1. Plenum - Lösungen **11.03.2011**

P1.1 Maxwell-Relationen

$\mathrm{d}E = T\,\mathrm{d}S - p\,\mathrm{d}V + \mu\,\mathrm{d}N$

$$\rightarrow \quad \frac{\partial^2 E}{\partial V \partial N} = \frac{\partial}{\partial V}\frac{\partial E}{\partial N} = \underline{\frac{\partial \mu}{\partial V} = -\frac{\partial p}{\partial N}} = \frac{\partial}{\partial N}\frac{\partial E}{\partial V}$$

P1.2 Legendretransformation

a)

$$f(x,v) = \frac{mv^2}{2} + \sin x \quad \text{mit} \quad \frac{\partial f}{\partial v} = p = mv$$

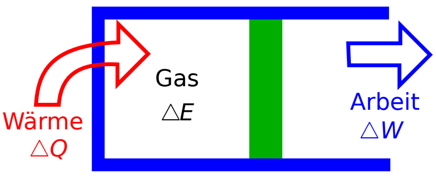
$$\rightarrow \quad g(x,p) = f - pv = -\frac{mv^2}{2} + \sin x = -\frac{p^2}{2m} + \sin x$$

b)

$$f(x,v) = \frac{mv^2}{2} + \sin x \quad \text{mit} \quad \frac{\partial f}{\partial x} = y = \cos x$$

$$\rightarrow \quad h(y,p) = f - yx = -\frac{mv^2}{2} + \sin(\arccos y) - y \arccos y$$

P1.3 Wärmekapazität



$\Delta Q = \Delta E - \Delta W = \Delta E + P\Delta V$

a) bei konstantem Volumen: $\Delta W = 0$

$$C_V = \left.\frac{\Delta Q}{\Delta T}\right|_{\Delta V=0} = \left.\frac{\Delta E}{\Delta T}\right|_{\Delta V=0} = \left(\frac{\partial E}{\partial T}\right)_V = \left(\frac{\partial E(V,T)}{\partial T}\right)_V$$

b) bei konstantem Druck

$$C_P = \left.\frac{\Delta E}{\Delta T}\right|_{\Delta P=0} - \left.\frac{\Delta W}{\Delta T}\right|_{\Delta P=0} = \left(\left(\frac{\partial E}{\partial V}\right)_T + P\right)\left(\frac{\partial V}{\partial T}\right)_P + \left(\frac{\partial E}{\partial T}\right)_V$$

wobei für $E = \tilde{E}(P,T) = E(V(P,T),T)$ verwendet wurde:

$$\left(\frac{\partial \tilde{E}}{\partial T}\right)_P = \left(\frac{\partial E}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P + \left(\frac{\partial E}{\partial T}\right)_V, \qquad \left(\frac{\partial W}{\partial T}\right)_P = -P\left(\frac{\partial V}{\partial T}\right)_P$$

c) bei konstantem $X = X(P,V) \quad \rightarrow \quad X = X(P(V,T),V) \quad \rightarrow \quad V = V(X,T)$

$$C_X = \left(\left(\frac{\partial E}{\partial V}\right)_T + P\right)\left(\frac{\partial V}{\partial T}\right)_X + \left(\frac{\partial E}{\partial T}\right)_V$$