} + \frac{N_c}{24\pi^2} F^{A}_{\mu\nu} \widetilde{F}^{\mu\nu}_{A} \\ \partial_\mu \bar{\mathcal{J}}^{\mu} &= 0 \end{aligned}$$

"covariant" anomaly

J.S. Bell, R.Jackiw, Nuovo Cim. A60, 47 (1969); S.L. Adler, Phys. Rev. 177, 2426 (1969)

- **conservation of vector current**
  & correct decay rate $\pi^0 \to 2\gamma$

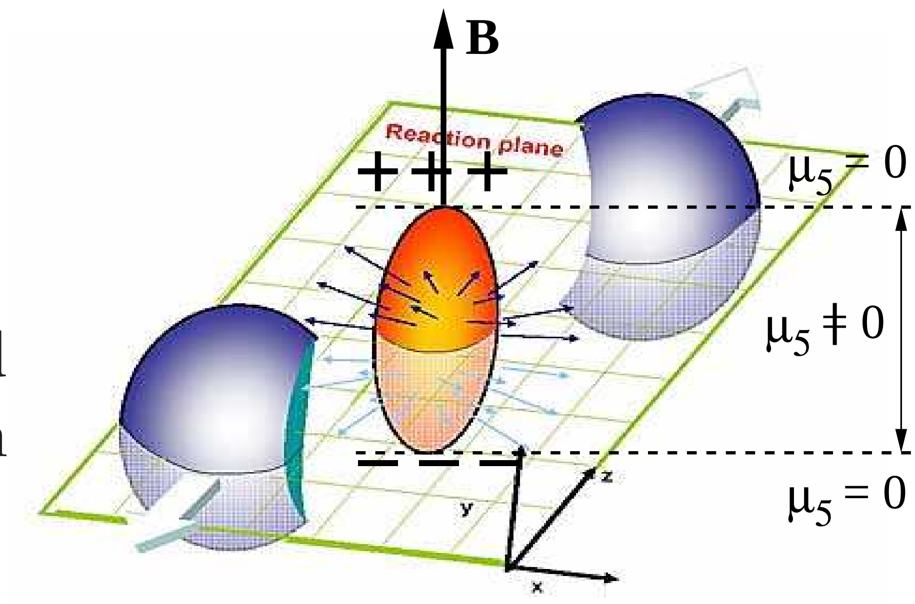
- **Use YM currents?**

  - notice: YM part alone gives

$$\partial_\mu \mathcal{J}^\mu_{\mathrm{YM},5} = \frac{N_c}{8\pi^2}\left(F^V_{\mu\nu}\widetilde{F}^{\mu\nu}_V + F^A_{\mu\nu}\widetilde{F}^{\mu\nu}_A\right)$$

$$\partial_\mu \mathcal{J}^\mu_{\mathrm{YM}} = \frac{N_c}{4\pi^2}F^V_{\mu\nu}\widetilde{F}^{\mu\nu}_A$$

  - seems OK for $F_A = 0$
  - $\mathcal{J}^\mu_{\mathrm{YM}} \neq \bar{\mathcal{J}}^\mu$ even if $F_A = 0$
  - $\mathcal{J}^\mu_{\mathrm{YM}}$ not strictly conserved (need $\nabla\mu_5$ for charge separation at RHIC!)



• Results for currents (page 1/2)

A. Rebhan, A. Schmitt, S.A. Stricker, JHEP 1001, 026 (2010)

	$\mathcal{J}_{\rm YM}$	$\mathcal{J}_{\rm YM}+\mathcal{J}_{\rm CS}$	$\mathcal{J}_{\rm YM}+\mathcal{J}_{\rm CS}+\Delta\mathcal{J}$
anomaly	"semi-covariant":	consistent:	covariant:
$\partial_\mu \mathcal{J}_5^\mu / \frac{N_c}{24\pi^2}$	$\underline{3F_V\widetilde{F}_V}+3F_A\widetilde{F}_A$	$F_V\widetilde{F}_V + F_A\widetilde{F}_A$	$\underline{3F_V\widetilde{F}_V + F_A\widetilde{F}_A}$
$\partial_\mu \mathcal{J}^\mu / \frac{N_c}{24\pi^2}$	$6F_V\widetilde{F}_A$	$2F_V\widetilde{F}_A$	$\underline{0}$
$\left.(\mathcal{J}_\parallel^5 / \frac{\mu B N_c}{2\pi^2})\right\|_{T>T_c}$	1	$\frac{2}{3}$	1
$\mathcal{J}_\parallel / \frac{\mu_5 B N_c}{2\pi^2}$	1	$\frac{2}{3}$	0
$\left.(\mathcal{J}_\parallel / \mathcal{J}_5^0)\right\|_{B\to\infty}$	1	$\frac{2}{3}$	0

axial current as expected

absence of CME

	$\bar{\mathcal{J}}_\parallel$	$\bar{\mathcal{J}}_\parallel^5$
$T > T_c$	0	$\frac{N_c}{2\pi^2}\mu B$
$T < T_c$	0	$\frac{N_c}{4\pi^2}\mu B \, \frac{\beta \coth \beta\pi}{\rho(\beta)}$

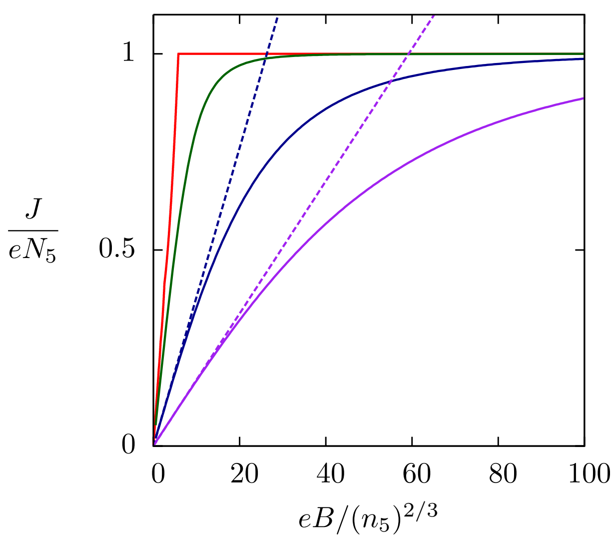
chirally **symmetric** phase

chirally **broken** phase

• Results for currents (page 2/2)

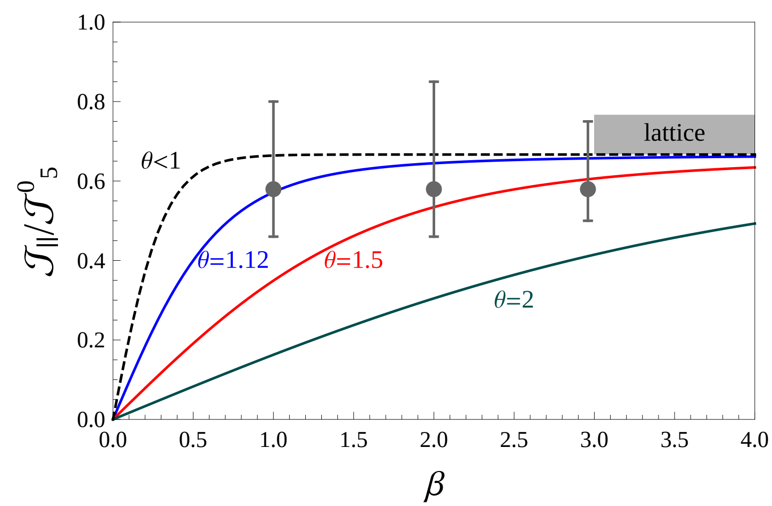
	$\mathcal{J}_{\rm YM}$	$\mathcal{J}_{\rm YM} + \mathcal{J}_{\rm CS}$	$\mathcal{J}_{\rm YM} + \mathcal{J}_{\rm CS} + \Delta\mathcal{J}$
anomaly	"semi-covariant":	consistent:	covariant:
$\partial_\mu \mathcal{J}_5^\mu / \frac{N_c}{24\pi^2}$	$3F_V\widetilde{F}_V + 3F_A\widetilde{F}_A$	$F_V\widetilde{F}_V + F_A\widetilde{F}_A$	$3F_V\widetilde{F}_V + F_A\widetilde{F}_A$
$\partial_\mu \mathcal{J}^\mu / \frac{N_c}{24\pi^2}$	$6F_V\widetilde{F}_A$	$2F_V\widetilde{F}_A$	0
$(\mathcal{J}_\parallel^5 / \frac{\mu B N_c}{2\pi^2})\big\vert_{T>T_c}$	1	$\frac{2}{3}$	1
$\mathcal{J}_\parallel / \frac{\mu_5 B N_c}{2\pi^2}$	1	$\frac{2}{3}$	0
$(\mathcal{J}_\parallel / \mathcal{J}_5^0)\big\vert_{B\to\infty}$	1	$\frac{2}{3}$	0

like in weak coupling:



K.Fukushima *et.al.*, PRD 78, 074033 (2008)

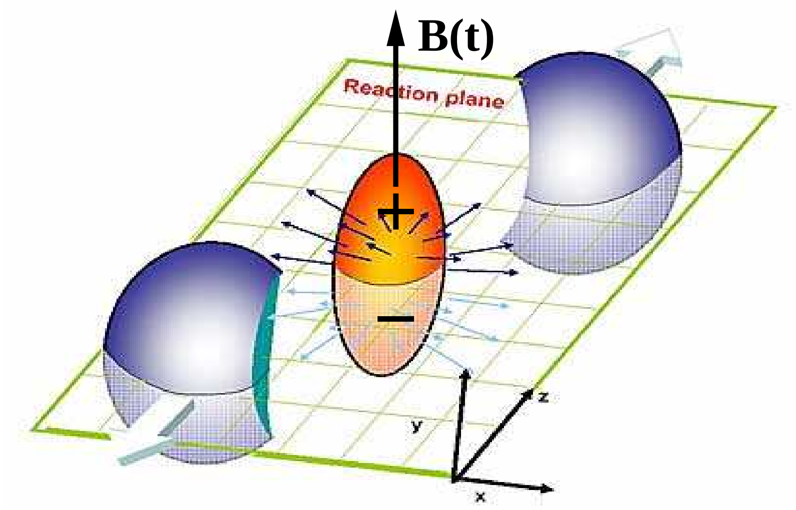
like on the lattice:



P. V. Buividovich *et.al.*, PRD 80, 054503 (2009)

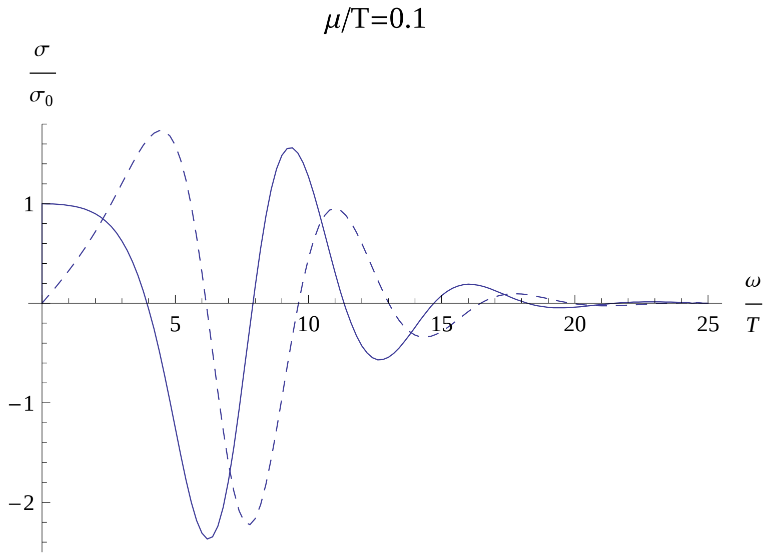
A. Rebhan *et.al.*, JHEP 1001, 026 (2010)

• Frequency-dependent conductivity

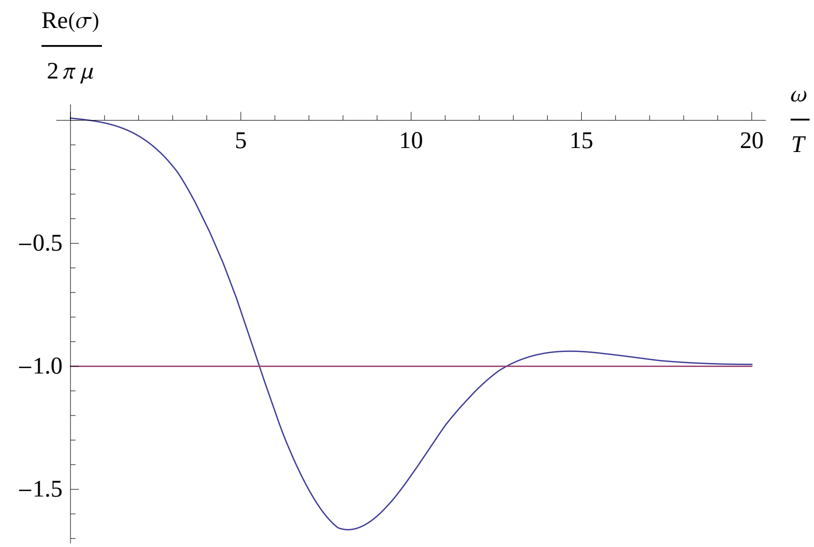


- time dependent B-field
- need $\sigma(\omega)$ for time-dependent $J(t)$
- however: charge separation only depends on $\sigma(0)$

- $\mathcal{J}_{\mathrm{YM}}$ alone
  H.U. Yee, JHEP 0911, 085 (2009)



- full current $\mathcal{J}_{\mathrm{YM}} + \mathcal{J}_{\mathrm{CS}} + \Delta\mathcal{J}$
  K. Landsteiner, F. Peña (preliminary)



- **Discussion (CME or no CME?)**

- **Something wrong/unphysical?**
  - definition of current within the model still incomplete?
    $\mathcal{J} = \mathcal{J}_{\mathrm{YM}} + \mathcal{J}_{\mathrm{CS}} + \Delta\mathcal{J} + \ldots ?$
  - Sakai-Sugimoto not interpretable as strong-coupling limit of QCD (for the case of the CME)?

- **If result to be taken seriously ...**
  - strong-coupling *vector* current differs from weak-coupling current (unlike *axial* current)
  - quasiparticle/Landau-level picture not applicable at strong coupling
  - if charge separation at RHIC due to CME, quark-gluon plasma at RHIC sufficiently weakly coupled?!

work in progress, in collaboration with K. Landsteiner, F. Peña (Madrid) and A. Gynther (Vienna)