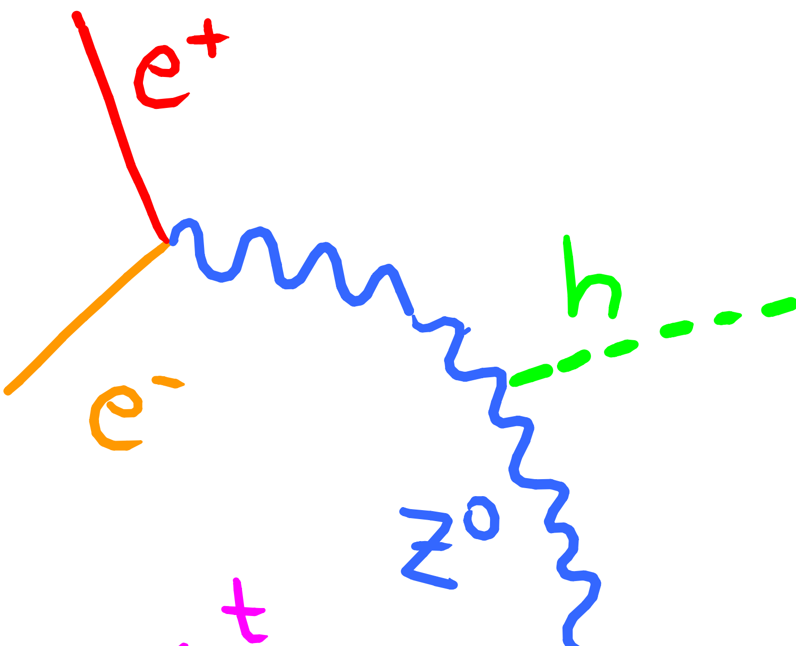
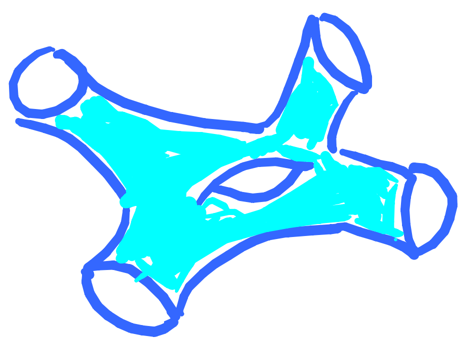
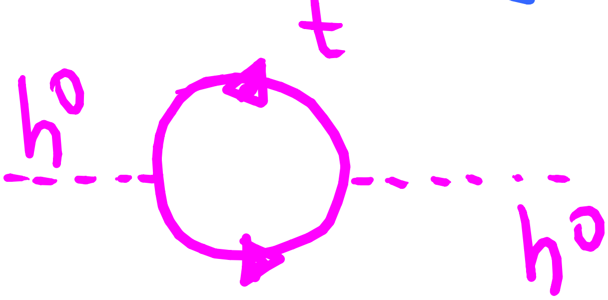
NEW HETEROTIC FLUX COMPACTIFICATIONS

VIENNA, OCTOBER 8, 2008





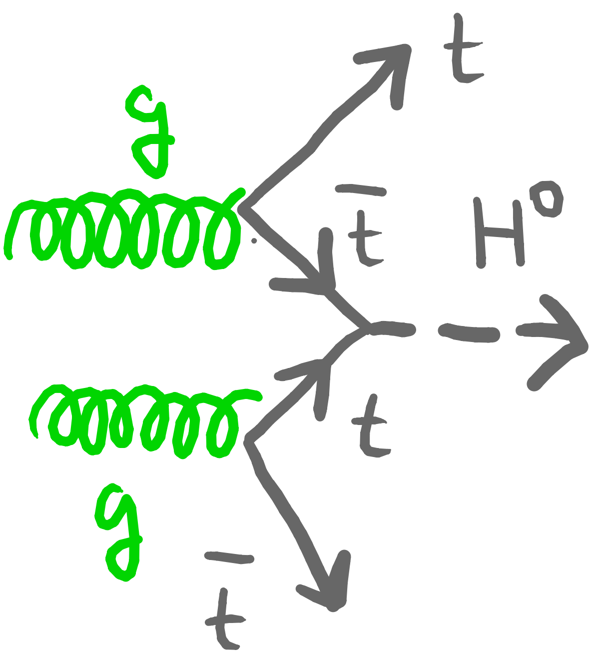


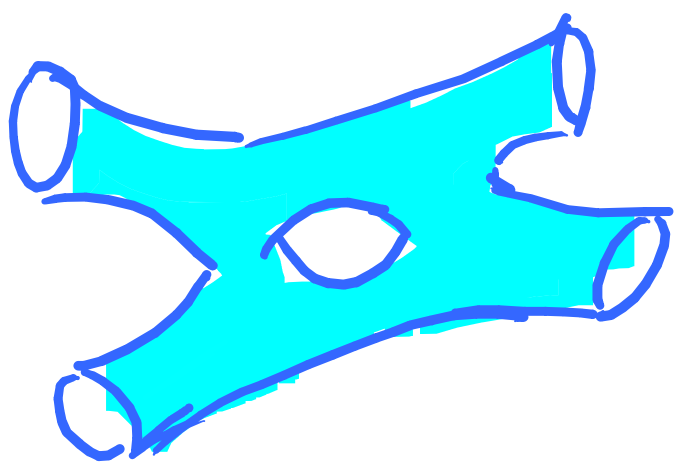
MELANIE BECKER
TEXAS A&M

NEW HETEROTIC FLUX COMPACTIFICATIONS

STRING THEORY $\leftrightarrow$ MSSM, GUT ?

- STRING THEORISTS HAVE TRIED TO MAKE THE CONNECTION TO THE MSSM SINCE CALABI-YAU'S WERE FOUND (1985)





- HETEROTIC STRING

THIS IS A TRICKY QUESTION

- EXOTIC PARTICLES !

MANY TIMES ADDITIONAL CHARGED MATTER EMERGES IN STRING THEORY MODELS THAT PLAYS NO ROLE IN MSSM

PROGRESS IN RECENT YEARS :

* HETEROTIC M-THEORY
* BRANES AT SINGULARITIES
* INTERSECTING D-BRANE MODELS

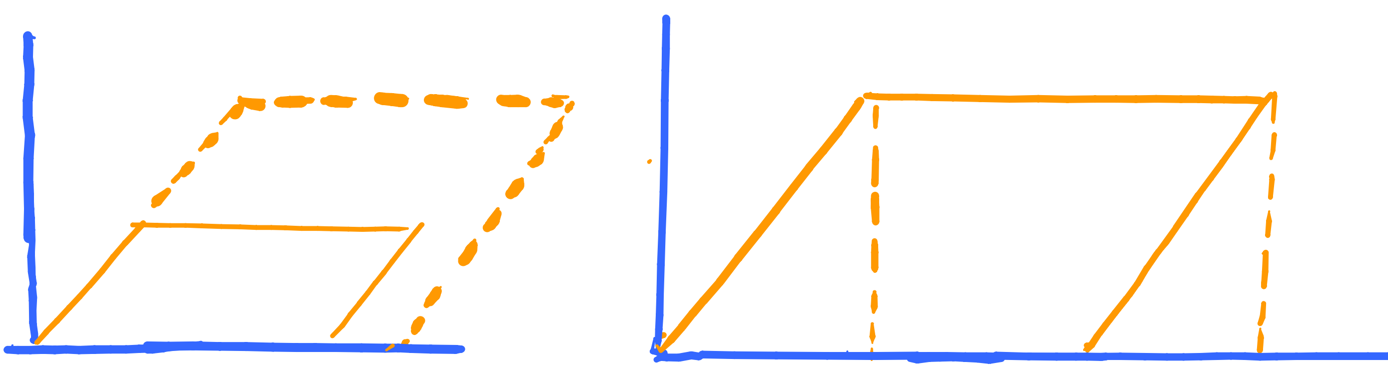
$\Big]$ TYPE II

* F-THEORY MODELS (GUT GROUPS)

- **MODULI FIELDS:** !

CONVENTIONAL CALABI-YAU COMPACTIFICATIONS OF STRING THEORY INVOLVE MODULI FIELDS (MASSLESS NEUTRAL SCALARS) DESCRIBING THE DEFORMATIONS OF THE SHAPE AND SIZE OF THE INTERNAL MANIFOLD

FLUXES ! HETEROTIC TORSIONAL GEOMETRIES



KÄHLER ST. COMPLEX ST.

TORSIONAL GEOMETRIES

GOAL: DESCRIBE TORSIONAL GEOMETRIES EMERGING IN HETEROTIC FLUX COMPACTIFICATIONS

→ SUMMARY OF WHAT WE KNOW

→ NEW ORBIFOLD MODELS

→ OPEN QUESTIONS & MATHEMATICAL CHALLENGES

NEW SOLUTIONS BY STRING DUALITY
(K. BECKER'S TALK ON MONDAY)

REFERENCES

* MB, TSENG & YAU, arXiv:0807.0827
"New heterotic flux compactifications"
* MB, TSENG & YAU, arXiv:0706.4290
"Heterotic Kähler / non-Kähler transitions"
* MB, TSENG & YAU, hep-th/0612290
"Moduli space of torsional manifolds"
* K. BECKER, MB, FU, TSENG & YAU
"Anomaly cancellation and smooth non-Kähler solutions in ..."
hep-th/0604137

OVERVIEW

- TORSIONAL CONSTRAINTS
- SMOOTH SOLUTIONS : $T^2 \ltimes K3$
- MODULI FIELDS & MODULI SPACE
- HETEROTIC KÄHLER / NON-KÄHLER DUALITY
- NEW TORSIONAL GEOMETRIES : ORBIFOLD MODELS
- CHALLENGES FOR THE FUTURE

Strominger
De Wit et al

• TORSIONAL CONSTRAINTS

THE (BOSONIC PART) OF 10D HETEROTIC STRING CONTAINS $(g_{\mu\nu}, \phi, H, F)$

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g}\, e^{-2\phi} \left(R + 4(\partial\phi)^2 - \frac{H^2}{2} - \frac{\alpha'}{4} \operatorname{tr} F^2 \right)$$

$$\delta\psi_M = \nabla_M \eta + \frac{1}{8} H_{MNP} \gamma^{NP} \eta = 0$$

$$\delta\lambda = \gamma^M \partial_M \phi\, \eta + \frac{1}{12} H_{MNP} \gamma^{MNP} \eta = 0$$

$$\delta\chi = \gamma^{MN} F_{MN} \eta = 0$$

SUSY BACKGROUND: $\exists\, \eta = \xi \otimes \varepsilon$

THE 6D SPINOR DEFINES

- $J^{(1,1)}_{m\bar{n}} = -i\,\varepsilon^{+}\gamma_{mn}\varepsilon$ HERMITIAN FORM

- $\Omega^{(3,0)}_{mnp} = e^{-2\phi}\,\varepsilon^{T}\gamma_{mnp}\varepsilon$

  HOLOMORPHIC 3-FORM

THESE FORMS DESCRIBE THE GEOMETRY

THEY SATISFY A SYSTEM OF EQS.
THAT IS A GENERALIZATION OF
CALABI'S EQN FOR RICCI-FLAT
METRICS ON KÄHLER MANIFOLDS

① HERMITIAN YANG-MILLS

$F_{20} = F_{02} = 0$ $\qquad$ $F_{m\bar{m}} J^{m\bar{m}} = 0$

HOLOMORPHIC $\qquad$ PRIMITIVE

② ANOMALY CANCELLATION

$$dH = 2i\,\partial\bar{\partial}J = \frac{\alpha'}{4}\left(\mathrm{tr}\,R\wedge R - \mathrm{tr}\,F\wedge F\right)$$

- RELATES GAUGE BUNDLE TO GEOMETRY
- NO "STANDARD EMBEDDING"
  $\Rightarrow$ H-FLUX WITHOUT BRANE SOURCES OR SINGULARITIES

BONUS !

FOR NON-STANDARD EMBEDDING

WE GET INTERESTING

GAUGE SYMMETRY BREAKING

$$E_8 \longrightarrow SU(3) \times E_6$$

$$E_8 \longrightarrow SU(4) \times SO(10)$$

$$E_8 \longrightarrow SU(5) \times SU(5)$$

GUT GROUPS

③ $d^{+}\partial = i(\partial - \bar{\partial}) \ln \|\Omega\|$

THESE EQS. FOLLOW FROM SPACE-TIME SUSY & THEY GUARANTEE THAT THE EQS. OF MOTION ARE SATISFIED

OBSTRUCTIONS ! $C_2(F) = C_2(M)$

GLOBALLY DEF. Ω : $h^{m,0} = 1$; $c_1(M) = 0$

- FU & YAU SHOWED THAT $T^2 \ltimes K3$ ARE SOLUTIONS

- STRING DUALITY & K. BECKER'S TALK

  STRING DUALITY & LARGE RADIUS EXPANSION

CALABI-YAU	TORSIONAL GEOMETRY
$dJ = 0$	$d(\|\Omega\|^2 J \wedge J) = 0$
KÄHLER	BALANCED
$d\Omega = 0 \quad c_1(M) = 0$	$d\Omega = 0 \quad c_1(M) = 0$

GIVEN A SOLUTION FOR (J, Ω) THE PHYSICAL FIELDS ARE

CALABI-YAU	TORSIONAL GEOMETRY
$H = 0$	$H = i(\bar{\partial} - \partial) J = d^c J$
$\phi_0 = \text{const}$	$e^{-2(\phi - \phi_0)} = \|\Omega\|$

PROMINENT EXAMPLE : $T^2 \ltimes K3$

Dasgupta, Rajesh & Sethi
K. Becker & Dasgupta
Becker^2, Dasgupta,
Green, Sharpe, Tseng
Goldstein, Prokushkin,
Fu & Yau, ...

TORUS BUNDLE

Calabi & Eckman

- **GEOMETRY**

$$J = e^{2\phi(y)} J_{K3} + \frac{i}{2}\, \theta \wedge \bar{\theta}$$

warp factor on base

$$\Omega = \Omega_{K3} \wedge \Theta \quad ; \quad d\Omega = 0$$

$$\Theta = dz + \alpha(y)$$ GLOBALLY DEFINED (1,0) FORM

$$\underset{\text{fiber}}{dz} + \underset{\text{base}}{\alpha(y)}$$

THE EXPLICIT FORM OF Θ IS CONSTRAINED BY SUPERSYMMETRY

1. $\mathcal{N}=1$ $D=4$: $\omega = d\Theta$ 2 COMPONENTS

$$\omega = \omega_1 + i\,\omega_2 = d\Theta = d\alpha = \\ = \omega^{2,0} + \omega^{1,1}$$

AS LONG AS ω IS PRIMITIVE wrt BASE

PRIMITIVE : $\omega \wedge J_{K3} = 0$

CHECK : $d(e^{-2\phi} J \wedge J) = i\, d(\theta \wedge \bar{\theta}) \wedge J_{K3} = 0$

2 $\mathcal{N} = 2$ $D = 4$: $\omega^{(2,0)} = 0$

GLOBALLY DEFINED METRIC :

$$\frac{\omega_i}{2\pi\sqrt{\alpha'}} \in \mathbb{Z}$$

• GAUGE BUNDLE

ALL STABLE BUNDLES OF THE 6D MANIFOLD ARE OBTAINED BY LIFTING STABLE BUNDLES OF K3 TO THE 6D MANIFOLD (UNDERSTOOD FROM MUKAI)

1. $F_{m\bar{m}} J_{K3}^{m\bar{m}} = 0 \qquad F_{mm}^{(2,0)} = F_{\bar{m}\bar{m}}^{(0,2)} = 0$

CHECK: $F_{m\bar{m}} J^{m\bar{m}} = *\left(F \wedge J \wedge J\right) =$

$= i\, e^{2\phi} \left(F \wedge J_{K3} \wedge \Theta \wedge \bar{\Theta}\right) \stackrel{!}{=} 0$

2. DIRAC QUANTIZATION: $\frac{F}{2\pi\sqrt{\alpha'}} \in \mathbb{Z}$

NUMBER OF GENERATION & EULER CHARACTERISTIC

AS FOR ORDINARY CY'S THE NET NUMBER OF GENERATIONS IS RELATED TO $C_3(F)$

* $N_g = \frac{1}{2} C_3(F) = 0$ for $T^2 \ltimes K3$

PULLBACK OF K3

SU(N) BUNDLES

$$\mathrm{tr}(F \wedge \cancel{F \wedge F}) + \cancel{\mathrm{tr}(F \wedge F) \wedge \mathrm{tr} F} + \cancel{(\mathrm{tr} F)^3}$$

* ALSO $\chi(M) = 0$ (UNRELATED)

MB, TSENG & YAU
SETHI, ADAMS

KÄHLER / NON-KÄHLER DUALS

($T^2 \ltimes K3$ WITH $\mathcal{N}=2$)

MODEL DESCRIBED BY (F, ω)
HOLOMORPHIC & PRIMITIVE

$F \wedge J_{K3} = 0$	$\omega \wedge J_{K3} = 0$
$F_{2,0} = F_{0,2} = 0$	$\omega_{2,0} = \omega_{0,2} = 0$
$H = 0 \quad F \neq 0$	$F = 0 \quad H \neq 0$
KÄHLER $T^2 \times K3$	NON-KÄHLER $T^2 \ltimes K3$

DUALITY ??

FOLLOW THE "DUALITY CHAIN"

DUALITY CHAIN

THE M-THEORY LIFT OF THESE GEOMETRIES REVEALS THAT INDEED A VERY INTERESTING DUALITY IS PRESENT!

$$
\begin{tikzcd}
\boxed{\begin{array}{c}\text{M-THEORY}\\ K3\times K3'\end{array}} \arrow[r, Rightarrow, "\text{ORBIFOLD}"] & \boxed{\begin{array}{c}\text{M-THEORY}\\ K3\times T^4/\mathbb{Z}_2\end{array}} \arrow[d, Rightarrow, "T^2\to 0"] \\
 & \boxed{\begin{array}{c}\text{TYPE IIB}\\ K3\times T^2/\mathbb{Z}_2\end{array}} \arrow[d, Rightarrow, "\text{T-DUALITY}"] \\
\boxed{\begin{array}{c}\text{HETEROTIC}\\ K3\ltimes T^2\end{array}} & \boxed{\begin{array}{c}\text{TYPE I}\\ K3\ltimes T^2\end{array}} \arrow[l, Rightarrow, "S"']
\end{tikzcd}
$$

NON-ZERO G-FLUX

THIS DUALITY CAN ALSO INCORPORATE A NON-ZERO 4-FORM FLUX G

TO PRESERVE $N=4$ SUSY $D=3$ THIS G-FLUX SATISFIES:

$$G^{(2,2)} \wedge J = 0 \qquad \text{PRIMITIVE}$$

$$G^{(4,0)} = G^{(0,4)} = G^{(1,3)} = G^{(3,1)} = 0$$

IN ADDITION ON $Y = K3 \times K3'$

$$G \in H^4(Y, \mathbb{Z})$$

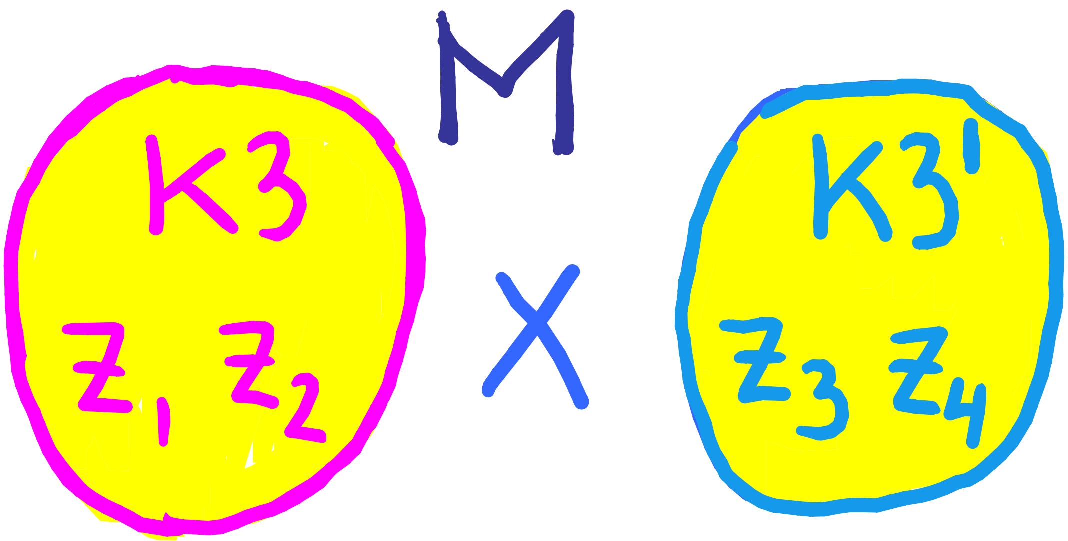
TADPOLE CONDITION

$$\frac{1}{2}\int_Y G \wedge G = \frac{\chi(Y)}{24} \quad (\text{no } M2\text{'s})$$

M-THEORY G $<$ F HETEROTIC, H

DUALITY CAN BE OBTAINED FROM THE CONSTRUCTION OF G !

DUALITY ARGUMENT



G CAN BE TAKEN TO BE THE PRODUCT OF TWO (1,1) FORMS, ONE FOR EACH K3

K3 HAS 19 PRIMITIVE (1,1) FORMS...

K3 FORMS

$\beta_i \quad i = 1 \ldots 16$

LOCALIZED

$\gamma_I \quad I = 1 \ldots 3$

NON-LOCALIZED

$$\gamma_{1,2} = \frac{1}{2}\left(dz_3 \wedge d\bar{z}_4 \pm d\bar{z}_3 \wedge dz_4\right)$$

$$\gamma_3 = \frac{1}{2}\left(dz_3 \wedge d\bar{z}_3 - dz_4 \wedge d\bar{z}_4\right)$$

WE CAN THEN WRITE

$$G = C_{ij}\, \beta_i \wedge \beta'_j + C_{Ij}\, \overbrace{\gamma_I}^{K3} \wedge \overbrace{\beta'_j}^{K3'} + D_{iJ}\, \beta_i \wedge \gamma'_J + D_{IJ}\, \gamma_I \wedge \gamma'_J$$

$$G = C_{ij}\,\beta_i \wedge \beta_j' + \overbrace{C_{Ij}\,\gamma_I \wedge \beta_j'}^{1} +$$

$$+ \underbrace{D_{ij}\,\beta_i \wedge \gamma_j'}_{2} + D_{IJ}\,\gamma_I \wedge \gamma_J'$$

T-DUALITY IS PERFORMED ALONG

$$K3' = T^2 \ltimes T^2/\mathbb{Z}_2$$

BUSCHER RULES: 1 $\rightarrow$ F

2 $\rightarrow$ H

DUALITY: $F \longleftrightarrow H \triangleq K3 \longleftrightarrow K3'$

HETEROTIC M-THEORY

MATH. CHALLENGE: MODULI SPACE

A MASSLESS 4D SCALAR EMERGES FOR EACH ALLOWED DEFORMATION OF BACKGROUND GEOMETRY (MODULUS)

$$g_{mn} \to g_{mn} + \delta g_{mn}$$

COMPLEX STRUCTURE: $\Omega_{ab}{}^{\bar{d}}\ \delta g_{\bar{d}\bar{c}}\ dz^a \wedge dz^b \wedge dz^c$

KÄHLER STRUCTURE: $\delta g_{a\bar{b}}\ dz^a \wedge d\bar{z}^{\bar{b}}$

UNLIKE FOR CALABI-YAU THE MODULI SPACES DO NOT DECOUPLE !

STRATEGY WE FOLLOWED

- FIX COMPLEX STRUCTURE & VARY

$$J_{a\bar{b}} \to J_{a\bar{b}} + \delta J_{a\bar{b}}$$

- DERIVE THE SUSIC DEFORMATIONS SOLVING TO LINEAR ORDER δJ

(1) $$d(\|\Omega\| J \wedge J) = 0$$

(2) $$F_{2,0} = F_{0,2} = 0 \,, \quad F_{a\bar{b}} J^{a\bar{b}} = 0$$

(3) $$i\partial\bar{\partial} J = \frac{\alpha'}{8}\left(\mathrm{tr} R \wedge R - \mathrm{tr} F \wedge F\right)$$

SOME MODULI ARE LIFTED...

CLASSICAL SUPERPOTENTIAL

Becker[2], Dasgupta
Green, Sharpe,
Lüst et. al

$$W = \int (H + i\, dJ) \wedge \Omega$$

SOME MODULI REMAIN ...

BALANCED IS PRESERVED :

$$\frac{\mathrm{Ker}(d) \cap \Lambda^{2,2}}{d(\beta \wedge J)}$$

← Closed (2,2) form

non-primitive 3-form

WE LISTED ALL CONDITIONS BUT GENERAL ANALYSIS IS TRICKY...

SPECIAL CASE : $T^2 \times K3$

- DILATON IS MODULUS

  $$\phi(y) \to \phi(y) + \delta\phi(y)$$

  SCALE TRANSFORMATION OF K3

- FROM THE $h^{11} = 20$ KÄHLER DEF. ONLY SOME REMAIN

  $$\omega \wedge \partial J_{K3} = 0 \qquad \text{BALANCED}$$

- NO RADIAL MODULUS (ANOMALY)

  $$dH = 2i\,\partial\bar{\partial} J = \mathrm{tr}\, R \wedge R$$

- NO COMPLEX ST. MODULI

NEW TORSIONAL GEOMETRIES
ORBIFOLDS

GOAL:
- CONSTRUCT NEW GEOMETRIES
- ORBIFOLDS $\mathcal{N} = 2, 1, 0$

IF X_6 HAS DISCRETE SYMMETRY Γ
WE CAN OBTAIN A NEW GEOMETRY

$$\boxed{X_6' = \frac{X_6}{\Gamma}}$$

WE DEMAND THE PHYSICAL FIELDS (g, H, F, ϕ) TO REMAIN INVARIANT $\Rightarrow$ EOM

PHYSICAL FIELDS

$$H = i(\partial - \bar{\partial})J$$

$$e^{4\Phi} = \|\Omega\|^2 \Leftarrow \text{ALLOWS PHASE}$$

$$F$$

$$\Omega \to e^{i\varphi}\Omega$$

BREAKS SUSY

WHY? : Ω IS BILINEAR IN η

$$\Omega_{mnp} = \bar{\eta}^{+} \gamma_{mnp} \eta$$

A NOWHERE VANISHING η (SUSY) IMPLIES THAT Ω MUST BE GLOBALLY DEFINED $\Rightarrow$ INVARIANT

TO CONSTRUCT ORBIFOLDS :

WE USED SPECIAL K3 BASE SURFACES :

1. KUMMER K3 WITH $T^4/\mathbb{Z}_2$ ORBIFOLD LIMIT

2. K3s WITH ALGEBRAIC DESCRIPTION :

BRANCHED COVERING DESCRIPTION

K3 WITH ORBIFOLD LIMIT

$$K3 \triangleq T^4/\mathbb{Z}_2$$

$$\sigma : (z_1, z_2) \to -(z_1, z_2)$$

$$J = \frac{i}{2} e^{2\Phi}(dz_1 \wedge d\bar{z}_1 + dz_2 \wedge d\bar{z}_2) + \frac{i}{2} \theta \wedge \bar{\theta}$$

$$\Omega = dz_1 \wedge dz_2 \wedge \theta$$

DISCRETE IDENTIFICATIONS :

$$z_i \sim z_i + 1 \sim z_i + i \quad , \quad i = 1, 2$$

$$z \sim z + 1 \sim z + i$$

GOAL : CONSTRUCT ORBIFOLDS

① $\Gamma : (z_1, z_2, z) \to (i z_1, -i z_2, z + \frac{1}{2})$

- FREELY ACTING , NO FIXED PTS
- PRESERVES $(J, \Omega) \Rightarrow$ SUSIC
- DEPENDING ON TWIST $\mathcal{N} = 1, 2$

$$\omega = \omega^{(2,0)} + \omega^{(1,1)}$$

TWIST :

$$\omega \in \begin{cases} dz_1 \wedge dz_2 \\ dz_1 \wedge d\bar{z}_1 \pm dz_2 \wedge d\bar{z}_2 \end{cases}$$

② ENRIQUES INVOLUTION

$$\Gamma : (z_1 z_2 z) \mapsto \left(-z_1 + \frac{1}{2} \;\; z_2 + \frac{1}{2} \;\; -z\right)$$

- SUSIC AS EVEN THOUGH

$$\Omega_{K3} \to -\Omega_{K3} \Rightarrow \Omega \to \Omega$$

$$\Theta = dz + A_1 (z_1 - \bar{z}_1)\, dz_2 + \\ + A_2 (z_2 - \bar{z}_2)\, dz_1$$

A_1, A_2 : Gaussian numbers

- FREELY ACTING

③ $\Gamma : (z_1 z_2 z) \to (i z_1\ i z_2\ z + \frac{1}{2})$

- FREELY ACTING
- $\mathcal{N} = 0$ AS $\Omega \to -\Omega$

FREELY ACTING ORBIFOLDS HAVE $\chi = 0$ AND $N_{gen} = c_3(F) = 0$

④ TORSIONAL GEOMETRIES $\chi \neq 0$ ARE ORBIFOLDS W. FIXED CURVES / PTS

$\Gamma : (z_1 z_2 z) \to (i z_1\ i z_2\ -z)$

- SUSIC $\quad \omega \simeq dz_1 \wedge dz_2$
- FIXED PTS & CURVES

MATHEMATICAL CHALLENGE: RESOLUTION OF SINGULARITIES

SINGULAR $\qquad\qquad$ SMOOTH

$\chi = 0$ $\qquad\qquad$ $\chi \neq 0$

- KÄHLER: $c_1(M_R) = 0 \Longrightarrow$ RICCI-FLAT METRIC

- NON-KÄHLER: METRIC NEEDS TO BE CONSTRUCTED!

MORE TORSIONAL GEOMETRIES

(1) ALGEBRAIC DESCRIPTION :

BRANCHED COVERINGS

$c_1(M) = 0$ (MB, Tseng & Yau)

(2) K3-FIBRATIONS & DUALITY

(K. BECKER'S MONDAY TALK)

(3) MODELS WITH NON-COMPACT BASE (Fu, Tseng & Yau)

$$ds^2 = e^{2\phi} ds^2_{EH} + |dz + \alpha|^2$$

Eguchi - Hanson

CHALLENGES FOR FUTURE

(1) MODULI SPACE DESCRIPTION : PREPOTENTIAL & SPECIAL GEOMETRY, INDEX

(2) RESOLUTION OF SINGULARITIES

(3) BUNDLE CONSTRUCTION

(4) THREE GENERATION MODEL

(5) PHENOMENOLOGY

THANKS!