Non-Geometric String Backgrounds

The landscape of “stringy” internal spaces

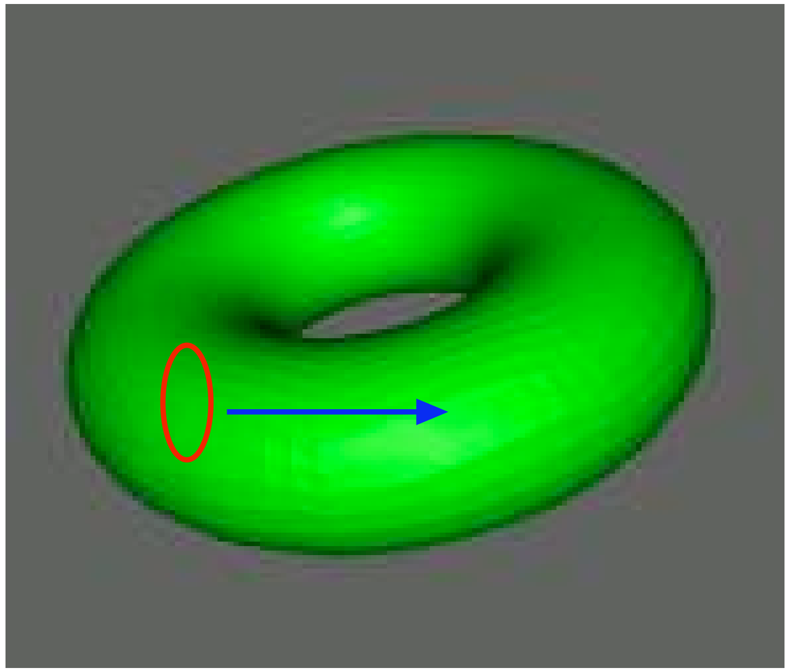
Vienna October 2008

Strings in Geometric Background

Manifold, background tensor fields G_{ij}, H_{ijk}, Φ

Fluctuations: modes of string

Treat background and fluctuations the same?



Non-Geometric Background?

Stringy geometry? Singularity resolution?

Dualities: mix geometric and stringy modes

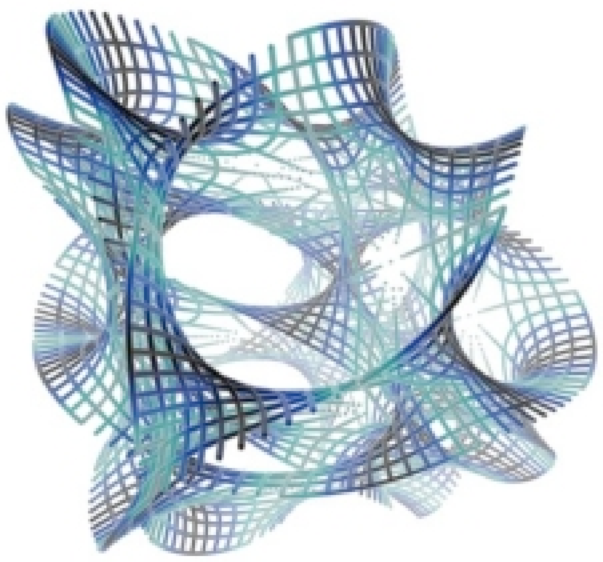
Use in transititions between patches

Sigma Model inadequate. String Field Theory?

Plan of Talk

- T-folds: T-duality transitions
- Doubled formalism, D-branes
- Example, Generalised T-duality
- Compactifications and gauged SUGRA
- Generalised geometry and U-folds: C_3
- How do we introduce fermions and supersymmetry in non-geometry?
- Generalised spinors, SUSY

- T-duals or mirrors of flux compactifications
- Stringy, not supergravity. Simplest extension
- Stringy Geometry, singularity resolution
- New “compactifications”, fix moduli
- Lift of generic D=4 supergravity to D=10 or 11 is non-geometric
- Generic solutions of string (field) theory?



Geometric Background

Manifold with tensor fields, fluxes and gauge fields

Duality ⇓

Non-Geometric Background

???

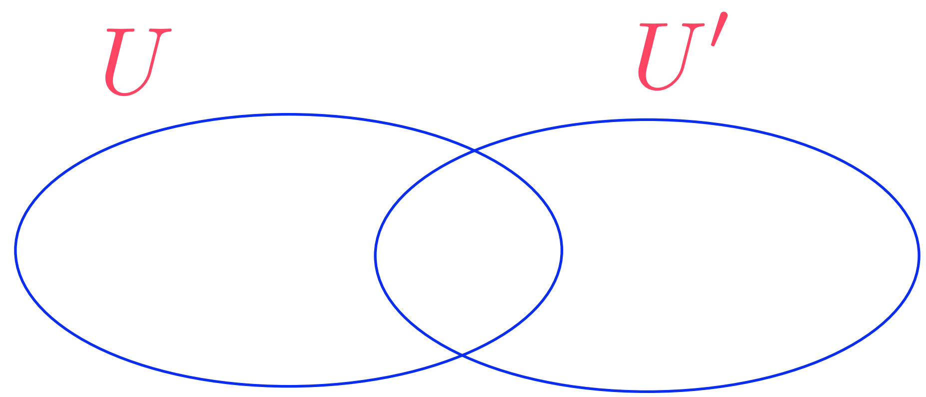
Dualities: stringy symmetries

Usually maps to another geometric background

But sometimes not:

Obstruction to duality?

Or non-geometric background?



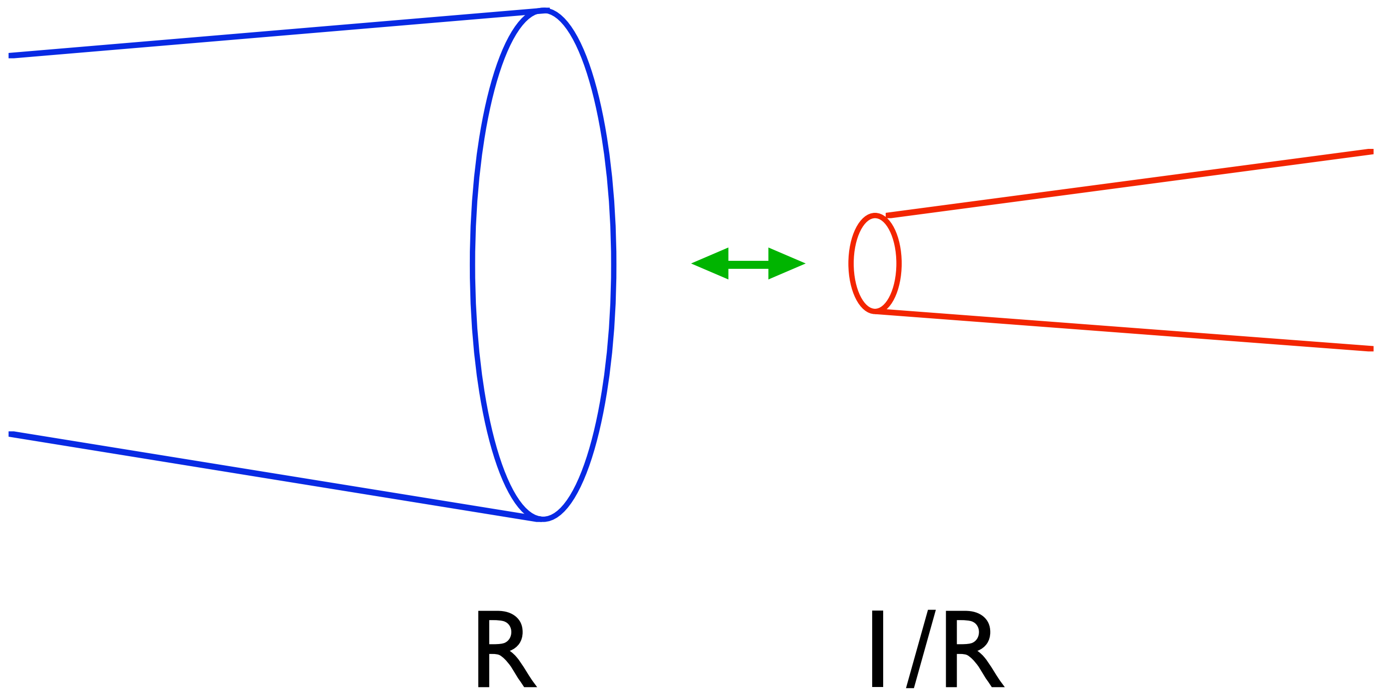
Patches glued using geometric symmetries:
Diffeomorphisms, gauge transformations
to construct geometric background

If toroidal fibration:

↓ T-duality

Glue using T-dualities also
T-fold: Patching uses T-duality
Physics smooth, as T-duality a symmetry

T-fold patching



Glue big circle (R) to small (1/R)

Glue momentum modes to winding modes

(or linear combination of momentum and winding)

Not conventional smooth geometry

Non-Geometric Backgrounds

Many consistent non-geometric string backgrounds

Orbifolds, asymmetric orbifolds:

arise in NGB at special points in moduli space

T-folds, U-folds, mirror-folds

spaces with torus fibration and T or U duality patching, or CY fibration and mirror symmetry patching

S. Kachru, M. B. Schulz, P. K. Tripathy and S. P. Trivedi
A. Dabholkar and C. Hull
S. Hellerman, J. McGreevy and B. Williams
A. Flournoy, B. Wecht and B. Williams
J. Shelton, W. Taylor and B. Wecht
S. Hellerman and J. Walcher; D. Vegh and J. McGreevy

T-Duality Monodromies Round Degenerate Fibres

B.Greene, A.Shapere, C.Vafa, S.T.Yau
S. Hellerman, J. McGreevy and B.Williams 2002

Torus fibration over base
Singularities in base where fibre degenerates
T-duality monodromies round singularities

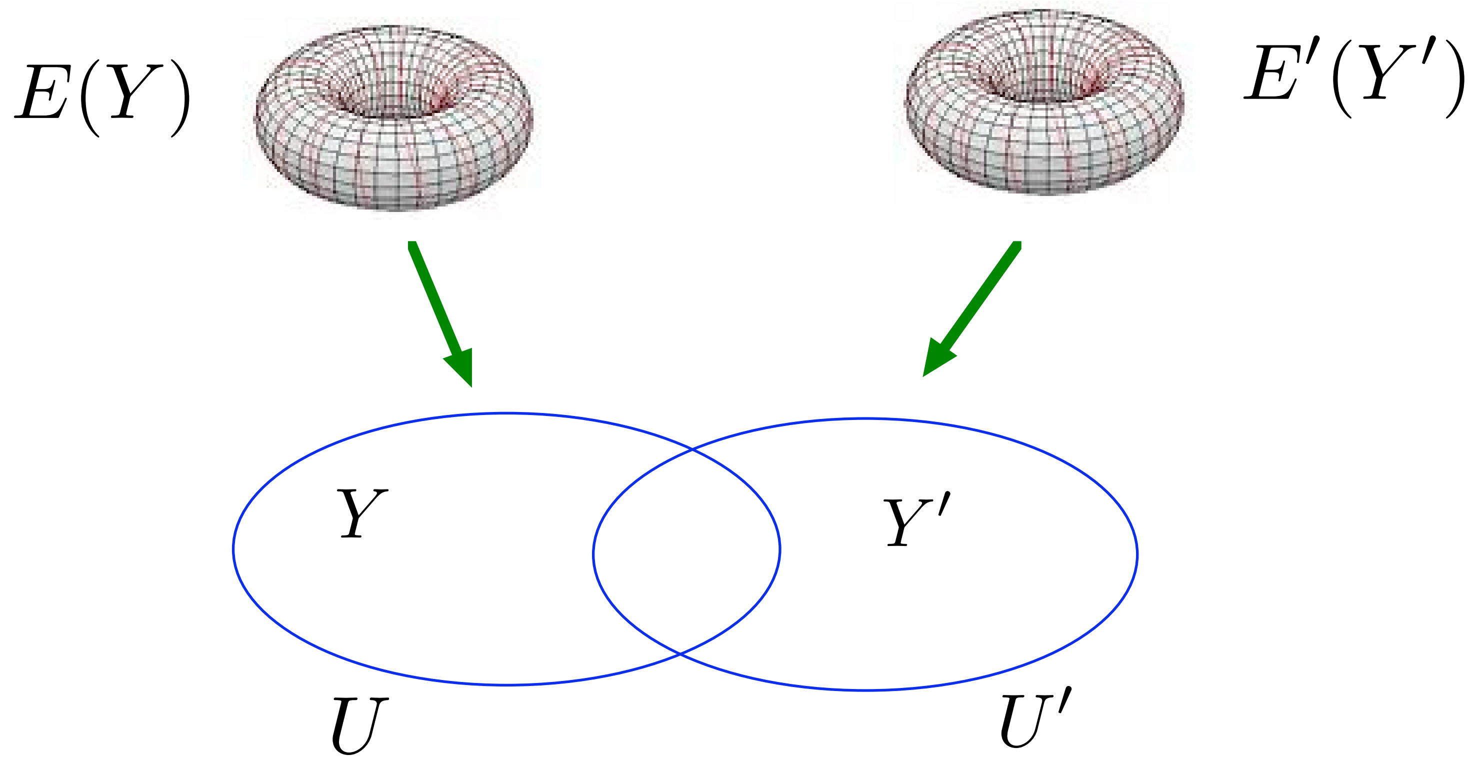
Reductions with Duality Twist

A. Dabholkar and C. Hull 2002

Compactify on circle (or torus)
T-duality or U-duality Monodromy round circle(s)
Stringy lift of Scherk-Schwarz
At special points in mod space: assym orbifolds

NG Phenomenology?

- Internal NG background, gives conventional 4-D field theory
- Lift of generic 4-D SUGRA
- Count in landscape? T-folds? More general NG spaces?
- Moduli Stabilisation
- Beckers, Vafa, Walcher: all moduli frozen



Torus fibration

Geometric background: E=G+B tensorial

Patches glued using geometric symmetries

$$E' = aEa^t \quad \text{in} \quad U \cap U' \qquad a \in GL(d, \mathbb{Z})$$

Duality: transition functions now T-dualities

Non-tensorial

$O(d, d; \mathbb{Z})$

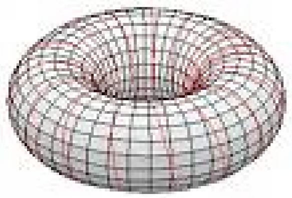
$$E' = (aE + b)(cE + d)^{-1} \quad \text{in} \quad U \cap U'$$

Glue using T-dualities also → **T-fold**

Physics smooth, as T-duality a symmetry

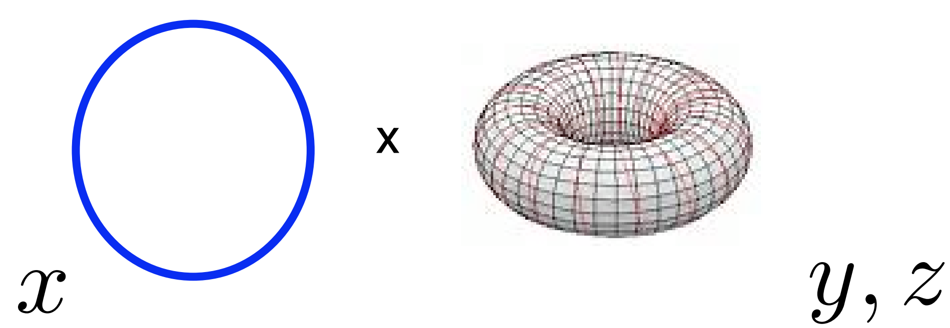
Example: T³ with H-flux

$$H = N \times (Vol)$$



$$H_{xyz} = N$$

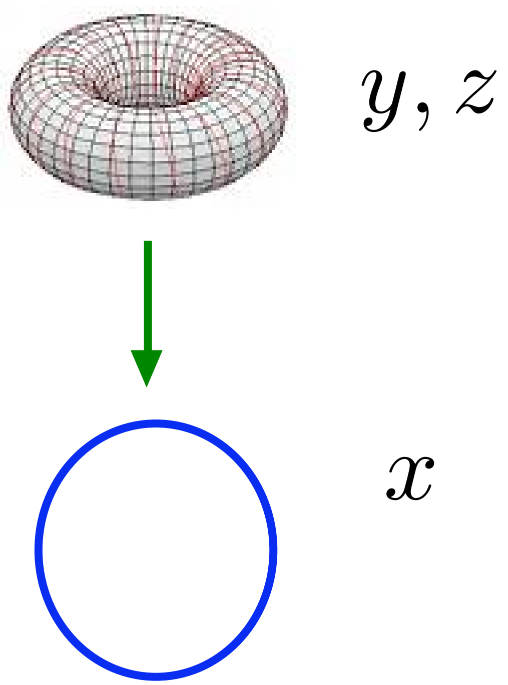
Regard as product $S^1 \times T^2$



$$B_{yz} = B_0 + \frac{1}{2\pi} N x$$

T-dual on z-circle:

Torus bundle over circle, H=0



$$\tau(x) = \tau_0 + \frac{1}{2\pi} N x$$

Nilfold: Heisenberg group manifold identified under discrete subgroup

Next, T-dual on y-circle?

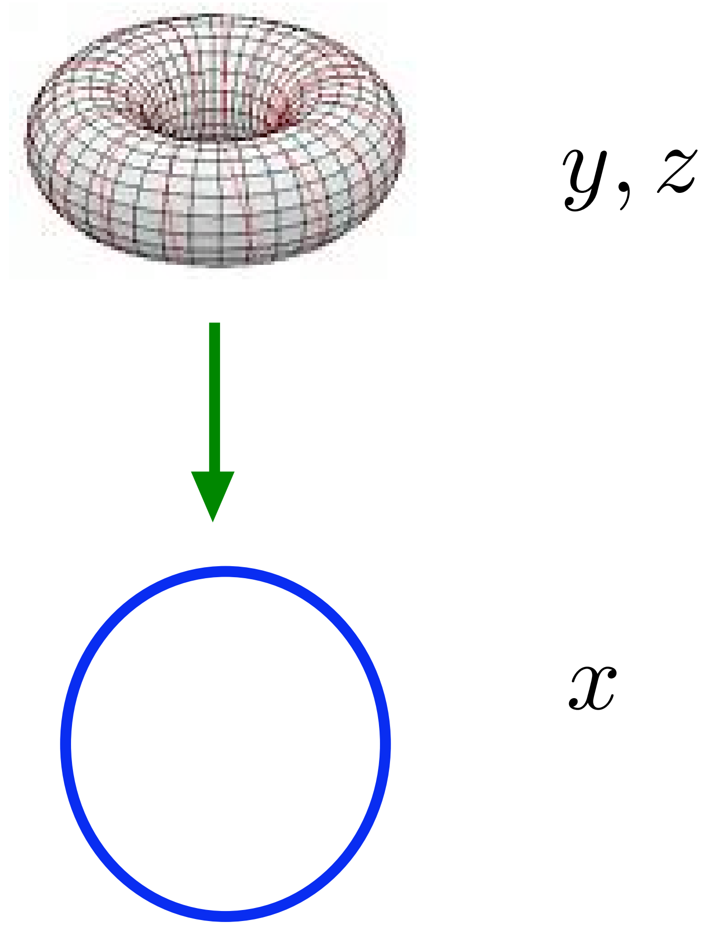
No global Killing vector. Do fibrewise duality, use Buscher rules locally

T-dual on y-circle:

Torus bundle over circle?

$$ds^2 = dx^2 + \frac{1}{1+N^2x^2}\left(dy^2 + dz^2\right)$$

$$B_{yz} = \frac{Nx}{1+N^2x^2}$$



But x periodic

$$E(x+2\pi) = (aE+b)(cE+d)^{-1}$$

Monodromy $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(2,2;Z)$ T-duality

T-fold. T-duality in x? No isometry - see later

Strings on T^d

$$X = X_L(\sigma + \tau) + X_R(\sigma - \tau), \qquad \tilde{X} = X_L - X_R$$

X conjugate to momentum, $\tilde{X}$ to winding no.

$$dX = *d\tilde{X} \qquad\qquad \partial_a X = \epsilon_{ab}\partial^b \tilde{X}$$

Need "auxiliary" $\tilde{X}$ for interacting theory

i) Vertex operators $e^{ik_L \cdot X_L}, \quad e^{ik_R \cdot X_R}$

ii) String field Kugo & Zwiebach $\Psi[X, \tilde{X}, a, \tilde{a}]$

Doubled Formalism

$$\mathbb{X}^I = \begin{pmatrix} X^i \\ \widetilde{X}_i \end{pmatrix} \qquad I = 1, ..., 2d$$

Transforms linearly under $O(d,d;\mathbb{Z})$

T-fold transition: mixes

No global way of separating "real" space coordinate X from "auxiliary" $\widetilde{X}$

Duality covariant formulation in terms of $\mathbb{X}$

Transition functions $O(d,d;\mathbb{Z}) \subset GL(2d;\mathbb{Z})$

can be used to construct bundle with fibres T^{2d}

Doubled Bundle

T^{2d} bundle: doubled fibre

Construct duality-covariant sigma model on doubled space $(\mathbb{X}^I, Y^m)$

Constraint to halve degrees of freedom on fibre:

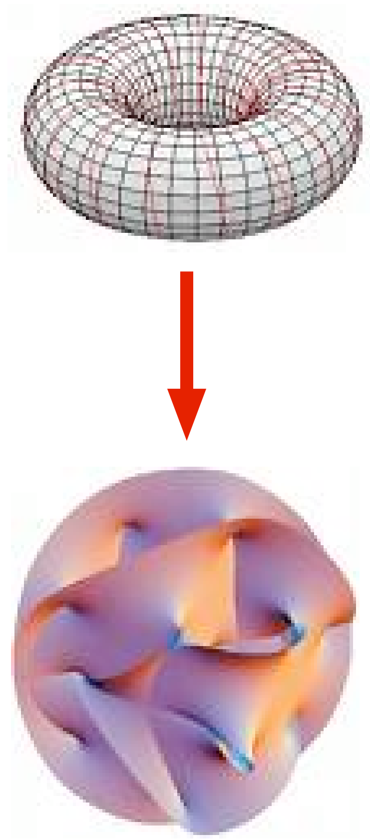
$$dX = *d\widetilde{X}$$ for free case

$$D\mathbb{X} = S(Y) * D\mathbb{X}$$ for general case

$$S(Y) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + O(Y) \qquad S^2 = 1$$

$\mathbb{X}^I$

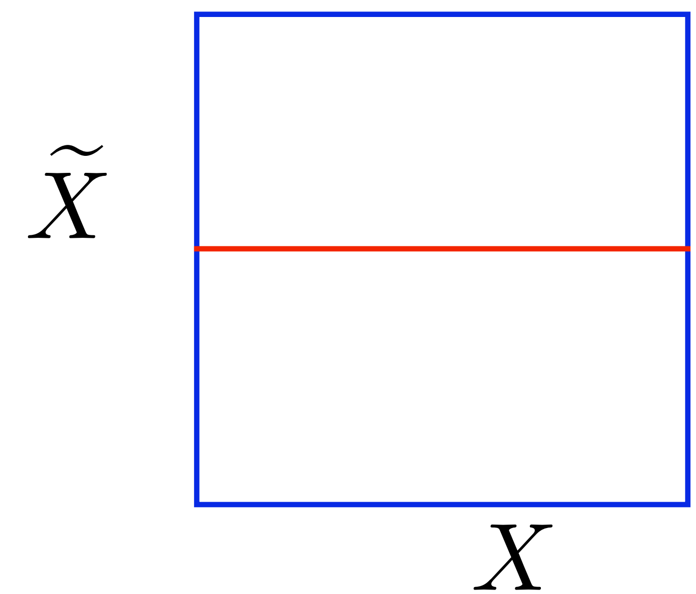
Y^m



Polarisation

To recover conventional formulation, split into "fundamental" and "auxiliary":

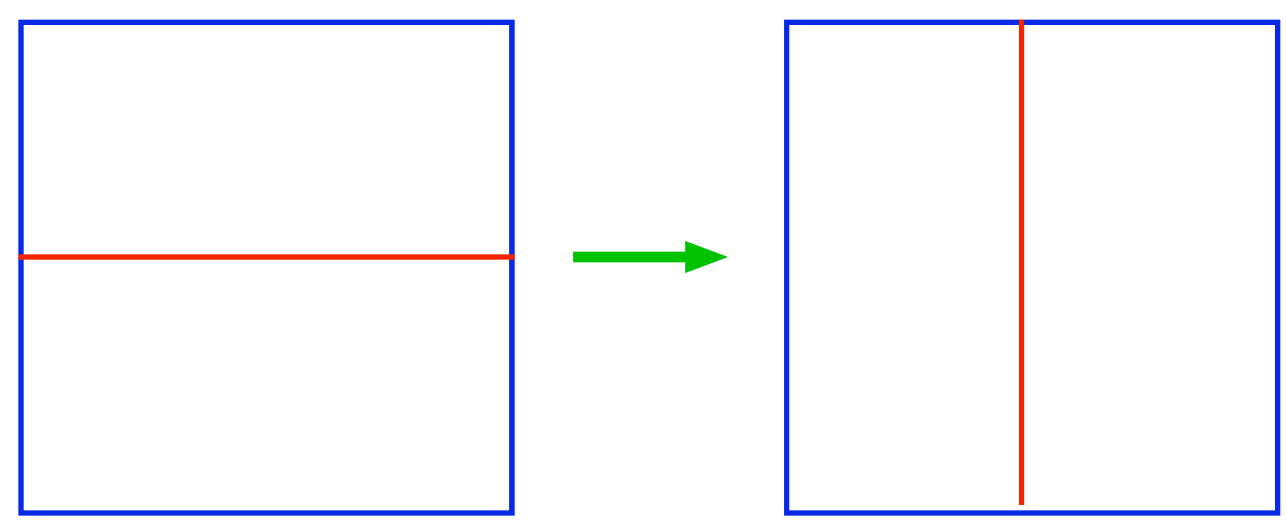
$$\mathbb{X} \to \{X^i, \tilde{X}_i\}$$



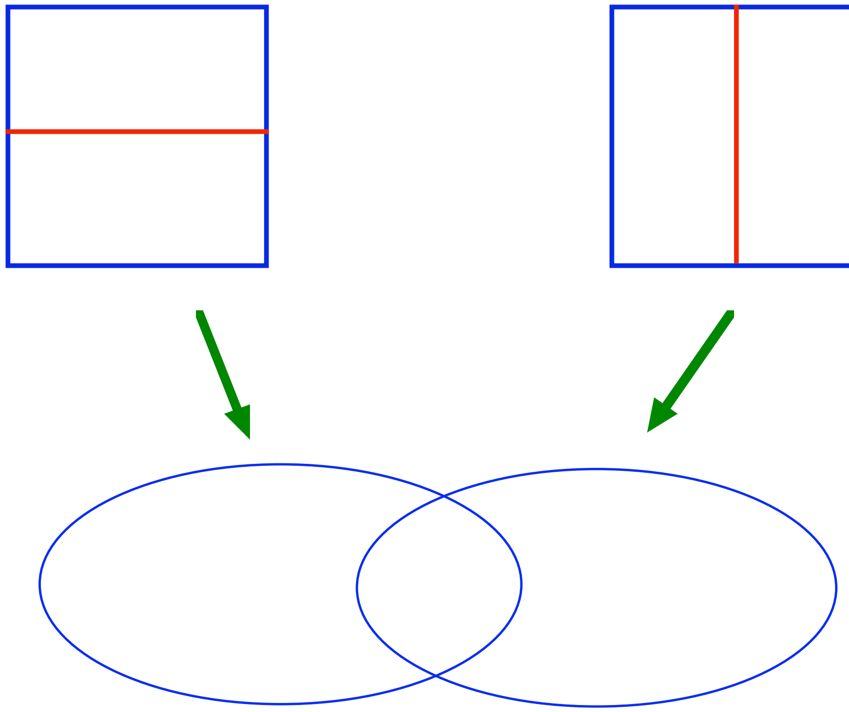
Pick "real spacetime", $T^d \subset T^{2d}$

T-duality rotates polarisation.

T-duality symmetry: physics independent of polarisation.



T-fold



Pick polarisation over each patch in base.

T-duality transitions: polarisation changes from patch to patch.

Geometric: there is global spacetime submanifold

Non-geometric if there is no global polarisation.

$$\mathcal{L}_k = \frac{1}{4}\mathcal{H}_{IJ}\,(d\mathbb{X}^I + \mathcal{A}^I) \wedge *(d\mathbb{X}^J + \mathcal{A}^J) \quad + \mathcal{L}(Y)$$

$$\mathcal{L}_{WZ} = -\frac{1}{2}L_{IJ}d\mathbb{X}^I \wedge \mathcal{A}^J \qquad \mathcal{L}_{top} = \frac{1}{2}\Omega_{IJ}d\mathbb{X}^I \wedge d\mathbb{X}^J$$

Generalised metric

$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$$

O(d,d) metric

$$L = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

2d connections

$$\mathcal{A}^I = \begin{pmatrix} A^i \\ \widetilde{A}_i \end{pmatrix} \qquad \begin{aligned} A^i &\sim G_{mi}dY^m \\ \tilde{A}^i &\sim B_{mi}dY^m \end{aligned}$$

O(d,d) Covariant

$$\mathcal{H} \to h^t\mathcal{H}h \quad \mathbb{X} \to h^{-1}\mathbb{X} \quad \mathcal{A} \to h^{-1}\mathcal{A}$$

Product structure

$$S^I{}_J = L^{IK}\mathcal{H}_{KJ} \qquad S^2 = 1$$

D-Branes and Open Strings

If X Neumann, T-dual $\widetilde{X}$ is Dirichlet
If $\widetilde{X}$ Dirichlet, T-dual X is Neumann

e.g. d=9, $R_{time} \times T^9$

$$\mathbb{X}^I = (X^i{}_D, X^i{}_N)$$

9 D coordinates, 9 N ones.
Universal 9-brane, lagrangian cycle

Polarisation chooses some number p of the Neumann directions as physical.
Interpret as p-brane

$X^1{}_N$, $X^2{}_N$, $X^3{}_N$ ....$X^9{}_N$, $X^1{}_D$,$X^2{}_D$,$X^3{}_D$...$X^9{}_D$

Polarisation chooses 9 of 18 coords as “physical”

$X^1_N, X^2_N, X^3_N \ldots X^9_N,\ X^1_D, X^2_D, X^3_D \ldots X^9_D$

Polarisation chooses 9 of 18 coords as "physical"

$X^1_D, X^2_D, X^3_D \ldots X^9_D$ All 9 coords Dirichlet, 0-brane

$X^1_N, X^2_D, X^3_D \ldots X^9_D$ 1-brane

$X^1_N, X^2_N, X^3_D \ldots X^9_D$ 2-brane

$X^1_N, X^2_N, X^3_N \ldots X^9_N$ 9-brane

$X^1_N, X^2_N, X^3_N \ldots X^9_N,\ X^1_D, X^2_D, X^3_D \ldots X^9_D$

Polarisation chooses 9 of 18 coords as "physical"

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$X^1_N, X^2_N, X^3_D \ldots X^9_D$ 2-brane

$X^1_N, X^2_N, X^3_N \ldots X^9_N$ 9-brane

T-fold transition: Glue Dp-brane to Dq-brane

Doubled picture: glue universal 9-branes together smoothly, but polarisation jumps

Quantisation

How should we impose constraint?

$$d\mathbb{X} + \mathcal{A} = S(Y) * (d\mathbb{X} + \mathcal{A})$$

1) Chose polarisation locally $\mathbb{X} \to \{X^i, \tilde{X}_i\}$

2) Constraint generates shifts in $\widetilde{X}$

Gauge these shifts: sigma-model $\mathcal{L}(Y, X)$

3)
$$\exp(i \int \mathcal{L}_{top}) = \exp(\pi i n \widetilde{n}) = \pm 1$$

Gives equivalence on arbitrary Riemann surface

4) Extends proof of T-duality to fibrewise case, with Killing vectors only locally defined

5) Canonical   Hackett-Jones

Compactification

Toroidal Reduction on T^d

$$\mathcal{L} = e^{-2\Phi}\left\{R + (\nabla\Phi)^2 - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda}\right\}$$

Gauge group $U(1)^{2d}$ $\quad A^i \sim G_{mi}dY^m \quad \tilde{A}^i \sim B_{mi}dY^m$

Moduli: scalar fields $\tau = G_{ij} + B_{ij} \in \frac{O(d,d)}{O(d)\times O(d)}$

Supergravity with global O(d,d) symmetry

Scherk-Schwarz Reduction

Gauged supergravity, non-abelian gauge fields $A, \widetilde{A}$

2d-dimensional gauge group $\quad \mathcal{G}_{2d} \subset O(d,d)$

Landscape of gauged supergravities

General gauged supergravities from gauging any

$$\mathcal{G}_{2d} \subset O(d,d)$$

Some lift to Scherk-Schwarz or other compactifications of D=10 supergravity. Many don't.

Which can arise from string theory? Generically, arise from non-geometric reductions.

Dabholkar and Hull

Shelton, Taylor and Wecht

Gauge Algebra

$$[Z_a, Z_b] = f_{ab}{}^c Z_c + H_{abc} X^c$$

$$[Z_a, X^b] = -f_{ac}{}^b X^c + Q_a^{bc} Z^c$$

$$[X^a, X^b] = Q_c^{ab} X^c + R^{abc} Z_c$$

Background, algebra specified by "fluxes"

$$f_{ab}{}^c, H_{abc}, Q_c^{ab}, R^{abc}$$

T-dualities

$$H_{abc} \underset{T_a}{\leftrightarrow} f_{bc}^a \underset{T_b}{\leftrightarrow} Q_c^{ab} \underset{T_c}{\leftrightarrow} R^{abc}$$

5 Classes of gauged supergravities, with lifts:

Dabholkar and Hull

- Twisted torus reductions with flux. Kaloper & Myers
  Lift of Scherk-Schwarz to string theory f,H
  CMH & Reid-Edwards
- Reduction with Duality Twist f,H,Q
- Asymmetric Orbifold f,H,Q
- Orbifolds with dual twists f,H,Q,R
- Reductions with dual twists f,H,Q,R

Origin of gauged sugras

1) Twisted torus reductions with flux f,H

Scherk-Schwarz reductions of supergravity use a symmetry to define a field theory truncation. This lifts to string theory as a compactification on G/Γ where G is a group manifold, in general non-compact, and Γ is a discrete subgroup.

CMH & Reid-Edwards

2) Reduction with Duality Twist   f,H,Q

Compactify on T^n, theory has T-duality symmetry

$$H = O(n,n;\mathbb{Z})$$

Compactify on further circle with H-twist. $\tau(x)$

Geometric twist: twist in $GL(n,\mathbb{Z}) \ltimes \mathbb{Z}^{n(n-1)/2}$

Gives torus bundle over circle.

T-duality twist: gives T-fold   Dabholkar and Hull

3) Asymetric Orbifold   f,H,Q

At special points in moduli space, $\tau = \tau_0$

DUALITY TWIST $\longrightarrow$ ASYMMETRIC ORBIFOLD

$\mathbb{Z}_m \subset O(n,n;\mathbb{Z})$ Symmetry of CFT

Shift on S^1 $x \to x + 2\pi/m$

4) Orbifolds with dual twists

f,H,Q,R

At special points in moduli space,

$\mathbb{Z}_m \times \mathbb{Z}_{\widetilde{m}} \subset O(n,n;\mathbb{Z})$ Symmetry of CFT

Shifts on "doubled" S^1

$$x \to x + 2\pi/m \qquad \widetilde{x} \to \widetilde{x} + 2\pi/\widetilde{m}$$

Fourier Trans to momentum p, winding number w

$\mathbb{Z}_m \times \mathbb{Z}_{\widetilde{m}}$ action on $\Big|p,w\Big\rangle$

$$\Big|p,w\Big\rangle \to \exp\left(2\pi i p/m\right)\exp\left(2\pi i w/\widetilde{m}\right)\Big|p,w\Big\rangle$$

5) Reductions with dual twists

f,H,Q,R

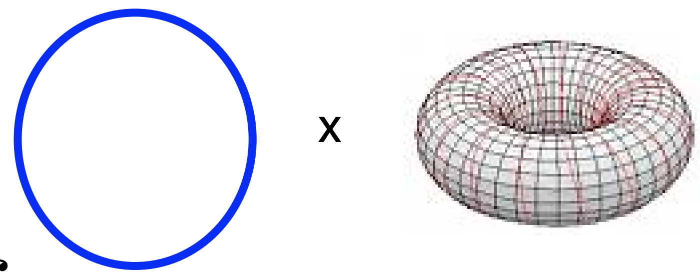
At orbifold points, x-twists reduce to x-shifts.
Suggests that moving away from orbifold points:

$$\begin{array}{ccc} x - \text{shifts} & \rightarrow & x - \text{twists} \\ \widetilde{x} - \text{shifts} & \rightarrow & \widetilde{x} - \text{twists} \end{array}$$

Dependence on dual coordinate!
Not even locally geometric

Example: T^3 with H-flux

Regard as product $S^1 \times T^2$ $H_{xyz} = N$



x y, z

Moduli $\tau, \rho = B_{yz} + iV$

$E = G + B$

$$B_{yz} = B_0 + \frac{1}{2\pi} N x$$

$$\rho = \rho_0 + \frac{1}{2\pi} N x$$

$$E = E_0 + \Omega x \qquad \Omega = \frac{N}{2\pi} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

T-duality in z-direction: gives T^2 bundle over S^1

$$\tau = \tau_0 + \frac{1}{2\pi} N x$$

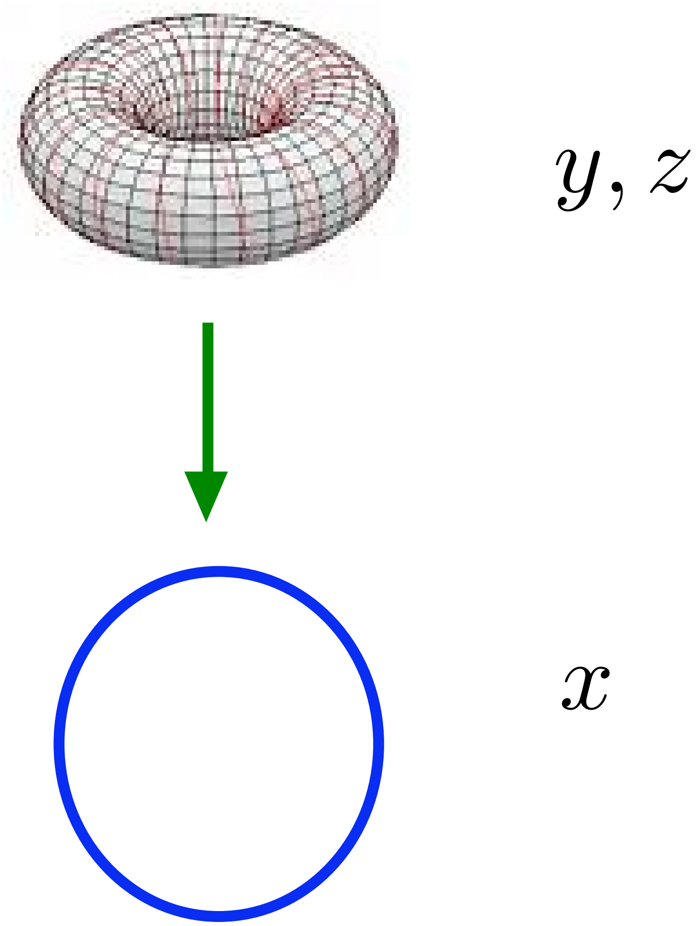
Geometric twist

T-dual on y-circle:

Torus bundle over circle?

$$ds^2 = dx^2 + \frac{1}{1+N^2x^2}\left(dy^2 + dz^2\right)$$

$$B_{yz} = \frac{Nx}{1+N^2x^2}$$



But x periodic

$$E(x+2\pi) = (aE+b)(cE+d)^{-1}$$

Monodromy $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(2,2;Z)$

T-duality

T-fold.

The duality in y-direction gives

$$E(x) = (E_0 + \Omega x)^{-1}$$

T-fold, T-duality twist in x-direction

No isometry in the x-direction.
Can we do the final T-duality?
This would give the T-dual of 3-torus with constant H-flux on all 3-directions. cf. mirror symmetry

The duality in y-direction gives

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T-fold, T-duality twist in x-direction

No isometry in the x-direction.
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T-duality swaps x with dual coordinate $x \to \widetilde{x}$

Suggests twist in $\widetilde{x}$-direction

Dabholkar and Hull

$$E(\widetilde{x}) = (E_0 + \Omega \widetilde{x})^{-1}$$

- Conventional T-duality requires isometry
- Generalisation suggested by doubled geometry
- Replace x dependence with $\widetilde{x}$ dependence
- General backgrounds with dependence on both $x, \widetilde{x}$?
- Not locally geometric, can't use σ-model. Use string field theory

U-Folds

For T^d Fibration, allow $E_d(Z)$ Transitions

Mix Momentum and brane wrapping mode

e.g. T^4, E_4=SL(5)

String: 4 momentum + 4 string winding

4+4 ~ 8 of $O(4,4;\mathbb{Z})$ $\quad T \oplus T^*$

M-Theory: 4 mom^m + 6 membrane wrapping

4+6 ~ 10 of $SL(5;\mathbb{Z})$ $\quad T \oplus \wedge^2 T^*$

SL(5) acts non-linearly on G_{ij}, C_{ijk}

Doubled torus T^8 replaced by M-Torus T^{10}

U-duality changes polarisation, $T^4 \subset T^{10}$.

Symmetries in D Dimensions

D	n	$G = E_n$	H_n	$\dim(E_n)$	$\dim(E_n/H_n)$
9	2	$SL(2,\mathbb{R}) \times \mathbb{R}$	$SO(2)$	4	3
8	3	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$SO(3) \times SO(2)$	11	7
7	4	$SL(5,\mathbb{R})$	$SO(5)$	24	14
6	5	$Spin(5,5)$	$(Sp(2) \times Sp(2))/\mathbb{Z}_2$	45	25
5	6	$E_{6(6)}$	$Sp(4)/\mathbb{Z}_2$	78	42
4	7	$E_{7(7)}$	$SU(8)/\mathbb{Z}_2$	133	70
3	8	$E_{8(8)}$	$Spin(16)/\mathbb{Z}_2$	248	128

Table 1: Symmetries of Maximal Supergravities in $D = 11 - n$ Dimensions. The U-duality groups $G = E_n$, their maximal compact subgroups H_n, and the dimensions of E_n and the cosets E_n/H_n.

Heterotic:G=O(d,d+16), H=O(d)xO(d+16), d=10-D

R-Symmetry: **Double cover** $\hat{H}$ of H

Generalised Geometry

Studies structures on a d-dimensional manifold M on which there is a natural action of O(d,d)

$T \oplus T^*$ Vector + 1-form

$$V = v + \xi \in T \oplus T^*$$

$$V^I = \begin{pmatrix} v^i \\ \xi_i \end{pmatrix}$$

O(d,d) Metric

$$\eta(v+\xi, v+\xi) = 2\iota_v \xi$$

$$\eta = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

O(d,d) $V \to gV$ $g^t \eta g = \eta$

Hitchin

$$\iota_v \xi = v^i \xi_i$$

Generalised Metric $\mathcal{H}_{IJ}$ Gualtieri

Positive definite metric on $T \oplus T^*$ compatible with η

$$\eta^{-1}\mathcal{H}\eta^{-1} = \mathcal{H}^{-1}$$

$S = \eta^{-1}\mathcal{H}$ satisfies $S^2 = \mathbb{1}$ Real structure

Parameterised by $G = G^t, \ B = -B^t$ $E = G + B$

$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}$$

O(d,d):

$\mathcal{H} \to g^t \mathcal{H} g$ $E \to (aE+b)(cE+d)^{-1}$

Parameterise coset $\dfrac{O(d,d)}{O(d) \times O(d)}$

Geometric Subgroup

$$g = \begin{pmatrix} M & 0 \\ \Theta & (M^t)^{-1} \end{pmatrix}$$

$G \to M^t G M, \qquad B \to M^t B M \qquad GL(d,\mathbb{R})$

$B \to B + \Theta$ B-shifts $\Gamma(\mathbb{R}) \subset O(d,d)$

Transition Functions

$T, \ T^*, \ T \oplus T^* \ldots.$

$GL(d,\mathbb{R}) \qquad \dfrac{\partial x'^i}{\partial x^j}$

Symmetries

Generalised Tangent Bundle:

Transition functions $\Gamma(\mathbb{R})$ Θ exact

$B' = B + d\lambda$

Rest of O(d,d) not symms, can't be used as transitions

Suggestive to think of $T \oplus T^*$ as O(d,d) bundle

Really $GL(d,\mathbb{R})$ bundle

Transition Functions $GL(d,\mathbb{R})$ $\frac{\partial {x'}^i}{\partial x^j}$

Generalised Tangent Bundle:

Transition functions $\Gamma(\mathbb{R})$

Rest of O(d,d) not symms, can't be used as transitions

If M has an n-torus fibration, there is an $O(n,n;\mathbb{Z})$ T-duality symmetry. Suggests using this as transition functions in more general space

T-fold

Type I Extended Geometry

Action of O(d,d)

$T \oplus T^*$

cf Brane charges

Generalised metric

G, B_2

$$\mathcal{H}_{IJ} \in \frac{O(d,d)}{O(d) \times O(d)}$$

Type I Extended Geometry

Action of O(d,d)

$T \oplus T^*$

cf Brane charges

Generalised metric

G, B_2 ⟷ $\mathcal{H}_{IJ} \in \dfrac{O(d,d)}{O(d) \times O(d)}$

Type II: O(d,d) ⟶ E_{d+1}, add RR fields

Type I Extended Geometry

Action of O(d,d)

$T \oplus T^*$

cf Brane charges

Generalised metric

G, B_2 ⟷

$$\mathcal{H}_{IJ} \in \frac{O(d,d)}{O(d) \times O(d)}$$

Type M: O(d,d) ⟶ E_d, B_2 ⟶ C_3

$$T \oplus T^* \longrightarrow T \oplus \Lambda^2 T^*$$

Type M Extended Geometry $d \leq 4$

Action of E_d

$T \oplus \Lambda^2 T^*$

cf Brane charges

Generalised metric

G, C_3

$$\mathcal{H}_{IJ} \in \frac{E_d}{H_d}$$

Type M Extended Geometry $d \leq 7$

Action of E_d

$$T \oplus \Lambda^2 T^* \oplus \Lambda^5 T^* \oplus \Lambda^6 T$$

cf Brane charges

$$G, C_3, \tilde{C}_6$$

Generalised metric

$$\mathcal{H}_{IJ} \in \frac{E_d}{H_d}$$

Type M Extended Geometry $d \leq 7$

Action of E_d

$T \oplus \Lambda^2 T^* \oplus \Lambda^5 T^* \oplus \Lambda^6 T$

cf Brane charges

$G, C_3, \tilde{C}_6$ ⟷ Generalised metric

$$\mathcal{H}_{IJ} \in \frac{E_d}{H_d}$$

Extended Tangent Bundle: Structure group E_d
Reducible to H-Bundle, reduction introduces $\mathcal{H}_{IJ}$

Extended Geometry for M-theory, Type II

CMH: hep-th 0701203; Waldram and Pacheco 0804.1362

Suggestive rewrite of familiar structures on manifold

G_{ij}, B_{ij}	$\mathcal{H}_{IJ} \in \dfrac{O(d,d)}{O(d)\times O(d)}$
$J_\pm$	$\mathcal{J}$
$C_0 + C_2 + C_4 + \ldots$	C^+

- Understand general features, prove theorems, construct new examples.....
- Actions: Hitchin functionals
- Natural action of O(d,d) or E_d but not a symmetry
- Discrete subgroups can be symmetry for toroidal fibrations, can be used in transtions

Conclusions

- Duality in general leads to T-folds and other non-geometric backgrounds
- Local spacetime patches, no global spacetime
- Field equations from conformal and Lorentz anomalies. Modular invariance?
- All symmetries in transitions, T- and U-dualities and R-symmetries

- T-folds: momentum & string winding mix
  U-folds: momentum & brane wrapping mix
- Generalised geometry: doubles tangent space O(d,d), not O(d,d;Z) [Hitchin]
  Doubled formalism: doubles spacetime
- M-Theory: Doubled torus e.g. $T^7 \to T^{14}$ becomes M-torus e.g. $T^7 \to T^{56}$. U-duality acts to change polarisation $T^7 \subset T^{56}$
- $\hat{H}$-symmetry transitions allow generalised spinors

- D-branes and supersymmetry: incorporated
- T-fold as "Quantum bundle": bundle of torus CFT's over base.
- Generic solutions of string field theory: fields $\psi(X, \widetilde{X})$ No spacetime even locally?
- General backgrounds, not torus fibrations?
- What is stringy/M geometry? What is string/M theory?