Generalized Geometry and Flux Compactifications

Part I by Paul Koerber

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Contents

- Supersymmetry conditions for compactifications of type II supergravity
- Generalized geometry
- Generalized calibrations

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Motivation

- Supersymmetric compactifications of type II supergravity
  - With fluxes: moduli stabilization
  - With four-dimensional flat Minkowski or AdS_4 factor: uplifting to dS in later phase

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  - Supersymmetry conditions: easier to solve than EOM
  - Supersymmetry conditions and Bianchi's form fields imply all EOM
    IIA: *Lüst,Tsimpis*, IIB: *Gauntlett, Martelli, Sparks, Waldram*
    With sources: *PK, Tsimpis*

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- Relation between susy conditions of type II in the presence of fluxes and GG
- Applications to AdS/CFT:
  find new susy solutions of supergravity
  $\Longrightarrow$ geometric dual of CFT

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Compactification ansatz I

- Metric:

$$ds^2 = e^{2A(y)} g_{(4)\mu\nu}(x) dx^\mu dx^\nu + g_{mn}(y) dy^m dy^n \ ,$$

with $g_{(4)}$ flat Minkowski or AdS$_4$ metric, A warp factor

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- RR-fluxes:
  - Democratic formalism: double fields, impose duality condition
  - Combine forms into one polyform

$$F_{\text{tot}} = \sum_l F_{(l)} = F + e^{4A} \mathrm{vol}_4 \wedge F_{\text{el}} \ , \qquad (F_{\text{el}} = \star_6 \sigma(F))$$

with l even/odd in type IIA/IIB

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Compactification ansatz II

- $N = 1$ ansatz for susy generators:

$$\epsilon^1 = \zeta_+ \otimes \eta_+^{(1)} + \zeta_- \otimes \eta_-^{(1)} \; ,$$
$$\epsilon^2 = \zeta_+ \otimes \eta_\mp^{(2)} + \zeta_- \otimes \eta_\pm^{(2)} \; ,$$

ζ: 4d spinor characterizes preserved susy
$\eta^{(1,2)}$: fixed 6d-spinor, property background

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Compactification ansatz II

- $\eta^{(1,2)}$: fixed 6d-spinor, property background
- Define polyforms

$$\not{\Psi}_{\pm} = -\frac{8i}{||\eta^{(1)}||^2}\eta_+^{(1)} \otimes \eta_\pm^{(2)\dagger}$$

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Fierzing, we find:

$$\Psi_{\pm} = -\frac{i}{||\eta^{(1)}||^2}\sum_l \frac{1}{l!}\,\eta_{\pm}^{(2)\dagger}\gamma_{i_1\ldots i_l}\eta_+^{(1)} dx^{i_l} \wedge \ldots \wedge dx^{i_1}$$

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- Clifford map between polyforms and operators on spinors
- Not every polyform $\Longleftrightarrow$ spinor bilinear, only pure spinors

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- Clifford map between polyforms and operators on spinors
- Not every polyform $\Longleftrightarrow$ spinor bilinear, only pure spinors
- Special case $SU(3)$-structure: $\eta^{(2)} = c\eta^{(1)}$

$$\Rightarrow \Psi_+ = -ic^{-1}e^{iJ}, \quad \Psi_- = \Omega$$

J two-form, Ω holomorphic three-form

Background susy conditions

Graña, Minasian, Petrini, Tomasiello

- Susy conditions type II sugra:

  Gravitino's

$$\delta\psi^1_M = \left(\nabla_M + \frac{1}{4}\not{H}_M\right)\epsilon^1 + \frac{1}{16}e^{\Phi}\not{F}_{\text{tot}}\,\Gamma_M\Gamma_{(10)}\epsilon^2 = 0$$

$$\delta\psi^2_M = \left(\nabla_M - \frac{1}{4}\not{H}_M\right)\epsilon^2 - \frac{1}{16}e^{\Phi}\sigma(\not{F}_{\text{tot}})\,\Gamma_M\Gamma_{(10)}\epsilon^1 = 0$$

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$\Longrightarrow$ can be concisely rewritten as ...

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- Susy equations in polyform notation:

$$\mathrm{d}_H\left(e^{4A-\Phi}\mathrm{Re}\,\Psi_1\right) = e^{4A}F_{\mathrm{el}}\,,$$
$$\mathrm{d}_H\left(e^{3A-\Phi}\Psi_2\right) = 0\,,$$
$$\mathrm{d}_H(e^{2A-\Phi}\mathrm{Im}\,\Psi_1) = 0\,,$$

  for Minkowski.

- F_{el}: external part polyform RR-fluxes, Φ: dilaton, A: warp factor, H NSNS 3-form, $d_H = d + H\wedge$
- $\Psi_1 = \Psi_\mp, \Psi_2 = \Psi_\pm$ for IIA/IIB

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$$\mathsf{d}_H\left(e^{4A-\Phi}\mathrm{Re}\,\Psi_1\right) = (3/R)\,e^{3A-\Phi}\mathrm{Re}\left(e^{i\theta}\Psi_2\right) + e^{4A}F_{\mathsf{el}}\,,$$
$$\mathsf{d}_H\left(e^{3A-\Phi}\Psi_2\right) = (2/R)i\,e^{2A-\Phi}e^{-i\theta}\mathrm{Im}\,\Psi_1\,,$$
$$\mathsf{d}_H(e^{2A-\Phi}\mathrm{Im}\,\Psi_1) = 0\,,$$

  for AdS: $\nabla_\mu\zeta_- = \pm\frac{e^{-i\theta}}{2R}\gamma_\mu\zeta_+$.

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$$d_H F = -j \,,$$

where j represents sources

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  Sources negative tension (orientifolds) necessary *Maldacena, Núñez*
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For AdS_4: solutions without sources possible
- New Minkowski solutions on nilmanifolds/solvmanifolds:
Graña, Minasian, Petrini, Tomasiello; *Andriot*
- New AdS_4 solutions on twistor bundles/coset manifolds:
Tomasiello; *PK, Lüst, Tsimpis*

Classification of structures

$N = 1$ ansatz susy generators:

$$\epsilon^1 = \zeta_+ \otimes \eta_+^{(1)} \, + \, \zeta_- \otimes \eta_-^{(1)}$$
$$\epsilon^2 = \zeta_+ \otimes \eta_\mp^{(2)} \, + \, \zeta_- \otimes \eta_\pm^{(2)}$$

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- Dynamic SU(3)×SU(3)-structure: type may change

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Generalized geometry

Hitchin; *Gualtieri*

- Interpretation of $\Psi_\pm$ in generalized geometry

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- Interpretation of $\Psi_\pm$ in generalized geometry
- Generalized geometry is based on: $TM \oplus T^\star M$
- Comes with natural metric:

$$\mathcal{I}(\mathbb{X}, \mathbb{Y}) = \frac{1}{2}(\eta(X) + \xi(Y))$$

for $\mathbb{X} = (X, \xi), \mathbb{Y} = (Y, \eta) \in \Gamma(TM \oplus T^\star M)$
$\Longrightarrow$ signature (6,6) $\Longrightarrow$ SO(6,6)-structure

Spinors of Spin(6,6)

- Action of generalized tangent bundle on polyforms:

$$\mathbb{X} \cdot \Psi = \iota_X \Psi + \xi \wedge \Psi \,,$$

for $\mathbb{X} = (X, \xi)$

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- Clifford algebra:

$$(\mathbb{X} \cdot \mathbb{Y} + \mathbb{Y} \cdot \mathbb{X}) \cdot \Psi = 2\,\mathcal{I}(\mathbb{X}, \mathbb{Y})\,\Psi$$

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- Spinor bilinear: Mukai pairing

$$\phi_1^T C \phi_2 = \langle \phi_1, \phi_2 \rangle = \phi_1 \wedge \sigma(\phi_2)|_{\text{top}}$$

Pure spinors

- Null space of polyform:

$$N_\Psi = \{\mathbb{X} \in \Gamma(TM \oplus T^\star M) : \mathbb{X} \cdot \Psi = 0\}$$

$\Longrightarrow$ isotropic: $\mathcal{I}(\mathbb{X}, \mathbb{Y}) = 0$ for all $\mathbb{X}, \mathbb{Y} \in N_\Psi$

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- Pure spinor $\Longleftrightarrow$ null space maximal
- Pure spinor $\Longleftrightarrow$ Spin(6)-spinor bilinear

Generalized almost complex structure

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$$\mathcal{J} : TM \oplus T^{\star}M \to TM \oplus T^{\star}M$$

so that

$$\begin{aligned} \mathcal{J}^2 &= -\mathbb{1} \\ \mathcal{I}(\mathcal{J}\mathbb{X}, \mathcal{J}\mathbb{Y}) &= \mathcal{I}(\mathbb{X}, \mathbb{Y}) \end{aligned}$$

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- Almost complex structure & symplectic structure examples

$$\mathcal{J}_J = \begin{pmatrix} J & \mathbf{0} \\ \mathbf{0} & -J^T \end{pmatrix} , \qquad \mathcal{J}_\omega = \begin{pmatrix} \mathbf{0} & \omega^{-1} \\ -\omega & \mathbf{0} \end{pmatrix}$$

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Generalized complex structure

- Generalized complex structure integrable if
  L_+ involutive: $[L_+, L_+]_H \subset L_+$
  where the H-twisted Courant bracket:

$$[X+\xi, Y+\eta]_H = [X,Y] + \mathcal{L}_X\eta - \mathcal{L}_Y\xi - \frac{1}{2}\mathrm{d}(\iota_X\eta - \iota_Y\xi) + \iota_X\iota_Y H$$

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- Properties Courant bracket:
  - Projects nicely to Lie bracket

$$\pi([\mathbb{X},\mathbb{Y}]_H) = [\pi(\mathbb{X}), \pi(\mathbb{Y})]$$

  - Under B-transform (off-diagonal part of SO(6,6))

$$e^B(X+\xi) = X + (\xi + \iota_X B)$$

  it transforms covariantly:

$$[e^B\mathbb{X}, e^B\mathbb{Y}]_{H+\mathrm{d}B} = e^B[\mathbb{X},\mathbb{Y}]_H$$

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Relation pure spinor and generalized almost complex structure

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- B-transform:

$$\mathbb{X} \longleftrightarrow e^{-B}\mathbb{X} \Longrightarrow [\cdot,\cdot]_H \longleftrightarrow [\cdot,\cdot]$$

corresponds to

$$\Psi \longleftrightarrow e^{B}\Psi \Longrightarrow \mathrm{d}_H \longleftrightarrow \mathrm{d}$$

$\Longrightarrow$ we can choose to work with H or B

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Susy conditions revisited

- Pure spinors Ψ_1, Ψ_2
  satisfying compatibility relation $\langle \Psi_2, \mathbb{X} \cdot \mathrm{Re}\,\Psi_1 \rangle = 0$
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- SO(6,6) structure reduces to SU(3)×SU(3)

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$\Longrightarrow$ exceptional generalized geometry *Hull*; *Waldram, Pacheco*

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Calibrations:

- A way to find minimal volume submanifolds in a curved space
- Second-order equations $\Rightarrow$ first-order equations
- Analogous to self-duality solves Yang-Mills equations
- Or more generally BPS equations solve equations of motion

Generalized calibrations:

- Submanifold $\Sigma \Longrightarrow$ D-brane $(\Sigma, \mathcal{F})$
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- In fact: extend self-duality YM to higher dimensions, combine with calibrations

Calibrations II

Calibration form ϕ:

- $d\phi = 0$ (1) (differential property)
- Bound: $\sqrt{g}|_{T_p} \geq \phi|_{T_p}$ (2) (algebraic property)
  for every subspace T_p of tangent space at point p
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Calibration forms from invariant spinors: e.g. $\Omega, \frac{1}{k!}J^k$ in CY

Generalized calibrations

PK; *Martucci, Smyth*

We have:

- bulk fluxes H and F
- $\mathcal{F}$ on the D-brane, where $\mathcal{F} = B + 2\pi\alpha' F_{\text{WV}}$ such that $\text{d}\mathcal{F} = H$

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- Corresponds to supersymmetric D-brane

Natural calibration forms

Martucci, Smyth

- Calibration forms are the polyforms:

$$\omega^{\mathsf{sf}} = e^{4A-\Phi}\mathrm{Re}\,\Psi_1\,,$$
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- Differential property is provided by the bulk susy equations:

$$d_H\big(e^{4A-\Phi}\mathrm{Re}\,\Psi_1\big) = e^{4A}F_{\mathsf{el}}\,, \qquad \text{space-filling D-brane}$$
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- Spoiled in the AdS case: interpretation *PK, Martucci*

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- Definition: $j_{(\Sigma,\mathcal{F})}$

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- Real pure spinor: null space is generalized tangent bundle *Gualtieri*

$$T_{(\Sigma,\mathcal{F})} = \{X + \xi \in T_\Sigma \oplus T^\star_M|_\Sigma : P_\Sigma[\xi] = \iota_X \mathcal{F}\}$$

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D-flatness and F-flatness conditions

Focus on space-filling D-brane
Saturating bound consists of two parts

- $e^{-\Phi}\sqrt{g+\mathcal{F}}|_\Sigma = e^{i\alpha}e^{4A-\Phi}\Psi_1|_\Sigma \wedge e^{\mathcal{F}}$
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  $\Rightarrow (\Sigma, \mathcal{F})$ is generalized complex submanifold with respect to $\mathcal{J}_2$
  This becomes an F-flatness condition in the 4d-effective theory

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- For interpretation bulk susy conditions as F- and D-flatness: see part II

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Conclusions

- Generalized geometry organizes supersymmetry conditions of type II with fluxes
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- Part II: 4D effective theory, susy breaking

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End of part I . . . Part II by Martucci

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