Dimensional reductions on SU(3)xSU(3) structures
and $\mathcal{N} = 1$ vacuum conditions

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Mathematical Challenges in String Phenomenology

Vienna, 6-15 October 2008

Based on arXiv:0707.3125 (with Adel Bilal) and 0804.0595

Plan of the talk

1. Motivation
2. Supergravity and SU(3)xSU(3) structures
3. Truncation to a finite set of modes
4. N=2 data from Generalized Geometry
   - Special Kähler geometry
   - Scalar potential
5. Lifting $N = 1$ vacua

Motivation

Effective actions from Flux compactifications

Start from type II supergravity

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Compactification on Calabi-Yau 3-folds

$\hookrightarrow N = 2$ sugra in 4d

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Beyond Calabi-Yau

?? most general flux compactifications
leading to $N = 2$ sugra in 4d **??**

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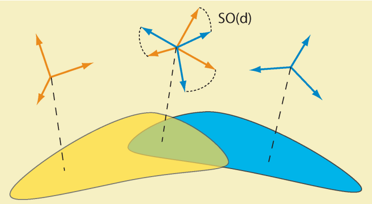
??How $N = 2$ data are determined by Generalized Geometry**??**

Supergravity and SU(3)xSU(3) structures

Need a couple of
(possibly coincident)
internal spinors η^1, η^2

$\Downarrow$

a couple of SU(3)
structures
for TM_6



$\uparrow$ *structure group*

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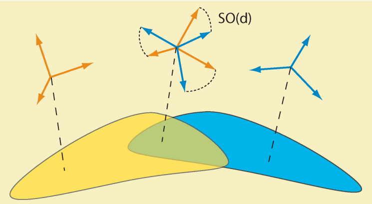

↑ *structure group*

Best seen as an SU(3)$\times$SU(3) structure on $TM_6 \oplus T^*M_6$

[Graña,Louis,Waldram '05,'06]

Structures on $T \oplus T^*$ are described by Generalized Geometry

[Hitchin '02]

Supergravity and SU(3)xSU(3) structures [recall talks by Witt, Koerber & Martucci]

Basic objects: $O(6,6)$ pure spinors Φ_+ and Φ_-

- polyforms : $\Phi_+ \in \wedge^{\text{even}} T^* M_6 \quad , \quad \Phi_- \in \wedge^{\text{odd}} T^* M_6$
- generalize J and Ω of a CY
- encode the whole *internal* NSNS sector $(g_{mn},\ B_{mn},\ \phi)$
- $\Phi_\pm$ can be built as $e^{-B}(\eta^1_+ \otimes \eta^{2\dagger}_\pm)$

$\hookrightarrow$ polyforms through fierzing

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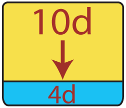
  $\hookrightarrow$ polyforms through fierzing

Polyforms natural also in the RR sector:

for IIA: $\mathbf{F} = F_0 + F_2 + \ldots + F_8 + F_{10}$ *(democratic formulation)*

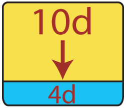
Expansion forms

When reducing



$\Rightarrow$ need to truncate to a finite set of modes

Expansion forms

When reducing 

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Truncation specified introducing a finite basis of (poly)forms

$$\Sigma_{+} = \begin{pmatrix} \tilde{\omega}^{A} \\ \omega_{A} \end{pmatrix} \quad , \quad \Sigma_{-} = \begin{pmatrix} \beta^{I} \\ \alpha_{I} \end{pmatrix}$$

and expanding $\Phi_{\pm}$ as:

$$\Phi_{+} = X^{A}\omega_{A} - \mathcal{F}_{A}\tilde{\omega}^{A} \quad , \quad \Phi_{-} = Z^{I}\alpha_{I} - \mathcal{G}_{I}\beta^{I} \ .$$

for a CY : $\Phi_{+} = e^{iJ}$, $\Phi_{-} = \Omega$ and the forms are harmonic

Expansion forms

Σ_+ and Σ_- have to satisfy several constraints for a 4d, $N = 2$ supergravity to be defined (and the reduction proceed analogously to the CY case)

[Graña,Louis,Waldram '05,'06; Minasian,Kashani-Poor'06; DC,Bilal'07; DC'08]

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E.g. basis forms have to preserve a symplectic structure:

$$\int_{M_6} \langle \Sigma_-, \Sigma_- \rangle = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}$$

where $\langle \; , \; \rangle$ is the antisymmetric Mukai pairing :

$$\langle \alpha, \beta \rangle = [\lambda(\alpha) \wedge \beta]_6 \qquad , \qquad \lambda(\alpha_k) = (-)^{[\frac{k+1}{2}]} \alpha_k$$

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Further condition :

$$\frac{\langle \Sigma_+, \Phi_+ \rangle}{\langle \Phi_+, \bar{\Phi}_+ \rangle} \quad \text{and} \quad \frac{\langle \Sigma_-, \Phi_- \rangle}{\langle \Phi_-, \bar{\Phi}_- \rangle} \quad \text{constant on } M_6$$

Expansion forms

In general $\Sigma_\pm$ are not closed :

$$d\Sigma_- = \mathbb{Q}\Sigma_+$$

$\mathbb{Q}$: geometric charges $\longrightarrow$ more gaugings w.r.t. CY with fluxes

$\mathbb{Q}$ also accommodates nongeometric fluxes [Graña,Louis,Waldram '06]

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Examples?

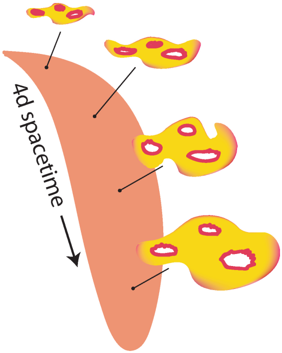
on coset spaces: $\frac{SU(3)}{U(1)\times U(1)}$, $\frac{G_2}{SU(3)}$, $\frac{Sp(2)}{S(U(2)\times U(1))}$, $\ldots$

[Caviezel,Koerber,Kors,Lüst, Tsimpis,Zagermann '08]

Not only: the reduction goes through *consistently* (solutions lift)

Special Kähler Geometry

Moduli space of CY manifolds



CY case :

$$\delta g_{mn} \;\leftrightarrow\; \delta J\,,\;\delta\Omega$$

(Kähler- & complex-structure deformations)

parameterize two Special Kähler manifolds.

Kähler potentials:

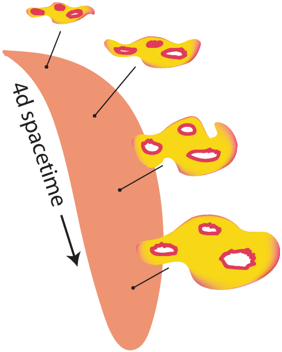
$K_+ \sim \log \int J \wedge J \wedge J$ and $K_- \sim \log i \int \Omega \wedge \overline{\Omega}$

$$\Downarrow$$

fits into 4d, $N = 2$ sugra

Special Kähler Geometry

Deformations of SU(3)×SU(3) structures



What about general SU(3)×SU(3) structures?

$\delta\Phi_+ \,,\ \delta\Phi_-$ *at a point* of M_6

$\updownarrow$

Special Kähler geometries [Hitchin'02]

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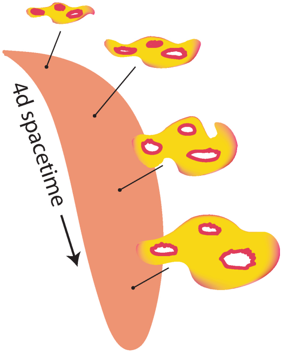

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Requirements on $\Sigma_\pm$ assure this is inherited by the truncated 4d theory

Kähler potentials : $K_\pm = -\log i \int \langle \Phi_\pm, \overline{\Phi}_\pm \rangle$

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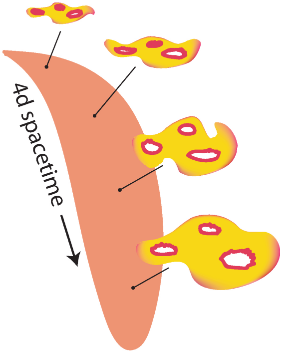

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Kähler potentials : $K_\pm = -\log i \int \langle \Phi_\pm, \overline{\Phi}_\pm \rangle$

We computed:

$$\underbrace{\frac{e^{2\varphi}}{8} \int vol_6 e^{-2\phi} g^{mn} g^{pq} (\delta g_{mp} \delta g_{nq} + \delta B_{mp} \delta B_{nq})}_{\text{metric on space of } g_{mn} \text{ and } B_{mn} \text{ deform.}} = \underbrace{\delta^{\text{holo}} \delta^{\text{anti}} K_+ + \delta^{\text{holo}} \delta^{\text{anti}} K_-}_{\text{sp. Kähler metrics for } \Phi_+ \text{ and } \Phi_- \text{ def.}}$$

$\Downarrow$

4d scalar kinetic terms

Special Kähler Geometry

Generalized diamond

Complex polyforms decompose in reps of SU(3)$\times$SU(3) :

$$
\begin{array}{ccccccc}
 & & & \mathbf{1},\overline{\mathbf{1}} & & & \\
 & & \mathbf{1},\mathbf{3} & & \overline{\mathbf{3}},\overline{\mathbf{1}} & & \\
 & \mathbf{1},\overline{\mathbf{3}} & & \overline{\mathbf{3}},\mathbf{3} & & \mathbf{3},\overline{\mathbf{1}} & \\
\mathbf{1},\mathbf{1} & & \overline{\mathbf{3}},\overline{\mathbf{3}} & & \mathbf{3},\mathbf{3} & & \overline{\mathbf{1}},\overline{\mathbf{1}} \\
 & \overline{\mathbf{3}},\mathbf{1} & & \mathbf{3},\overline{\mathbf{3}} & & \overline{\mathbf{1}},\mathbf{3} & \\
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 & & & \overline{\mathbf{1}},\mathbf{1} & & &
\end{array}
$$

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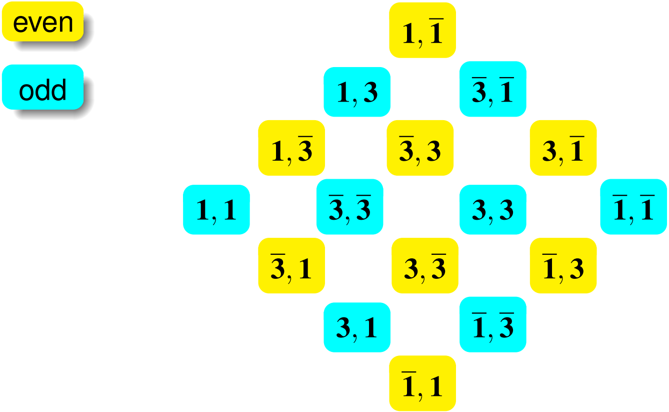

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SU(3)$\times$SU(3) invariant polyforms :

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\end{array}
$$

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SU(3)$\times$SU(3) invariant polyforms :

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SU(3)×SU(3) invariant polyforms :

$$\Phi_+$$

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$$\mathbf{1},\bar{\mathbf{3}} \qquad \bar{\mathbf{3}},\mathbf{3} \qquad \mathbf{3},\bar{\mathbf{1}}$$

$$\Phi_- \qquad \bar{\mathbf{3}},\bar{\mathbf{3}} \qquad \mathbf{3},\mathbf{3} \qquad \bar{\mathbf{1}},\bar{\mathbf{1}}$$

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$$\overline{\mathbf{3}},\mathbf{1} \qquad \mathbf{3},\overline{\mathbf{3}} \qquad \overline{\mathbf{1}},\mathbf{3}$$

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$$\overline{\Phi}_+$$

Special Kähler Geometry

Generalized diamond

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$$\eta^1_+\eta^{2\dagger}_+$$

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$$\eta^1_+\eta^{2\dagger}_- \qquad \bar{\mathbf{3}},\bar{\mathbf{3}} \qquad \mathbf{3},\mathbf{3} \qquad \eta^1_-\eta^{2\dagger}_+$$

$$\bar{\mathbf{3}},\mathbf{1} \qquad \mathbf{3},\bar{\mathbf{3}} \qquad \bar{\mathbf{1}},\mathbf{3}$$

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$$\eta^1_-\eta^{2\dagger}_-$$

acting with (anti)holomorphic Spin(6) gamma matrices one can build a basis for the repr space (easy to include B)

Special Kähler Geometry

Deformations of SU(3)×SU(3) structures

Deformations of Φ_+ (analogous for Φ_-) :

			Φ_+			
		$\mathbf{1},\mathbf{3}$		$\overline{\mathbf{3}},\overline{\mathbf{1}}$		
	$\mathbf{1},\overline{\mathbf{3}}$		$\overline{\mathbf{3}},\mathbf{3}$		$\mathbf{3},\overline{\mathbf{1}}$	
$\mathbf{1},\mathbf{1}$		$\overline{\mathbf{3}},\overline{\mathbf{3}}$		$\mathbf{3},\mathbf{3}$		$\overline{\mathbf{1}},\overline{\mathbf{1}}$
	$\overline{\mathbf{3}},\mathbf{1}$		$\mathbf{3},\overline{\mathbf{3}}$		$\overline{\mathbf{1}},\mathbf{3}$	
		$\mathbf{3},\mathbf{1}$		$\overline{\mathbf{1}},\overline{\mathbf{3}}$		
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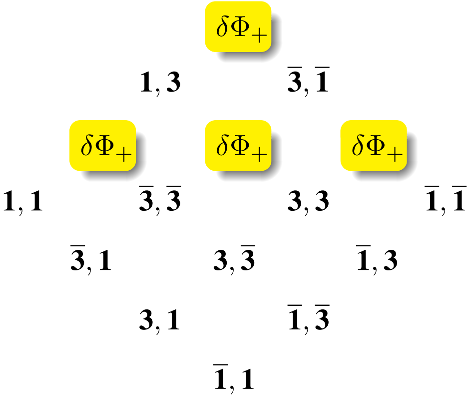

Deformations of Φ_+ (analogous for Φ_-) :

$$\boxed{\delta\Phi_+}$$

$$\mathbf{1},\mathbf{3} \qquad \bar{\mathbf{3}},\bar{\mathbf{1}}$$

$$\boxed{\delta\Phi_+} \qquad \boxed{\delta\Phi_+} \qquad \boxed{\delta\Phi_+}$$

$$\mathbf{1},\mathbf{1} \qquad \bar{\mathbf{3}},\bar{\mathbf{3}} \qquad \mathbf{3},\mathbf{3} \qquad \bar{\mathbf{1}},\bar{\mathbf{1}}$$

$$\bar{\mathbf{3}},\mathbf{1} \qquad \mathbf{3},\bar{\mathbf{3}} \qquad \bar{\mathbf{1}},\mathbf{3}$$

$$\mathbf{3},\mathbf{1} \qquad \bar{\mathbf{1}},\bar{\mathbf{3}}$$

$$\bar{\mathbf{1}},\mathbf{1}$$

$\hookrightarrow$ relation with δg_{mn} , δb_{mn} **?**

compatible $\Phi_+ \ , \ \Phi_-$

compatible $\Phi_+ \;,\; \Phi_-$

$\downarrow\downarrow\downarrow$

$$\mathcal{J}_{\pm}{}^{\Lambda}{}_{\Sigma} \;=\; 4i\frac{\langle\, \mathrm{Re}\,\Phi_\pm \;,\; \Gamma^{\Lambda}{}_{\Sigma}\mathrm{Re}\,\Phi_\pm \,\rangle}{\langle\, \Phi_\pm \;,\; \overline{\Phi}_\pm \,\rangle} \qquad \text{with} \qquad [\mathcal{J}_+, \mathcal{J}_-] = 0$$

$$\mathcal{J}_\pm \;: T\oplus T^* \to T\oplus T^* \quad , \quad (\mathcal{J}_\pm)^2 = -id_{T\oplus T^*}$$

generalized almost complex structure

where $\Gamma^{\Lambda} \;=\; \begin{pmatrix} dx^m \wedge \\ \iota_{\partial_m} \end{pmatrix}$: $O(6,6)$ gamma matrices

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compatible $\Phi_+ \ , \ \Phi_-$

$$\downarrow\downarrow\downarrow$$

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$\mathcal{J}_{\pm} \ : T \oplus T^* \to T \oplus T^* \quad , \quad (\mathcal{J}_{\pm})^2 = -id_{T \oplus T^*}$ generalized almost complex structure

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Metric on $T \oplus T^*$: $\mathcal{G} = -\mathcal{J}_+\mathcal{J}_-$

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compatible Φ_+ , Φ_-

$\downarrow\downarrow\downarrow$

$$\mathcal{J}_{\pm}{}^{\Lambda}{}_{\Sigma} = 4i\frac{\langle \mathrm{Re}\,\Phi_{\pm}\,,\,\Gamma^{\Lambda}{}_{\Sigma}\mathrm{Re}\,\Phi_{\pm}\,\rangle}{\langle\,\Phi_{\pm}\,,\,\overline{\Phi}_{\pm}\,\rangle} \qquad \text{with} \qquad [\mathcal{J}_+,\mathcal{J}_-]=0$$

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Metric on $T\oplus T^*$: $\mathcal{G} = -\mathcal{J}_+\mathcal{J}_- = \begin{pmatrix} g^{-1}B & g^{-1} \\ g - Bg^{-1}B & -Bg^{-1} \end{pmatrix}$

compatible $\Phi_+ \; , \; \Phi_-$

$$\downarrow\downarrow\downarrow$$

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Deformations :

$$g^{mn}g^{pq}(\delta g_{mp}\delta g_{nq} + \delta B_{mp}\delta B_{nq}) = -\tfrac{1}{2}\text{Tr}\left[\delta\mathcal{G}\delta\mathcal{G}\right]$$

compatible $\Phi_+ \ , \ \Phi_-$

$$\downarrow\downarrow\downarrow$$

$$\mathcal{J}_\pm{}^\Lambda{}_\Sigma \;=\; 4i\frac{\langle\, \text{Re}\,\Phi_\pm \;,\; \Gamma^\Lambda{}_\Sigma \text{Re}\,\Phi_\pm \,\rangle}{\langle\, \Phi_\pm \;,\; \overline{\Phi}_\pm \,\rangle} \qquad \text{with} \qquad [\mathcal{J}_+, \mathcal{J}_-] = 0$$

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Deformations :

$$g^{mn}g^{pq}(\delta g_{mp}\delta g_{nq} + \delta B_{mp}\delta B_{nq}) \;=\; -\tfrac{1}{2}\text{Tr}\big[\delta\mathcal{G}\delta\mathcal{G}\big]$$

use : $\delta\mathcal{G} = -\delta\mathcal{J}_+\mathcal{J}_- - \mathcal{J}_+(\delta\mathcal{J}_-)$

Special Kähler Geometry

Deformations of SU(3)×SU(3) structures

Deformations of Φ_+ (analogous for Φ_-) :

$\delta\Phi_+$

$\mathbf{1},\mathbf{3}$ $\quad$ $\bar{\mathbf{3}},\bar{\mathbf{1}}$

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$\mathbf{1},\mathbf{1}$ $\quad$ $\bar{\mathbf{3}},\bar{\mathbf{3}}$ $\quad$ $\mathbf{3},\mathbf{3}$ $\quad$ $\bar{\mathbf{1}},\bar{\mathbf{1}}$

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$$\delta\Phi_+ \quad \longrightarrow \quad \delta\mathcal{J}_+ = 0 \quad \Rightarrow \quad \delta\mathcal{G} = 0$$

$$\mathbf{1},\mathbf{3} \qquad \overline{\mathbf{3}},\overline{\mathbf{1}}$$

$$\delta\Phi_+ \qquad \delta\Phi_+ \qquad \delta\Phi_+$$

$$\mathbf{1},\mathbf{1} \qquad \overline{\mathbf{3}},\overline{\mathbf{3}} \qquad \mathbf{3},\mathbf{3} \qquad \overline{\mathbf{1}},\overline{\mathbf{1}}$$

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Special Kähler Geometry

Deformations of SU(3)×SU(3) structures

Deformations of Φ_+ (analogous for Φ_-) :

$$\delta\Phi_+ \longrightarrow \delta\mathcal{J}_+ = 0 \quad \Rightarrow \quad \delta\mathcal{G} = 0$$

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$\delta\mathcal{J}_+ \neq 0$ but $\delta\mathcal{G} = 0 \longleftarrow \delta\Phi_+ \qquad \delta\Phi_+ \qquad \delta\Phi_+ \longrightarrow \delta\mathcal{J}_+ \neq 0$ but $\delta\mathcal{G} = 0$

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Matching : yes, provided we truncate these deformations

Special Kähler Geometry

Period matrices

Important ingredient : $\mathcal{G}_I = \mathcal{M}_{IJ} Z^J \quad , \quad D\mathcal{G}_I = \overline{\mathcal{M}}_{IJ} DZ^J$

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↖ period matrix ↗

$\mathcal{R}$=Re$\mathcal{M}$, $\mathcal{I}$=Im$\mathcal{M}$

$$\mathbb{M} \equiv \begin{pmatrix} \mathcal{I} + \mathcal{R}\mathcal{I}^{-1}\mathcal{R} & -\mathcal{R}\mathcal{I}^{-1} \\ -\mathcal{I}^{-1}\mathcal{R} & \mathcal{I}^{-1} \end{pmatrix} = \begin{pmatrix} -\int\langle\alpha, *_B\alpha\rangle & \int\langle\alpha, *_B\beta\rangle \\ \int\langle\beta, *_B\alpha\rangle & -\int\langle\beta, *_B\beta\rangle \end{pmatrix}$$

- uses $*_B \bullet := e^{-B} * \lambda(e^B \bullet)$
- generalizes a result valid for the harmonic 3-forms of CY
- parallel expression for even forms $\longrightarrow \mathcal{N}$ & $\mathbb{N}$

(valid for CY as well)

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- $*_B$ on the generalized diamond :

$$\begin{matrix} & & & i & & & \\ & & i & & -i & & \\ & i & & -i & & i & \\ i & & -i & & i & & -i \\ & -i & & i & & -i & \\ & & i & & -i & & \\ & & & -i & & & \end{matrix}$$

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  (valid for CY as well)
- In e.g. IIA:
  - Im$\mathcal{N}$ and Re$\mathcal{N}$ define kinetic & top. terms for gauge fields
  - $\mathbb{M}$ enters in the hyperscalar kinetic terms
  - Both $\mathbb{M}$ and $\mathbb{N}$ appear in the scalar potential

Scalar potential

NSNS sector$\longrightarrow$ $\mathcal{V}_{\mathrm{NS}} \sim \int_{M_6} vol_6 e^{-2\phi}\left(R_6 + 4\partial_m\phi\partial^m\phi - \frac{1}{12}H_{mnp}H^{mnp}\right)$

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Recast in Generalized Geometry language:

- $[D_m, D_n]\,\eta \sim R_{mnpq}\gamma^{pq}\eta$
- derive formula relating R_6 and $\Phi_\pm \sim \eta^1_+ \otimes \eta^{2\dagger}_\pm$
- 'dress' it with ϕ and B

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$$\mathcal{V}_{\rm NS} \;=\; \frac{e^{4\varphi}}{4}\int_{M_6}\langle\, d\Phi_+, *_B(d\overline{\Phi}_+)\,\rangle + \langle\, d\Phi_-, *_B(d\overline{\Phi}_-)\,\rangle$$

$$-\; e^{4\varphi}\int_{M_6}\frac{\left|\langle d\Phi_+, \Phi_-\rangle\right|^2 + \left|\langle d\Phi_+, \overline{\Phi}_-\rangle\right|^2}{i\langle\Phi, \overline{\Phi}\rangle}$$

[DC, 0804.0595]

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$$\mathcal{V} = \mathcal{V}_{\rm NS} + \mathcal{V}_{\rm RR} \;=\; \frac{e^{4\varphi}}{4}\int_{M_6} \langle\, d\Phi_+, *_B(d\overline{\Phi}_+)\,\rangle + \langle\, d\Phi_-, *_B(d\overline{\Phi}_-)\,\rangle$$
$$- e^{4\varphi}\int_{M_6} \frac{\left|\langle d\Phi_+, \Phi_-\rangle\right|^2 + \left|\langle d\Phi_+, \overline{\Phi}_-\rangle\right|^2}{i\langle\Phi, \overline{\Phi}\rangle}$$
$$+ \frac{e^{4\varphi}}{2}\int_{M_6} \langle G, *_B G\rangle$$

[DC, 0804.0595]

where $G = G_0 + G_2 + G_4 + G_6$: internal RR field strengths

Scalar potential

$$
\begin{aligned}
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- Put the reduction ansatz in $\mathcal{V}$ $\longrightarrow$

$\longrightarrow$ find symplectically invariant 4d $N = 2$ potential

[D'Auria,Ferrara,Trigiante '07]

$$\mathcal{V} = -2e^{2\varphi}\left[e^{K_+}X^T\mathbb{Q}^T\mathbb{M}\mathbb{Q}\bar{X} + e^{K_-}Z^T\widetilde{\mathbb{Q}}^T\mathbb{N}\widetilde{\mathbb{Q}}\bar{Z}\right]$$
$$-8e^{2\varphi}e^{K_+ + K_-}\bar{Z}^T\mathbb{S}_-\mathbb{Q}(X\bar{X}^T + \bar{X}X^T)\mathbb{Q}^T\mathbb{S}_-Z$$
$$-\frac{e^{4\varphi}}{2}G^T\mathbb{N}G \quad ,$$

where $X=\binom{X^A}{\mathcal{F}_A}$, $Z=\binom{Z^I}{\mathcal{G}_I}$, $G=\binom{G^A}{\tilde{G}_A}$, $\mathbb{S}_\pm=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$

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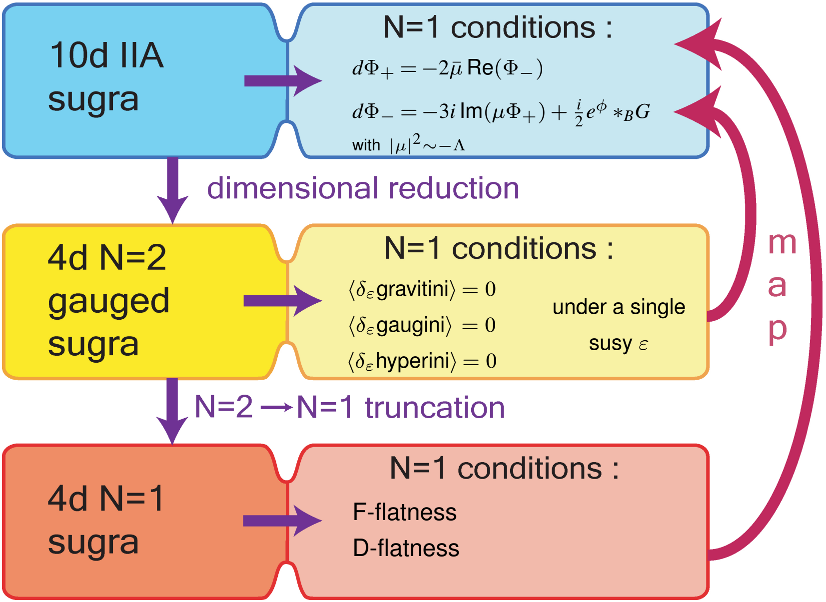
- $\mathcal{V}$ invariant under $\Phi_+ \leftrightarrow \Phi_-$ ('mirror' symmetry)
- $\mathcal{V}$ above is relevant for $N = 2$ reductions.
  Admits $N < 2$ generalization [Lüst,Marchesano,Martucci,Tsimpis '08]

Lifting $N = 1$ vacua



Lifting $N = 1$ vacua

10d level

FIRST $$d\Phi_+ = -2\bar{\mu}\,\mathrm{Re}(\Phi_-)$$ with $|\mu|^2 \sim -\Lambda$

SECOND $$d\Phi_- = -3i\mathrm{Im}(\mu\Phi_+) + \frac{1}{2}e^{\phi}(c_- G + i *_B G)$$

[Graña,Minasian,Petrini,Tomasiello '04,'05]

make contact with 4d :

- expand on the basis forms $\Sigma_\pm$
- separate in components

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$$\begin{array}{c} \langle\delta_\varepsilon \text{hyperini}\rangle = 0 \\ \langle\delta_\varepsilon \text{gravitini}\rangle = 0 \end{array} \quad \Longleftrightarrow \quad \text{FIRST}$$

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Lifting $N = 1$ vacua

4d $N = 2 \to N = 1$ truncation (induced e.g. by O6 plane)

two $N = 2$ gravitini $\longrightarrow$ $N = 1$ gravitino

$$n_V \;\; N = 2 \text{ vector mult.} \longrightarrow \begin{cases} n_C \leq n_V & \text{chiral mult. 'A'} \\ n_V - n_C & N = 1 \text{ vector mult.} \end{cases}$$

hypermultiplets $\longrightarrow$ chiral mult. 'B'

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- Translate in $N = 1$ language identifying the $N = 1$ variables
- From susy variations $\longrightarrow$ read F- and D-terms

$N = 1$ conditions in $N = 1$ language

F-flatness for chiral mult. ‘B’ $\iff$ FIRST

F-flatness for chiral mult. ‘A’, D-flatness $\iff$ SECOND

Summary : Comparison with Calabi-Yau case

	CY & no fluxes	SU(3)×SU(3) + fluxes
4d action	$N=2$ ungauged sugra	$N=2$ gauged sugra charges: RR, NSNS-fluxes non-CYness $d\Sigma_- = \mathbb{Q}\Sigma_+$
Geometric moduli	$\delta J\ ,\ \delta\Omega$	$\delta\Phi_+\ ,\ \delta\Phi_-$ (include $\delta B\,,\ \delta\phi$)
Kähler potentials	$K_+ \sim \log\int J\wedge J\wedge J$ $K_- \sim \log i\int \Omega\wedge\overline{\Omega}$	$K_\pm = \log i\int\langle\Phi_\pm,\overline{\Phi}_\pm\rangle$
Scalar potential	$\mathcal{V}=0$	$\mathcal{V}=\mathcal{V}(d\Phi_\pm,\text{fluxes})$
Susy vacua	trivially $N=2$	nontrivial $N=1$ conditions. Consistent with 10d eqs

Conclusions

- Type II reductions to 4d, $N = 2$ sugra require SU(3)$\times$SU(3) str.
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first step towards proof of consistency
- Future directions:
  - apply this general formalism to further explicit examples
  - first principles characterization of the expansion forms