D7 brane moduli and their flux stabilization via F-theory

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Based on

- [A.B.-Hebecker-Triendl] *0801.2163 [hep-th]*
- [A.B.-Gerigk-Hebecker-Triendl] *081?.????*
- [A.B.-Hebecker-Lüdeling-Valandro] *081?.????*

Motivation

- Study type IIB orientifolds with D7 branes and their stabilization in global and explicit models.
- **F-theory** is the right language to approach this problem $\longrightarrow$ elliptic Calabi-Yau.
- M-theory on an elliptically fibred CY is dual to F-theory in the limit of vanishing fibre.

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- **F-theory** is the right language to approach this problem $\longrightarrow$ elliptic Calabi-Yau.
- M-theory on an elliptically fibred CY is dual to F-theory in the limit of vanishing fibre.

- We can use M-theory to study the stabilization of this elliptic CY and translate the result back to some brane configuration. See e.g. the review by Denef [0803.1194]

Motivation

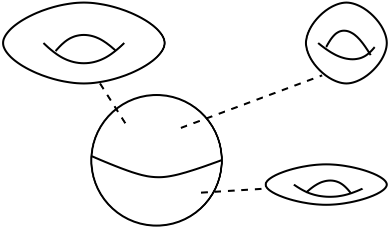
$$y^2 = x^3 + fx + g \quad \overset{?}{\longleftrightarrow} \quad \int_{\gamma_i} \Omega$$

- Brane moduli are complex structure deformations. They are very explicit in the Weierstrass model.
- On the M-theory side, flux potentials are given in terms of period integrals [Gukov, Vafa, Witten; Haack, Louis].
- How do the periods relate to brane configurations ?
- We want to attack this question and use the results to study the stabilization of branes by fluxes (without solving Picard-Fuchs equations).

Outline

- **Cycles vs. branes for K3**
- **Elliptic Threefolds**
- **Brane stabilization by fluxes on K3$\times$K3**

Elliptic K3 surfaces are fibrations of T^2 over $\mathbb{P}^1$.



$$y^2 = x^3 + f_8 x + g_{12}$$

$$\Delta = 4f_8^3 + 27g_{12}^2$$

$$j(\tau) = \frac{4(24f)^3}{\Delta}$$

The polynomials f_8 and g_{12} control the complex structure = brane positions

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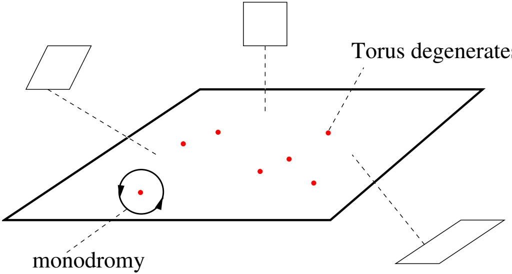
The torus degenerates over 24 points.



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weak coupling limit

- To construct the cycles and to make contact with IIB on T^2/Z_2 we use Sen's weak coupling limit [hep-th/9702165]:
- $f = C\eta - 3h^2, \quad g = Ch\eta - 2h^3 + C^2\chi, \quad C \to 0$

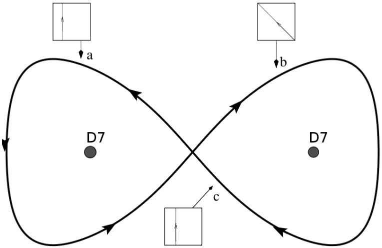
  In this limit: $\Delta = C^2 h^2(\eta^2 + 12h\chi)$

There exists an $SL(2, Z)$ frame in which all monodromies are

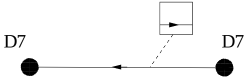
$-T^{-4}$	around	$h = 0$:	O7 planes
T	around	$\eta^2 + 12h\chi = 0$:	D7 branes

For K3 this means we have 4 O-planes and 16 D-branes.

K3 cycles

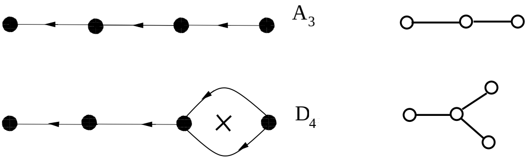


- We can now construct a two-cycle by following the fibre torus as we move in the base.
- This cycle is equivalent to a relative one-cycle in the base which has a fibre part vanishing at the D brane positions.



Intersection pattern

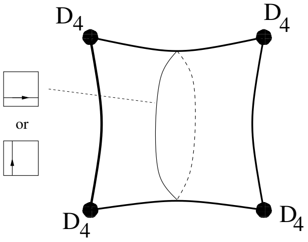
The intersection pattern gives the Cartan matrices of A_N and D_N, corresponding to the expected gauge enhancement.



- This is expected because the vanishing of these cycles produces a singularity in K3, so that we have reproduced the known dictionary between singularities and gauge enhancement. [Witten; Aspinwall]
- Gauge enhancement occurs if an integral form of self-intersection -2 is orthogonal to Ω and J.

More cycles

Besides the 16 cycles that move the branes relative to the O7 planes, there are four more.



These cycles surround two O-planes and can wrap an arbitrary direction in the fibre.

The 20 cycles we have found account for the 18 complex structure deformations of an elliptic K3.

Global picture

From the duality to the heterotic string, it is known how to expand

$$\Omega_{SO(8)^4} = \frac{1}{2}\left(\alpha + We_2 + S\beta - SWe_1\right)$$

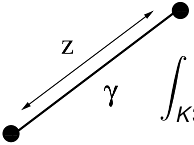
in some basis of $H^2(K3, \mathbb{Z})$. (J is fixed to fibre and base)

The forms α, β, e_1 and e_2 are dual to the four two-cycles of T^4/Z_2, W and S are the complex structures of the base and fibre torus, respectively.

We can find the lattice $2U^2 \otimes D_4^4$ that corresponds to the 20 cycles we have constructed in $H^2(K3, Z) \setminus \mathrm{Pic}(K3) = U^2 \otimes -E_8^2$.
We can now deform the complex structure and study how the brane positions are changed when the periods change.

Global picture

The periods measure the (complex) size of the various cycles.



$$\int_{K3} \Omega \wedge \gamma = z$$

$$\Omega = \frac{1}{2}\left(\alpha + Ue_2 + S\beta - (US - z^2)e_1\right) + \hat{E}_I z_I$$

We can choose the $\hat{E}_i$ such that z_I are coordinates of the D7 branes in the covering space of the base. Can check singularities, periodicities, etc...

This gives us the desired map (and some intuition) which will be exploited later.

Threefolds

In the case of an elliptic threefold, branes are complex curves that move in a complex two-dimensional space.

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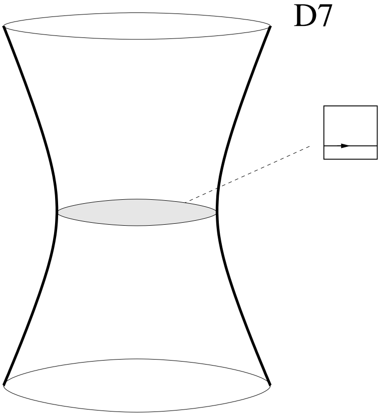

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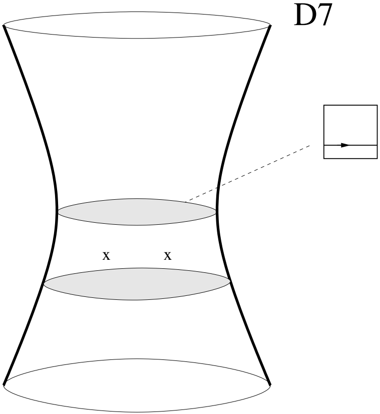

Expect $2g(D7) + I_{D7 \cdot O7} - 2$ cycles

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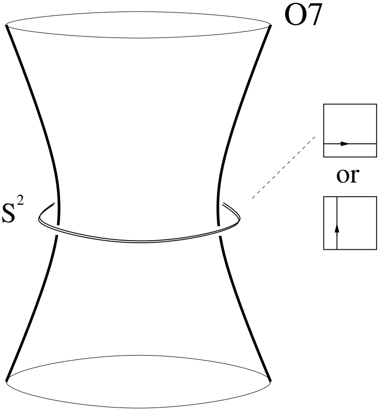


Expect $4g(O7)$ cycles

Threefolds

Degrees of freedom vs. cycles: $h^{(2,1)} = \frac{1}{2}(b_3 - 2)$

In the Orientifold limit:

- The threefold is given as $Z = \left(K3 \times T^2\right)/Z_2$.
- Can use Nikulin's classification to show:

$$b_3(Z) = 4g_{O7} + 2b_2(K3/Z_2)$$

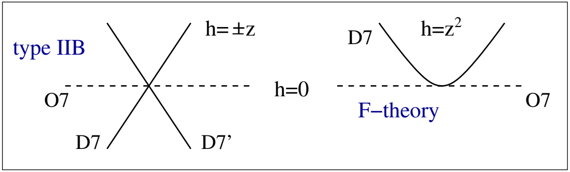
Move a D-brane$\simeq nK_B$ off the O plane,

Deformations: $\frac{n(n+1)}{2}K_B^2$

Cycles: $2g = n(n-1)K_B^2 + 2$ $\quad I_{D7 \cdot O7} - 2 = 2nK_B^2 - 2$

Obstructions

In the weak coupling limit, the D7 brane is NOT a generic hypersurface ! $D7 : \eta^2 + 12h\chi = 0.$



This reduces the degrees of freedom by $I_{D7 \cdot O7}/2$

- In F-theory this forces the D-brane to touch the O7 plane.
- This means that the D7 brane can only have double intersections in the type IIB orientifold. [A.B.,Hebecker,Triendl;Collinucci,Denef,Esole]

Conjecture: there is monodromy acting on the one-cycles of the D7 brane, similar to the appearance of non-simply laced groups in F-theory.

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- Studied by various authors, see e.g.: [Greene, Schalm, Shiu; Lüst, Mayr, Reffert, Stieberger;Görlich, Kachru, Tripathy, Trivedi;Aspinwall, Kallosh;Dasgupta, Rajesh, Sethi].

Starting point:
$$V = \frac{1}{4\nu^3}\left(\int_{\tilde{K3}\times K3} G_4 \wedge * G_4 - G_4 \wedge G_4\right)$$

- The geometric moduli of K3 are given by a spacelike threeplane Σ in $H^2(K3)$. $H^2(K3)$ has signature $(3, 19)$.
- The three orthonormal forms w_i that span this plane give

$$J = \sqrt{2\nu}\, w_1 , \qquad \Omega = w_2 + i w_3 .$$

M-theory on K3×K3

The flux can be expanded as: $$G_4 = \sum_{I\Lambda} G^{I\Lambda} \eta_I \wedge \tilde{\eta}_\Lambda$$

Using the intersection form, $G^{I\Lambda}$ can be viewed as a map between the cohomology groups of $K3$ and $\tilde{K3}$:

$$H^2(\tilde{K3}) \xrightarrow{G^a} H^2(K3), \qquad H^2(K3) \xrightarrow{G} H^2(\tilde{K3}).$$

The flux potential can now be expressed as

$$V = -\frac{1}{2(\nu\tilde{\nu})^3} \left(\sum_i \|\tilde{\mathbb{P}}[G w_i]\|^2 + \sum_j \|\mathbb{P}[G^a \tilde{w}_j]\|^2 . \right)$$

Here $\mathbb{P}$ and $\tilde{\mathbb{P}}$ project onto the directions orthogonal to Σ and $\tilde{\Sigma}$. V is explicitly invariant under the $SO(3)$ rotating the three w_i and is positive definite.

Vacua

We find Minkowski vacua iff

$$G^a\tilde{\Sigma} \subset \Sigma \quad \text{and} \quad G\Sigma \subset \tilde{\Sigma}.$$

This is equivalent to the diagonalizability of G^aG (not for free as $H^2(K3)$ is non-definite) with non-negative eigenvalues.
In this case also G and G^a can be bidiagonalized.

Tadpole

Without D3 branes: $\int G_4 \wedge G_4 = \mathrm{tr} G^a G = \chi/12 = 48$
$\rightarrow$ cannot switch on fluxes on all cycles.

Stability

Flat directions occur iff, for $G^aG = diag(a_1, a_2, a_3, b_i)$ the sets of eigenvalues a_i and b_i are pairwise distint.

How to choose a given brane configuration

In the F-theory limit, $J = bB + fF + j_I \eta_I \rightarrow fF.$

- We have to find two positive norm eigenvectors of $G^a G$ to find Ω (in a space of signature $(2,18)$).
- The intersetion product between Ω and the elements of the lattice $H^2(K3, Z)$ determines the gauge enhancement.
- By giving $G^a G$ the right block structure, a desired gauge enhancement can be achieved.

$$G^a G = \begin{pmatrix} A_{(2,n)} & 0 \\ 0 & B_{(0,16-n)} \end{pmatrix} \quad \rightarrow \quad \Omega = \begin{pmatrix} \omega \\ 0 \end{pmatrix}$$

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Can reproduce the attractive solutions of Aspinwall and Kallosh.

Supersymmetry

The conditions [Becker, Becker]

$$G_4 \wedge J = 0 \quad \text{and} \quad G_4 = G_4^{(2,2)}$$

put constraints on $G\big|_{\tilde{\Sigma}}$ and $G^a\big|_{\Sigma}$:

- If the kernel has dimension three, we find $N = 2$ in $4D$.
- If the kernel is one-dimensional and $G^a G$ acts as the identity on the direction orthogonal to the kernel, we find $N = 1$ in $4D$.
- Otherwise, Susy is completely broken.

Conclusions

- We have understood in detail how D7 brane configurations are encoded in the geometry of an elliptically fibred $K3$.
- This allows us to choose the gauge symmetry by turning on appropriate fluxes.
- We have demonstrated that the ideas used for $K3$ can be extended to the threefold case.

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Example

$SO(8)^3 \times SO(6)$

We first choose a basis in which the intersection form is $2U^2 \otimes A \otimes D_4^3$, where

$$A = \begin{pmatrix} -4 & 0 & 0 & 0 \\ 0 & -2 & 1 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & -2 \end{pmatrix}.$$

We choose the flux to be

$$G^{I\Lambda} = \begin{pmatrix} 1 & 1 & & & & \\ 1 & 1 & & & & \\ & & 1 & 1 & 0 & \\ & & 1 & 3 & 1 & \\ & & 0 & 1 & 0 & \\ & & & & & \mathbf{0}_{15} \end{pmatrix}$$

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This leaves the lattice $D_4^4 \otimes A_3$ orthogonal to Ω.
We have deleted one root from the Dynkin diagram: