Heterotic String Model Building

Volker Braun

Dublin Institute for Advanced Studies (DIAS)

6. October 2008

Heterotic String Theory

Ten-dimensional massless fields:

- $N = 1$ Supergravity, and
- $E_8 \times E_8$ or $Spin(32)/\mathbb{Z}_2$ gauge fields.

$E_8 \times E_8$ is particularly promising, and I will focus on this case in the following.

Supersymmetric gauge field configurations have to satisfy the Hermitian Yang-Mills equations with slope zero,

$$F_{\mu\nu} = 0 = F_{\bar{\mu}\bar{\nu}}, \quad g^{\mu\bar{\nu}} F_{\mu\bar{\nu}} = 0.$$

One solution is to use the spin connection, but I will not use this in the following. The general case is called “nonstandard embedding”.

$E_8 \times E_8$ Heterotic Strings

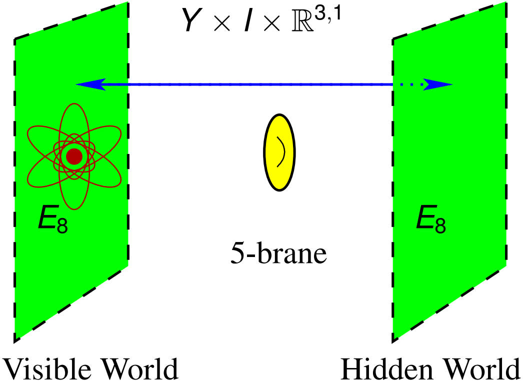
Desirable features of $E_8 \times E_8$ heterotic strings:

- Favourable group theory:

$$\begin{array}{ccccc} \text{Standard model} & \subset & Spin(10) & \subset & E_8 \\ \text{Quarks \& leptons} & \subset & \underline{\mathbf{16}} & \subset & \underline{\mathbf{248}} \end{array}$$

- Straightforward to break E_8 gauge symmetry
- Matter & Yukawa couplings from perturbative strings.
- Gauge unification without Yukawa unification
- Natural hidden sector, ideal place for SUSY breaking and/or dark matter

Heterotic M-theory



Y Calabi-Yau manifold (6-dimensional)

I Interval (1-dimensional)

$\mathbb{R}^{3,1}$ 4-d spacetime

Geometric Data

Generic E_8 bundles would break gauge symmetry completely.

Only give vevs to gauge bosons in suitable subgroups:

Gauge instanton	Unbroken gauge group
$SU(3) \subset E_8$	E_6
$SU(4) \subset E_8$	$Spin(10)$
$SU(5) \subset E_8$	$SU(5)$

- $U(n)$ bundle = vector bundle of rank n.
- $SU(n)$ bundle = vector bundle with vanishing c_1.

Geometric Data

We need:

- A Calabi-Yau manifold Y.

  Polynomial equations in projective spaces.
  SUSY $\Leftrightarrow$ Kähler and $c_1(TY) = 0$.

- Two vector bundles.

  Three basic constructions:

  - Linear sigma model $\Leftrightarrow$ Monad bundles. (See other talks)
  - Spectral covers.
  - Extensions.

  SUSY $\Leftrightarrow$ slope-stable of slope 0.

Bundles on a Torus

Lets look at a simple example:

Torus = Elliptic curve = Calabi-Yau 1-fold.

Atiyah classified the indecomposable bundles:

- Line bundles
- Extensions of line bundles

Line Bundles on a Torus

Recall the Jacobi theta function

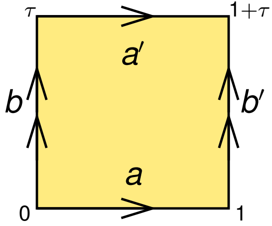
$$\vartheta(z;\tau) = \vartheta_{00}(z;\tau) = \sum_{n=-\infty}^{\infty} e^{2\pi i n z + \pi i n^2 \tau}$$

It satisfies

$$\begin{aligned} \vartheta(z+1,\tau) &= \vartheta(z,\tau) \\ \vartheta(z+\tau,\tau) &= e^{-\pi i \tau - 2\pi i z}\vartheta(z,\tau) \end{aligned}$$

Line Bundles on a Torus

$T^2 =$



ϑ is a section of a line bundle with transition functions

$$\phi_{a,a'} = e^{-\pi i \tau - 2\pi i z}, \quad \phi_{b,b'} = 1$$

Line Bundles on a Torus

In the fundamental region, $\vartheta(z,\tau)$ has

- no poles, and
- a single zero at $z = \frac{1+\tau}{2}$.

Theta functions and line bundles

$\vartheta(z;\tau)$ is a section of $\mathcal{O}_{T^2}\left(\frac{1+\tau}{2}\right)$

Note: $c_1\left(\mathcal{O}_{T^2}\left(\frac{1+\tau}{2}\right)\right) = 1.$

Line Bundles on a Torus with $c_1 = 0$

If we want vanishing first Chern class, then we need as many zeros as poles.

- $\vartheta(z-p,\tau)/\vartheta(z,\tau)$ has a zero at $\frac{1+\tau}{2}+p$ and a simple pole at $\frac{1+\tau}{2}$.
- $\vartheta(z-p-\frac{1+\tau}{2},\tau)/\vartheta(z-\frac{1+\tau}{2},\tau)$ has a zero at p and a simple pole at 0.
- Both are (meromorphic) sections of the same line bundle with transition functions

$$\phi_{a,a'} = e^{2\pi i p}, \quad \phi_{b,b'} = 1$$

Line Bundles on a Torus with $c_1 = 0$

Theta functions and line bundles

$\vartheta(z-p,\tau)/\vartheta(z,\tau)$ is a section of $\mathcal{O}_{T^2}(p-0)$

Note: Divisor = formal linear combination of points.

Every line bundle (with vanishing first Chern class) is of this form. Position of $p \in T^2$ = modulus.

Line Bundles on a Torus

For simplicity: $c_1(\mathcal{L}) = 0 \;\Leftrightarrow\; \int F = 0$.

First Chern class fixes topology of the line bundle, but not its complex structure.

Theorem (Donaldson-Uhlenbeck-Yau)

$$\left\{ \begin{array}{c} \textit{Slope-stable} \\ \textit{holomorphic bundle} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} \textit{Solution to the} \\ \textit{Hermitian-Yang Mills} \\ \textit{equation} \end{array} \right\}$$

Line Bundles on a Torus

All line bundles (with $c_1 = 0$) are of the form

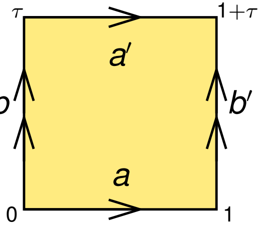
$$\mathcal{L} = \mathcal{O}_{T^2}(p - 0)$$

where p is a point in T^2.

- Algebraic point of view: There is a meromorphic section with a zero at 0 and a pole at p.
- Geometric point of view: The HYM connection has two $U(1)$ holonomies, parametrize a point $p \in T^2$.

Extension of Bundles

$T^2 =$



$\mathcal{O}_{T^2} = \mathcal{O}_{T^2}(0 - 0)$ has transition functions

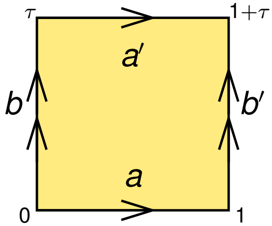
$$\phi_{a,a'} = 1, \quad \phi_{b,b'} = 1$$

$\mathcal{O}_{T^2} \oplus \mathcal{O}_{T^2}$ has transition functions

$$\phi_{a,a'} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \phi_{b,b'} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Extension of Bundles

$T^2 =$



Define a new bundle with transition functions

$$\phi_{a,a'} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad \phi_{b,b'} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Extension of Bundles

Define a new bundle with transition functions

$$\hat{\phi}_{a,a'} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad \hat{\phi}_{b,b'} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- Embedding $f : (x) \mapsto (x, 0)$
- Projection $g : (y_1, y_2) \mapsto (y_2)$

Defines "extension" bundle

$$0 \longrightarrow \mathcal{O}_{T^2} \xrightarrow{f} \mathcal{A}^{(2)} \xrightarrow{g} \mathcal{O}_{T^2} \longrightarrow 0$$

For the experts: $\mathrm{Ext}^1(\mathcal{O}_{T^2}, \mathcal{O}_{T^2}) = \mathbb{C}$

Classification of Bundles on T^2

A rank-2 bundle[1] on T^2 is one of the following two possibilities:

- $\mathcal{O}_{T^2}(p_1 - 0) \oplus \mathcal{O}_{T^2}(p_2 - 0)$

  (Spectral cover)

- $\mathcal{O}_{T^2}(p - 0) \otimes \mathcal{A}^{(2)}$

  (Extension)

[1] with $c_1 = 0$ and a HYM connection

Spectral Covers on Calabi-Yau Threefolds

Use elliptically fibered Calabi-Yau threefold over a base surface. Patch together sums of line bundles on each fiber.

Local coordinates (x, y) on the base

$$\mathcal{V}|_{f_{(x,y)}} = \mathcal{O}_{T^2}\Big(p_1(x, y) - 0\Big) \oplus \mathcal{O}_{T^2}\Big(p_2(x, y) - 0\Big) \oplus \cdots$$

Bundle defined by the surface

$$\mathcal{C}_V = \big\{p_1(x, y) = 0\big\} \cup \big\{p_2(x, y) = 0\big\} \cup \cdots$$

Slope and Stability

Slope of a vector bundle

$$\mu(\mathcal{V}) = \frac{1}{\operatorname{rk}\mathcal{V}} \int \omega^2 c_1(\mathcal{V})$$

Slope-stability: $\mu(\mathcal{W}) < 0$ for all subbundles $\mathcal{W}$.

Linebundle	Slope
$\mathcal{O}\Big(p(x,y)\Big)$	$1 + O\big(\operatorname{Vol}(f)\big)$
$\mathcal{O}\Big(p(x,y) - 0\Big)$	$O\big(\operatorname{Vol}(f)\big)$
$\mathcal{O}\Big(-p(x,y)\Big)$	$-1 + O\big(\operatorname{Vol}(f)\big)$

Slope-Stability of Spectral Covers

$$\mathcal{V}|_{f_{(x,y)}} = \mathcal{O}_{T^2}\Big(p_1(x,y) - 0\Big) \oplus \mathcal{O}_{T^2}\Big(p_2(x,y) - 0\Big)$$

Subbundle	Slope	
$\mathcal{O}_{T^2}\Big(p_i(x,y) - 0\Big)$	$O\big(\mathrm{Vol}(f)\big)$	Not a bdle
$\mathcal{O}_{T^2}\Big(-q(x,y)\Big)$	$-1 + O\big(\mathrm{Vol}(f)\big)$	< 0

- If $\mathrm{Vol}(f) \ll 1$, and
- if $\mathcal{C}_{\mathcal{V}}$ is irreducible (p_1 and p_2 exchanged at branch points)

then $\mathcal{V}$ is slope-stable.

Extensions on Calabi-Yau Threefolds

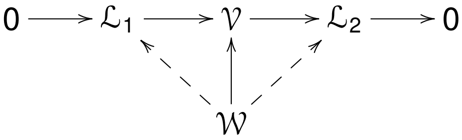
Rank-1 ingredients:

- Line bundles, defined by divisors (codimension 1)
- Sheaves, like bundles but with codimension ≥ 2 degenerations.

Extensions give higher rank bundles:

$$0 \longrightarrow \mathcal{L}_1 \longrightarrow \mathcal{V}^{(2)} \longrightarrow \mathcal{L}_2 \longrightarrow 0$$

Stability of Extensions



- If the slope of sub-linebundles of $\mathcal{L}_2$ is negative, and
- $\mu(\mathcal{L}_1) < 0$

then the extension bundle $\mathcal{V}$ is slope-stable.

$\Rightarrow$ Finite number of inequalities for the Kähler class

Constructions

- Our "Heterotic Standard Model", MSSM via $SO(10)$ [Braun-HuiHe-Pantev-Ovrut]
- MSSM via $SU(5)$, using extensions of spectral covers [Bouchard-Donagi]
- Standard embedding + intermediate scale breaking [Greene-Kirklin-Miron-Ross]
- Orbifolds [Buchmuller-Hamaguchi-Lebedev-Ratz, Wingerter]
- U(n) bundles instead of SU(n) [Blumenhagen-Honecker-Weigand]
- Free fermions [Faraggi]

Group Theory

The smallest representation that contains one generation of quarks and leptons:

Before 1998: $\underline{\bar{\mathbf{5}}} \oplus \underline{\mathbf{10}}$ of $SU(5)$

Since 1998: $\underline{\mathbf{16}}$ of $Spin(10)$

But note: GUT is never a good description, Wilson line breaks gauge group at the GUT scale. Only "organizing principle".

Particle Spectrum

The massless fields can be counted without knowing the metric and gauge connection explicitly, by computing “sheaf cohomology groups”.

We found a Calabi-Yau manifold and slope-stable bundles with “nice” 4*d* low energy effective action:

- $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$
- 3 families of quarks and leptons
- No anti-families
- Anti-fivebrane breaks SUSY in hidden sector

Calabi-Yau Threefold

$\widetilde{X}$ is a complete intersection of 2 equations of degree $(3,0,1)$ and $(0,1,3)$ in $\mathbb{P}^2 \times \mathbb{P}^1 \times \mathbb{P}^2$.

$\widetilde{X}$ is double elliptic fibration over $\mathbb{P}^1$ with[2] free $\mathbb{Z}_3 \times \mathbb{Z}_3$ group action.

$$\big\downarrow \text{quotient}$$

$$X = \widetilde{X}/\left(\mathbb{Z}_3 \times \mathbb{Z}_3\right) = \text{Calabi-Yau with } \pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$$

[2] For suitable equations

The Group Action

Homogeneous coordinates

$$\Big([x_0 : x_1 : x_2], [t_0 : t_1], [y_0 : y_1 : y_2]\Big) \in \mathbb{P}^2 \times \mathbb{P}^1 \times \mathbb{P}^2$$

$\mathbb{Z}_3 \times \mathbb{Z}_3$ group action

$$g_1 : \begin{cases} [x_0 : x_1 : x_2] \mapsto [x_0 : \zeta x_1 : \zeta^2 x_2] \\ [t_0 : t_1] \mapsto [t_0 : \zeta t_1] \\ [y_0 : y_1 : y_2] \mapsto [y_0 : \zeta y_1 : \zeta^2 y_2] \end{cases}$$

$$g_2 : \begin{cases} [x_0 : x_1 : x_2] \mapsto [x_1 : x_2 : x_0] \\ [t_0 : t_1] \mapsto [t_0 : t_1] \text{ (no action)} \\ [y_0 : y_1 : y_2] \mapsto [y_1 : y_2 : y_0] \end{cases}$$

Line Bundles on X

Sections of line bundles on $\mathbb{P}^2 \times \mathbb{P}^1 \times \mathbb{P}^2$ are homogeneous polynomials

$$H^0\Big(\mathcal{O}(a_1, b, a_2)\Big) = \mathbb{C}[x_0, x_1, x_2]_{a_1} \otimes \mathbb{C}[t_0, t_1]_b \otimes \mathbb{C}[y_0, y_1, y_2]_{a_2}$$

Three $\mathbb{Z}_3 \times \mathbb{Z}_3$-invariant divisors τ_1, τ_2, ϕ on $\widetilde{X}$. Define invariant line bundles $\mathcal{O}_{\widetilde{X}}(a_1\tau_1 + b\phi + a_2\tau_2)$.

$$\begin{aligned} &H^0\Big(\mathcal{O}_{\widetilde{X}}(a_1\tau_1 + b\phi + a_2\tau_2)\Big) \\ &\qquad\qquad = H^0\Big(\mathcal{O}(a_1, b, a_2)\Big)\Big/\Big\langle p_{(3,1,0)} = 0,\ p_{(0,1,3)} = 0\Big\rangle \end{aligned}$$

Calabi-Yau Threefold

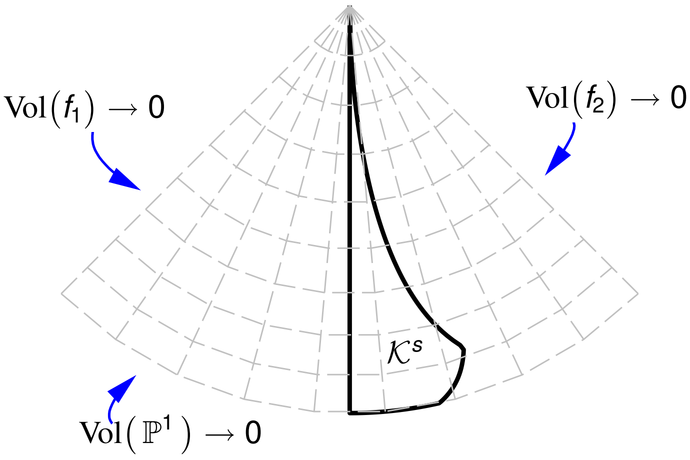
Hodge diamond

$$h^{p,q}(X) = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 3 & & 0 & \\ 1 & & 3 & & 3 & & 1 \\ & 0 & & 3 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

Three Kähler moduli $t_1, t_2, t_3 \geq 0$ on X.

- One overall volume (radial part). Ignore in the following...
- Two-dimensional “cross-section” of Kähler cone (angular part).

Cross-section of the Kählercone: $\mathrm{Vol}(X) = t_1 t_2 (t_1 + t_2 + 6 t_3)$



Visible Sector Gauge Group

- Rank 4 vector bundle $\mathcal{V}$ breaks visible E_8 to $Spin(10)$.

$$\underline{\mathbf{248}} = (\underline{\mathbf{1}}, \underline{\mathbf{45}}) \oplus (\underline{\mathbf{15}}, \underline{\mathbf{1}}) \oplus (\underline{\mathbf{4}}, \underline{\mathbf{16}}) \oplus (\underline{\overline{\mathbf{4}}}, \underline{\overline{\mathbf{16}}}) \oplus (\underline{\mathbf{6}}, \underline{\mathbf{10}})$$

$$n_{\underline{\mathbf{16}}} = \dim H^1\big(\widetilde{X}, \mathcal{V}\big), \quad n_{\underline{\overline{\mathbf{16}}}} = \dim H^1\big(\widetilde{X}, \mathcal{V}^\vee\big)$$

- $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson line breaks

$$Spin(10) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

Representations of $\mathbb{Z}_3 \times \mathbb{Z}_3$ are characters $\chi_1^i \chi_2^j$, $0 \leq i, j < 3$

$$\underline{\mathbf{16}} = \chi_1 \chi_2^2 (\underline{\mathbf{3}}, \underline{\mathbf{2}}, 1, 1) \oplus \chi_2^2 (\underline{\mathbf{1}}, \underline{\mathbf{1}}, 6, 3) \oplus \chi_1^2 \chi_2^2 (\underline{\overline{\mathbf{3}}}, \underline{\mathbf{1}}, -4, -1) \oplus$$
$$\oplus (\underline{\mathbf{1}}, \underline{\mathbf{2}}, -3, -3) \oplus \chi_1^2 (\underline{\overline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, -1) \oplus \chi_2 (\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0, 3).$$

Quarks and Leptons

$$\widetilde{X} \longrightarrow X$$

$$Spin(10) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

$$\underline{\mathbf{16}} \longrightarrow \text{one quark or lepton, including } \nu_R$$

$$27 \times \underline{\mathbf{16}} \longrightarrow 3 \text{ generations} = 3 \cdot 6 \text{ particles}$$

$$0 \times \underline{\overline{\mathbf{16}}} \longrightarrow 0 \text{ anti-generations}$$

Each quark and lepton corresponds to a different $\underline{\mathbf{16}}$.

Doublet-Triplet Splitting

$$Spin(10) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

$$\underline{\mathbf{10}} \longrightarrow \underbrace{\begin{matrix}(\underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 0) \\ (\underline{\mathbf{1}}, \underline{\overline{\mathbf{2}}}, -3, 0)\end{matrix}}_{\text{Higgs}} \begin{matrix}\oplus \\ \oplus\end{matrix} \underbrace{\begin{matrix}(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -2, -2) \\ (\underline{\overline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, 2)\end{matrix}}_{\text{Triplets}} \oplus$$

On $\widetilde{X}$, there are

$$h^1\big(\widetilde{X}, \wedge^2 \mathcal{V}\big) = 4$$

$\underline{\mathbf{10}}$'s. Depending on the $\mathbb{Z}_3 \times \mathbb{Z}_3$ group action on each $\underline{\mathbf{10}}$, get either Higgs, triplet, or nothing.
In our "heterotic standard model", all triplets are projected out while one Higgs (with its conjugate) is kept.

Doublet-Triplet Splitting

All triplets are projected out while one Higgs (with its conjugate) is kept. How so?

$$\underline{\mathbf{248}} = \left(\underline{\mathbf{6}}, \underline{\mathbf{10}}\right) \oplus \cdots$$

Note that $\underline{\mathbf{6}} = \wedge^2 \underline{\mathbf{4}} = \wedge^2 \overline{\underline{\mathbf{4}}}$, so the number of $\underline{\mathbf{6}}$-charged zero modes is

$$n_{\underline{\mathbf{10}}} = \dim H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right)$$

$$\underline{\mathbf{10}} = \chi_2^2\left(\underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 0\right) \oplus \chi_1^2\chi_2^2\left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -2, -2\right) \oplus \chi_2\left(\underline{\mathbf{1}}, \overline{\underline{\mathbf{2}}}, -3, 0\right) \oplus \chi_1\chi_2\left(\overline{\underline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, 2\right)$$

Cohomology on X = Invariant cohomology on $\widetilde{X}$

Doublet-Triplet Splitting

Cohomology on X = Invariant cohomology on $\widetilde{X}$

- Cohomology groups on $\widetilde{X}$ transform
- Phase factor from Wilson line

$$n_H = \dim\left(\chi_2^2 H^1(X, \wedge^2 \mathcal{V})\right)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \qquad n_{\bar{H}} = \dim\left(\chi_2 H^1(X, \wedge^2 \mathcal{V})\right)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$$

$$n_C = \dim\left(\chi_1^2 \chi_2^2 H^1(X, \wedge^2 \mathcal{V})\right)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \qquad n_{\bar{C}} = \dim\left(\chi_1 \chi_2 H^1(X, \wedge^2 \mathcal{V})\right)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$$

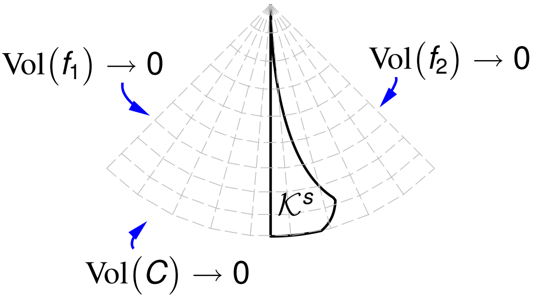
For our bundle,

$$H^1\left(\widetilde{X},\ \wedge^2 \mathcal{V}\right) = \chi_2 \oplus \chi_2^2 \oplus \chi_1 \chi_2^2 \oplus \chi_1^2 \chi_2$$

$\Rightarrow$ One Higgs Higgs-conjugate pair and no color triplets.

Hidden Sector

- Unbroken E_8 ($\Leftrightarrow$ trivial bundle is slope-stable)
- Anti-fivebranes wrapped on curve $C \simeq \mathbb{P}^1$.
- C is rigid curve, shrinks at "bottom" face of Kähler cone.



- Flip of $C \Leftrightarrow$ Decay into SUSY vacuum.

Anomaly Cancellation

Anomaly Cancellation

The tangent bundle TX, visible bundle $\mathcal{V}_{\text{vis}}$, hidden bundle $\mathcal{V}_{\text{hid}}$, 5-brane curve $\mathcal{W}$, and anti-fivebrane curve $\bar{\mathcal{W}}$ must satisfy

$$c_2(TX) - c_2(\mathcal{V}_{\text{vis}}) - c_2(\mathcal{V}_{\text{hid}}) = [\mathcal{W}] - [\bar{\mathcal{W}}]$$

- The anti-fivebrane must wrap a curve whose volume can shrink while keeping Vol(X) finite (rigid curve at the boundary of Mori cone).
- Stability region for $\mathcal{V}_{\text{vis}}$ and $\mathcal{V}_{\text{hid}}$ must allow shrinking $\bar{\mathcal{W}}$.
- Anomaly equation is in $H^4(X, \mathbb{Z})$, torsion matters.

Yukawa Textures

The “massless” 4*d* fields can have Yukawa couplings.

We cannot (yet) compute the numeric values of the Yukawa couplings. But we can determine which are (classically) zero.

In our “heterotic standard model”:

- Only two of the three families have Yukawa couplings $\Rightarrow$ One light family.
- No μ-term, avoids hierarchy problem.

The μ-Problem

Without SUSY, quadratic divergencies of the Higgs scalar destabilize EW scale.

SUSY introduces new cubic coupling $W_\mu = \lambda \phi H \bar{H}$

Expect $\lambda \approx 1$ and $\langle \phi \rangle \gg M_{\mathrm{EW}} \quad \Rightarrow$ EW scale destabilized.

Usual solution: postulate discrete symmetry.

But this is not necessary here; λ happens to be zero.

Note: Giudice-Masiero mechanism still generates required small Higgs mass.

Another Model

- Same Calabi-Yau manifold as before
- Slightly different sheaves in the bundle construction (Does not change Chern classes).

Leads to:

- Again 3 generations, no anti-generations
- Triplets still projected out
- But: 2 Higgs and 2 conjugate Higgs.

Massless Fields

Massless fields $\Leftrightarrow$ Bundle-valued one-forms.

Yukawa coupling

$$Y_{ijk} = \int_X \mathrm{Tr}\left(\alpha_i \wedge \alpha_j \wedge \alpha_k\right)$$

In order for $Y_{ijk} \neq 0$, the three forms α_i, α_j, α_k must have legs in

- fiber 1 direction,
- fiber 2 direction, and
- base $\mathbb{P}^1$ direction.

Massless Fields

Origin	Direction	Field
$\underline{\mathbf{16}}$	Fiber 1	$Q_1, u_1, d_1, L_1, e_1, \nu_1$
$\underline{\mathbf{16}}$	Base	–
$\underline{\mathbf{16}}$	Fiber 2	$Q_{2,3}, u_{2,3}, d_{2,3}, L_{2,3}, e_{2,3}, \nu_{2,3}$
$\underline{\mathbf{10}}$	Fiber 1	–
$\underline{\mathbf{10}}$	Base	$H_1, \bar{H}_1$
$\underline{\mathbf{10}}$	Fiber 2	$H_2, \bar{H}_2$

Yukawa Couplings

For example, up-quark mass matrix

$$\begin{pmatrix} 0 & \lambda_{u,12}\langle H_1\rangle & \lambda_{u,13}\langle H_1\rangle \\ \lambda_{u,21}\langle H_1\rangle & 0 & 0 \\ \lambda_{u,31}\langle H_1\rangle & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda_1\langle H_1\rangle & 0 \\ 0 & 0 & \lambda_2\langle H_1\rangle \end{pmatrix}$$

$\Rightarrow$ One light generation of quarks and leptons.

Second Higgs Pair

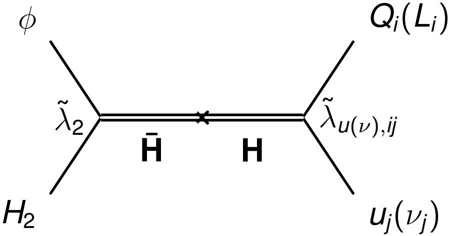
H_2, $\bar{H}_2$ has no Yukawa couplings except μ-term

$$W_\mu = \lambda_1 \phi H_1 \bar{H}_2 + \lambda_2 \phi H_1 \bar{H}_2$$

Beyond Tree-Level

The superpotential can have higher order terms:

- Integrating out massive Kaluza-Klein modes, for example



- Non-perturbative corrections.

But: Suppressed by $\frac{1}{M_c}$.

μ-Term and Moduli Vevs

Classical μ-term:

$$W_\mu = \lambda_1 \phi H_1 \bar{H}_2 + \lambda_2 \phi H_1 \bar{H}_2$$

Not all moduli ϕ appear, but those that do must have really small vev. Standard μ-problem, assume it is solved.

Quartic term

$$W_\mu = \frac{\lambda_{mnij}}{M_c} \phi_m \phi_n H_i \bar{H}_j$$

must be electroweak scale, hence

$$\frac{\left(\tilde{\lambda}^2\right)^2}{M_c} \langle \phi_m \rangle \langle \phi_n \rangle \lesssim M_{\mathrm{EW}} \quad \Leftrightarrow \quad \tilde{\lambda}^2 \frac{\langle \phi \rangle}{M_c} \lesssim \sqrt{\frac{M_{\mathrm{EW}}}{M_c}} \approx 10^{-7}$$

Simplified Model

Take normal (non-SUSY) Standard Model and add

- Single scalar ϕ
- second Higgs H_2
- $\mathbb{Z}_2$ symmetry $\phi \mapsto -\phi$, $H_2 \mapsto -H_2$.
- Dimension 5 operators

$$\mathcal{L}_5 = \tilde{\lambda}_{u,ij} \frac{\phi}{M_c} \bar{Q}_i H_2^* u_j + \tilde{\lambda}_{d,ij} \frac{\phi}{M_c} \bar{Q}_i H_2 d_j + \\ + \tilde{\lambda}_{\nu,ij} \frac{\phi}{M_c} \bar{L}_i H_2^* \nu_j + \tilde{\lambda}_{e,ij} \frac{\phi}{M_c} \bar{L}_i H_2 e_j + \text{h.c.}$$

with couplings $\lesssim 10^{-7}$.

Result

We obtain FCNC-induced mass splittings of mesons

F^0	$\Delta M_F^{SM}/GeV$	$\Delta M_F^{Exp}/GeV$	$\Delta M_F^{2-\text{Higgs}}/GeV$
K^0	$1.4 - 4.6 \times 10^{-15}$	3.51×10^{-15}	4.72×10^{-19}
B_d^0	$10^{-13} - 10^{-12}$	3.26×10^{-13}	$.88 \times 10^{-20}$
D^0	$10^{-17} - 10^{-16}$	$< 1.32 \times 10^{-13}$	4.56×10^{-21}

(Assuming the same mass for both Higgs)

Flavor-changing neutral currents are far below the Standard Model contributions.

Conclusion

- Heterotic strings are exciting.
- Can get suitable particle spectra.
- Yukawa couplings are not generic, in particular
  - μ-problem can be solved without extra symmetries.
  - Extra Higgs-pairs need not be dangerous.