The Inflationary Universe

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- Inflation solves the horizon and flatness problems and predicts $\Omega = 1$

- generates density perturbations $\delta\rho/\rho$ and gravitational waves $\delta g_{\mu\nu}$ from quantum fluctuations

- provides scale-invariant spectra $P(k) = Ak^n, \ n \sim 1$

$c = \hbar = 1, \ \kappa \equiv 8\pi G = 8\pi/M_{\mathrm{P}}^2$

Standard Cosmology: Basics

Robertson-Walker line element:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right]$$

a scale factor; k/a^2 spatial curvature with $k = 0, \pm 1$

Friedmann equations: (from ${G^0}_0 = \kappa {T^0}_0$ and $\mathrm{d}U = -p\mathrm{d}V$)

$$H^2 + \frac{k}{a^2} = \frac{\kappa}{3}\rho\ , \qquad \dot{\rho} = -3H(\rho + p)$$

$H \equiv \dot{a}/a$ Hubble expansion rate; ρ energy density; $p = p(\rho)$ pressure

$\Omega \equiv \rho/\rho_{\mathrm{c}}$ ($\rho_{\mathrm{c}} = 3H^2/\kappa$); $\Omega = 1$ for a flat universe
today: $0.1 < \Omega_0 < 2$

Standard Cosmology: Observations

Hubble expansion:

$$H_0 d_{\mathcal{L}} = z + \mathcal{O}(z^2) \qquad z \ldots \text{redshift} \quad (1 + z = a_0/a)$$

$H_0 = 100h$ km/s/Mpc with $h = 0.5 - 0.8$ 1Mpc $\approx 3 \times 10^6$ light-yrs

Cosmic microwave radiation: $T_{\text{dec}} \simeq 0.25$eV
Planck spectrum with $T_0 = 2.728 \pm 0.004$ K Fixsen et al. 1996
Isotropic up to $(\Delta T/T)_{\text{rms}} = (1.06 \pm 0.04) \times 10^{-5}$ Banday et al. 1996

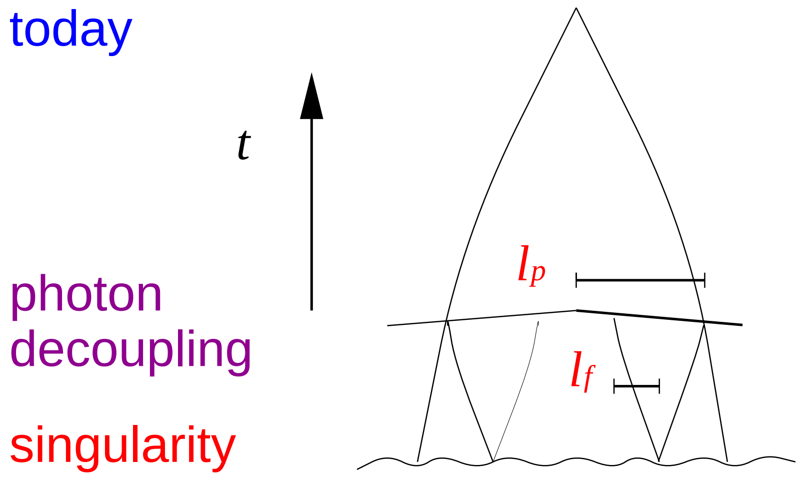
Primordial nucleosynthesis: ($T_{\text{nuc}} \simeq 0.2$MeV)
^{4}He/H $= 0.23 - 0.24$, D/H $= (2 - 20) \times 10^{-5}$
$\Rightarrow 0.005 < \Omega_B h^2 < 0.024$ Olive & Schramm 1997

Standard Cosmology: Problems I

Horizon problem:
$\ell_p(t)$ past causal horizon
$\ell_f(t)$ future causal horizon

$$(\ell_p/\ell_f)(z_{\text{dec}}) \simeq \sqrt{z_{\text{dec}}} \gg 1 \quad (z_{\text{dec}} \simeq 1100)$$

10^3 causally disconnected patches do have the same temperature. Why?



Flatness problem: Why is $\Omega_0 = \mathcal{O}(1)$?

$$|1-\Omega(z)| = |1-\Omega_0| \begin{cases} (1+z)^{-1} & \text{matter dominated} \\ (1+z)^{-2} & \text{radiation dominated} \end{cases}$$

$$\Rightarrow |1-\Omega(z_{\text{dec}})| = \mathcal{O}(10^{-3}) \;, \quad |1-\Omega(z_{\text{GUT}})| = \mathcal{O}(10^{-60}) \quad (z_{\text{GUT}} \sim 10^{30})$$

Standard Cosmology: Problems II

Origin of primordial fluctuations?

large scale structure (distribution of galaxies and clusters of galaxies)

redshift surveys: CfA, LCRS, SDSS, ...

anisotropies of the microwave radiation COBE DMR, MAP, PLANCK, ...

A singularity ($a \to 0; \rho \to M_{\mathrm{P}}^4$) does exist, if $\rho + 3p > 0$.

(strong energy condition; satisfied in matter and radiation dominated universe)

Proof: $\ddot{a} < 0$ from

$$-3\frac{\ddot{a}}{a} = \frac{\kappa}{2}(\rho + 3p) \qquad \text{(equation of geodesic deviation)}$$

if $\rho + 3p > 0$. Thus, $a \to 0$ for $t \ll t_0$. •

Is quantum-gravity necessary to solve the problems above?

A possible solution: the (quantum-) initial conditions are indeed fine tuned to provide the (observed) Universe

OR the problems of standard cosmology are solved dynamically:

Inflation — Main Idea

Starobinskii 1979; Guth 1980: initial vacuum dominated epoch
vacuum expectation of some field φ:

$$\langle 0|T_{\mu\nu}(\varphi)|0\rangle \simeq g_{\mu\nu}V(\varphi) \Rightarrow p \simeq -\rho$$

from 1st law of thermodynamics $\Rightarrow$ $\dot{\rho} \simeq 0$

from Friedmann equation $H_{\mathrm{inf}} \simeq$ constant and $a(t) \propto \exp(H_{\mathrm{inf}}t)$

Inflation: $\ddot{a} > 0$ accelerated expansion
from equation of geodetic deviation $\Rightarrow$ inflation occurs if $\rho + 3p < 0$
(violation of strong energy condition, but weak energy condition holds true, i.e. $\rho + p \geq 0$)

Inflation: Causality and Flatness

horizon problem is solved:

$\ell_p/\ell_f \sim z_{\mathrm{GUT}} \exp(-N) \ll 1$

if $N \equiv H_{\mathrm{inf}} \Delta t > 70$

flatness problem disappears:

during inflation

$|1 - \Omega(t)| \propto \exp(-2H_{\mathrm{inf}} t)$

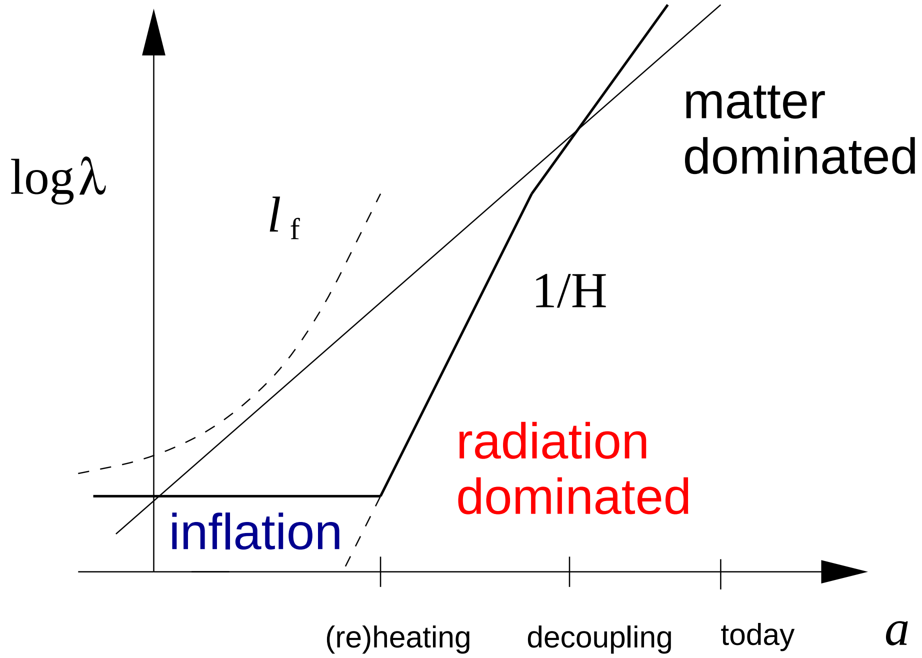
after inflation

$\Omega = 1 + \mathcal{O}(\exp[-2N])$

inflation should last for

at least 70 e-folds

$\Rightarrow$ inflation predicts $\Omega_0 = 1$



Inflation: Scenarios — History

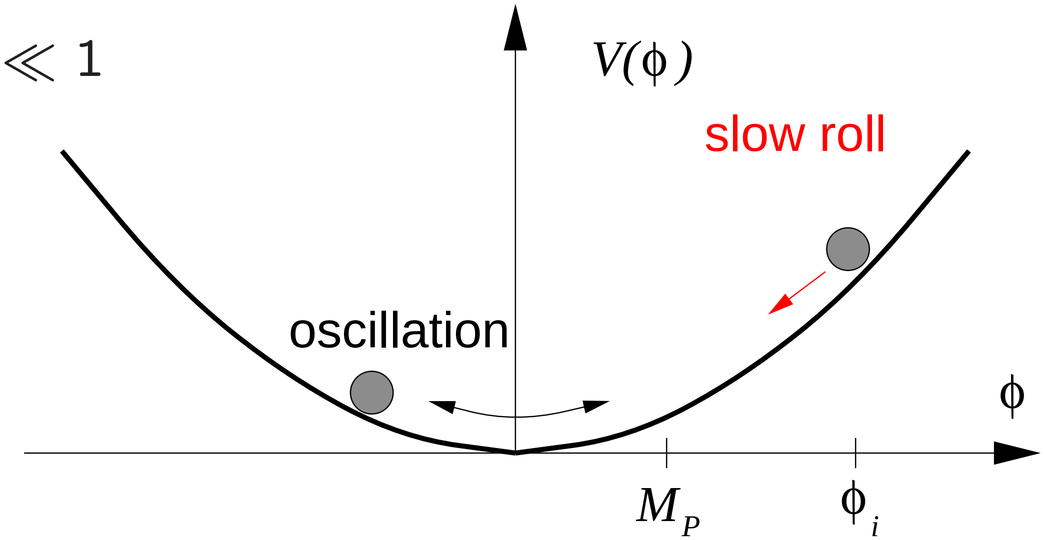
Starobinskii 1979	R^2-inflation (quantum gravity corrections)
Guth 1980	old inflation (first order GUT transition)
	never stops, because bubbles do not merge
Linde 1982	new inflation (flat potential, slow roll)
Albrecht & Steinhardt 1982	needs special initial conditions
Linde 1983	chaotic inflation (slow roll)
	arbitrary $V(\varphi)$, random initial conditions $\varphi_i, \dot{\varphi}_i$
La & Steinhardt 1989	(hyper-)extended inflation (two scalar fields)
Linde 1993	hybrid inflation (two scalar fields)
...	

Chaotic Inflation: Slow Roll

Linde 1983

simple example $V = \lambda\varphi^4/4,\ \lambda \ll 1$

a single scale: M_{P}



equations of motion:

$$H^2 = \frac{8\pi}{3M_{\mathrm{P}}^2}\left(\frac{1}{2}\dot\varphi^2 + V\right)$$

$$\ddot\varphi + 3H\dot\varphi + V_{,\varphi} = 0$$

generic initial conditions

at $t \sim t_{\mathrm{P}}$: $\dot\varphi_i^2 \sim M_{\mathrm{P}}^4$ and $V(\varphi_i) \sim M_{\mathrm{P}}^4 \Rightarrow$ $\varphi_i \sim \lambda^{-1/4} M_{\mathrm{P}} \gg M_{\mathrm{P}}$

slow roll: motion of φ is slowed down quickly by the Hubble drag ($H\dot\varphi \gg V_{,\varphi}$)

$\Rightarrow \frac{1}{2}\dot\varphi^2 \ll V$ and $\ddot\varphi \ll -3H\dot\varphi$ $\Rightarrow a(t) \propto \exp(H[\varphi(t)]t)$

with $H(\varphi) \simeq [8\pi V(\varphi)/3M_{\mathrm{P}}^2]^{1/2}$ and $\varphi(t) \simeq \varphi_i \exp[-(\lambda/6\pi)^{1/2} t M_{\mathrm{P}}]$

Chaotic Inflation: Reheating

Dolgov & Linde 1982; Abbott, Fahri & Wise 1982

inflation terminates at $\varphi \sim M_{\mathrm{P}}$: φ oscillates around its minimum

coherent oscillations decay into other particles

e.g. Yukawa coupling $\frac{1}{2}g^2 v\varphi\chi^2$ to a bosonic particle χ

$$\ddot{\chi}_k + 3H\dot{\chi}_k + [k_{\mathrm{ph}}^2 + m_\chi^2 + g^2\sigma\varphi(t)]\chi_k = 0$$

might be very efficient due to parametric resonance $\chi_k \sim \exp(\mu t)$

Traschen & Brandenberger 1990; Kofman, Linde & Starobinskii 1994

these decays produce entropy and (re)heat the universe to T_{rh}

T_{rh} should be high enough to allow baryogenesis

(probably GUT scale; in any case $T_{\mathrm{rh}} > T_{\mathrm{nuc}}$)

Structure Formation: Quantum Fluctuations

accelerated expansion provides energy to produce classical fluctuations from vacuum fluctuations

$$\widehat{\varphi}(\eta,\vec{x}) = \frac{1}{a}\int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\frac{1}{\sqrt{2k}}[\widehat{c}_k f_k(\eta)\exp(\imath\vec{k}\vec{x}) + \mathrm{h.c.}]$$

with $\widehat{c}_k|0\rangle = 0$ and $[\widehat{c}_k, \widehat{c}^\dagger_{k'}] = \delta(\vec{k}-\vec{k}')$ $[\eta \equiv \int \mathrm{d}t/a(t)]$

$$f_k'' + (k^2 - \frac{a''}{a})f_k = 0$$

subhorizon scales $k_{\mathrm{ph}} \equiv k/a \gg H$: harmonic oscillator

superhorizon scales $k_{\mathrm{ph}} \ll H$: $f_k \simeq a$ rapid amplification of fluctuations

rms amplitude at the moment $k_{\mathrm{ph}} = H$: $\delta\varphi(k=H) \simeq \dfrac{H(\varphi)}{2\pi}$

spectrum is almost scale-invariant (Harrison-Zel'dovich)

Structure Formation: Density perturbations

Chibisov & Mukhanov 1981; Hawking 1982; Guth & Pi 1982

fluctuations in $\delta\varphi$ imply fluctuations in the metric

($\phi(\eta,\vec{x}), \psi(\eta,\vec{x})$ . . . metric potentials of longitudinal sector)

$$\mathrm{d}s^2 = a^2(\eta)[-(1+2\phi)\mathrm{d}\eta^2 + (1-2\psi)\mathrm{d}\vec{x}^2] \qquad \text{(longitudinal gauge)}$$

and imply fluctuations in the energy density

$$\delta\rho(\eta,\vec{x}) = \frac{1}{a^2}(\varphi'\delta\varphi' - \varphi'^2\phi) + V_{,\varphi}\delta\varphi$$

linearized equations: $\phi = \psi$, $\mathcal{H}\phi + \phi' = \kappa/2\varphi'\delta\varphi$ and $\qquad [\mathcal{H} \equiv aH]$

$$\phi'' + 2(\mathcal{H} - \varphi''/\varphi)\phi' + [k^2 + 2(\mathcal{H}' - \mathcal{H}\varphi''/\varphi')\phi] = 0$$

first integral for $k_{\mathrm{ph}} \ll H$: $(\mathcal{H}^{-1}\phi' + \phi)/(1 - \mathcal{H}'/\mathcal{H}^2) + \phi \simeq \text{const.}$ $\Rightarrow$
large amplification $\phi_0 \simeq \frac{5}{2}\frac{\rho_i}{\rho_i + p_i}\phi_i$ with $p_i \simeq -\rho_i$ Martin & Schwarz 1997

Structure Formation: CMR

at photon decoupling metric fluctuations induce temperature fluctuations in the cosmic microwave radiation Sachs & Wolfe 1967

besides longitudinal metric perturbations, gravitational waves contribute to $\delta T/T$

gravitational waves with scale-invariant spectrum and amplitude h are produced in the same way as perturbations of the scalar field

observations of CMR COBE, MAP, PLANCK, ... allow to test inflation

def: the power spectrum $P_Q(k)$ of some observable Q is defined by

$$\langle Q(\vec{0}), Q(\vec{r})\rangle = \int \mathrm{d}(\ln k) \mathrm{j}_0(kr) k^3 P_Q(k) \quad \text{and} \quad Q_{\mathrm{rms}} \simeq \sqrt{k^3 P_Q(k)}$$

Q_{rms} is the root mean square amplitude in the interval$(k, k + \mathrm{d}k)$

Predictions — Open Problems

for large scale CMR observations $[k_{\text{ph}} = \mathcal{O}(H_0)]$ inflation predicts:

1 $\phi_{\text{rms}} = A_S k^{n-1}$ with $n \simeq 1$

2 $h_{\text{rms}} = A_T k^{n_T}$ with $n_T \leq 0$

3 $h_{\text{rms}}/\phi_{\text{rms}} \simeq A_T/A_S \simeq 6.6\sqrt{-n_T}$ consistency check!

COBE measured $(\Delta T/T)_{\text{rms}} \simeq 10^{-5}$ and $n = 1.2 \pm 0.3$ for the sum of both contributions

inflation predicts $\Omega_0 = 1$

Why is $A_S, A_T \leq 10^{-5}$?
Why is today's vacuum energy density just $10^{-122} M_{\text{P}}^4$?
...