Heterotic Compactifications and Phenomenology

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Vienna, September 2008

Overview

- Effective supergravities in d=10 and d=11
- Calabi-Yau compactifications
- Calabi-Yau model building
- Moduli stabilisation
- Beyond Calabi-Yau manifolds
- Conclusion

Effective supergravities in d=10 and d=11

Weakly coupled theory in d=10

N=1 supergravity multiplet: $(g_{AB}, \phi, B_{AB}, \lambda, \psi_A)$

d=10 SYM multiplet: (A^a_B, χ^a), gauge group $E_8 \times E_8$ or $SO(32)$

Weakly coupled theory in d=10

focus mostly on
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NS form field strength: $$\begin{aligned} H &= dB + \alpha' \left(\omega_{\rm L} - \omega_{\rm YM}\right) \\ dH &= \alpha' \left(\mathrm{tr}(F\wedge F) - \mathrm{tr}(R\wedge R)\right) \end{aligned}$$

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gauge invariant — transforms under YM and L

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(Bosonic) action as α' expansion and $g_S = e^{\phi}$:

$$S_{10} = -\frac{1}{2\kappa_{10}^2} \int \Big[\sqrt{-g}\, R + 4 d\phi \wedge \star d\phi + \frac{1}{2} e^{-\phi} H \wedge \star H$$
$$+\alpha' e^{-\phi/2} \left(\mathrm{tr}\, F^2 - \mathrm{tr}(R^2)\right)\Big]$$
$$+\mathrm{fermions} + \mathcal{O}({\alpha'}^2)$$

fermion supersymmetry transformations:

$$\begin{aligned}
\delta\psi_A &= D_A\eta + \frac{1}{96}e^{-\phi}\left(\Gamma_A^{BCD} - 9\delta_A^B\Gamma^{CD}\right)H_{BCD}\eta + \mathrm{fermi}^2 \\
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Some higher order terms: $\sim \frac{1}{\alpha'}\int\sqrt{-g}\left(t_8^2 - \frac{1}{2\sqrt{2}}e^{(10)}B\right)W_8$

W_8 is a quartic polynomial in R and F

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Branes: only NS two-form B, so string and NS 5-brane

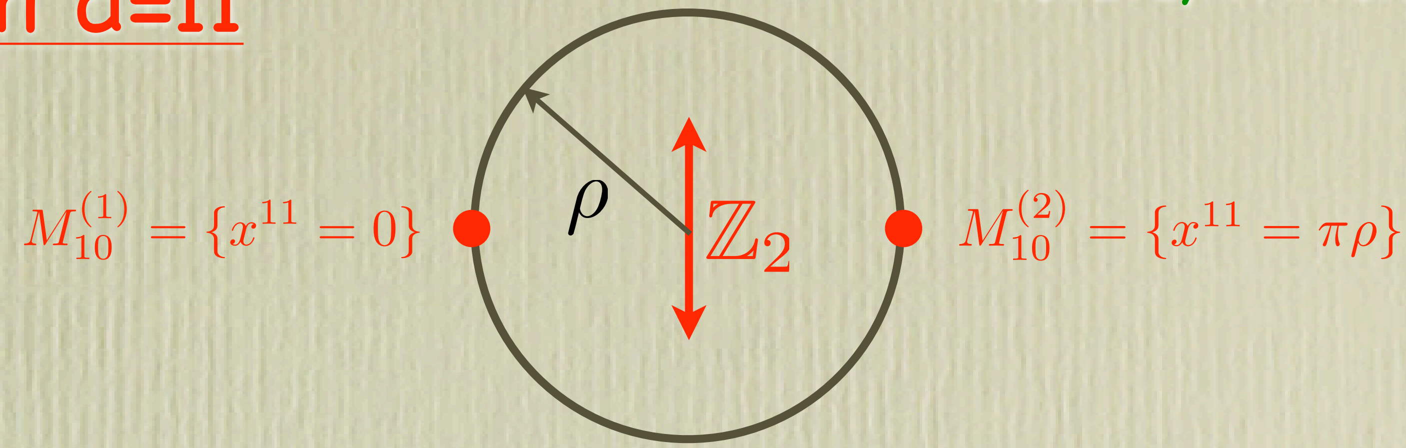
5-brane world-volume M_5: $dH = \alpha'\left(\mathrm{tr}(F\wedge F) - \mathrm{tr}(R\wedge R) + \delta(M_5)\right)$

Strongly coupled theory in d=11

Horava, Witten '96

M-theory on

$$M_{11} = S^1/\mathbb{Z}_2 \times M_{10}$$



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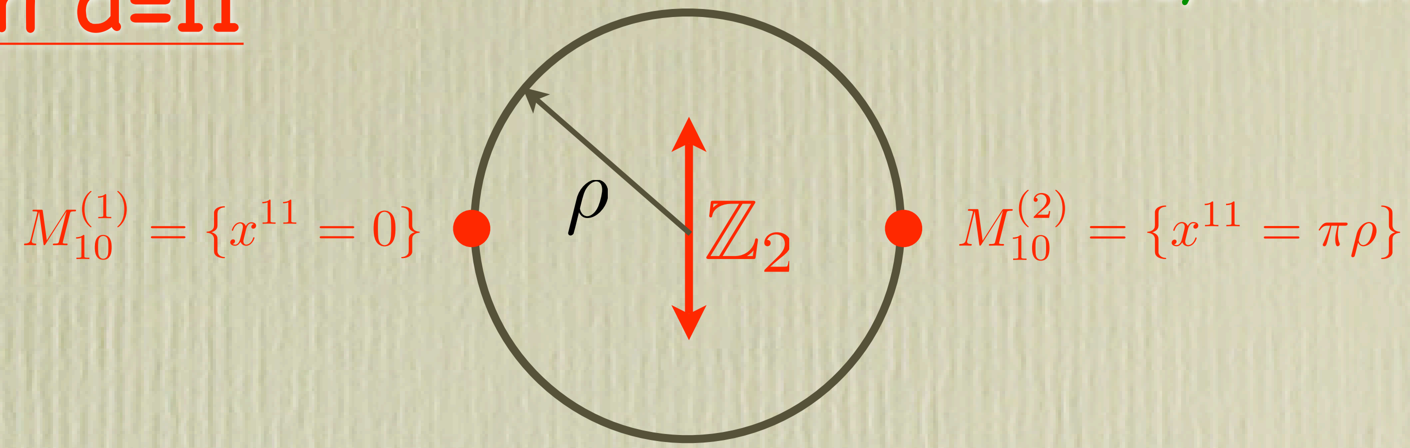

d=11, N=1 bulk supergravity multiplet: $(g_{IJ}, \psi_I, C_{IJK})$

$\mathbb{Z}_2$ even: $g_{AB}, g_{11,11}, \psi_A, C_{11AB}$ $\quad$ $\mathbb{Z}_2$ odd: $g_{A11}, \psi_{11}, C_{ABC}$

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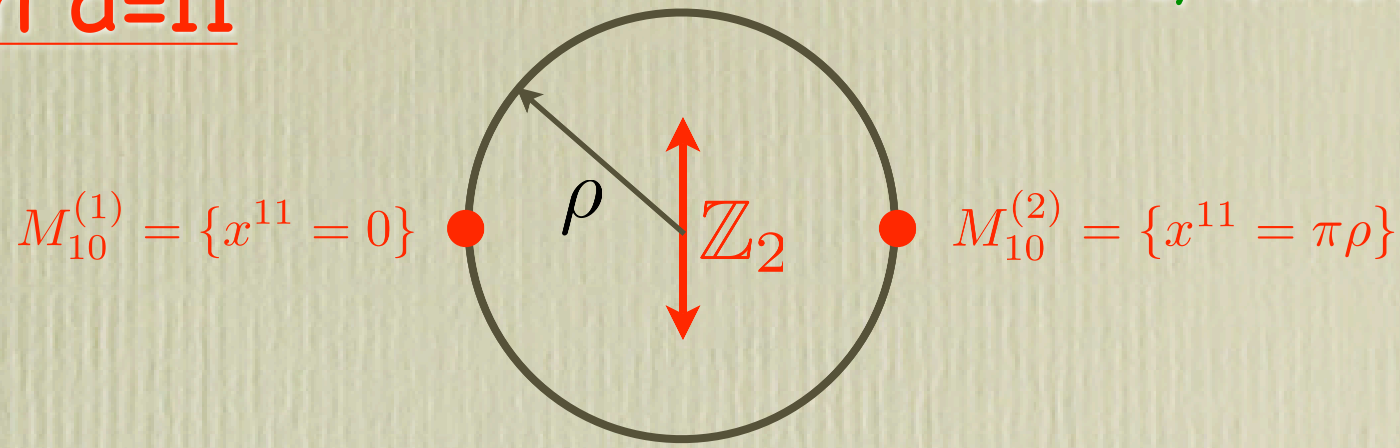

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Two d=10, N=1 E_8 SYM multiplets at $M_{10}^{(1)}$ and $M_{10}^{(2)}$:

$(A_B^{(1)}, \chi^{(1)a})$ and $(A_B^{(2)}, \chi^{(2)a})$

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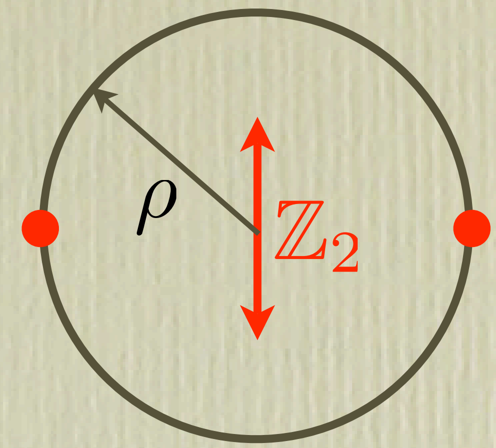
$M_{10}^{(1)} = \{x^{11} = 0\}$  $M_{10}^{(2)} = \{x^{11} = \pi\rho\}$

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Four-form field strength: $dG \sim \kappa^{2/3}\left(J^{(1)} \wedge \delta(x^{11}) + J^{(2)} \wedge \delta(x^{11} - \pi\rho)\right)$

$$J^{(i)} = \mathrm{tr} F^{(i)} \wedge F^{(i)} - \frac{1}{2} \mathrm{tr} R \wedge R$$

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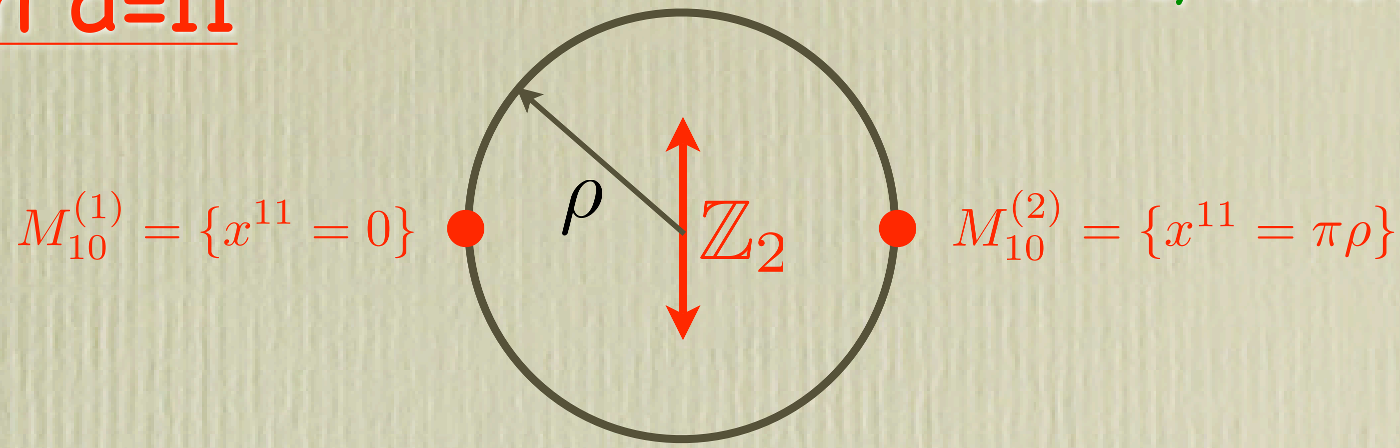

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$\kappa^{2/3}$ plays role similar to α'. Three-form C transforms under YM and L.

bosonic action as expansion in $\kappa^{2/3}$:

$$S_{\text{bulk}} = -\frac{1}{2\kappa^2}\int \left[\sqrt{-g}\,R + \frac{1}{2}G \wedge \star G + \frac{1}{6}C \wedge G \wedge G + \text{fermions} + \mathcal{O}(\kappa^{4/3})\right]$$

$$S_{\text{YM}} = -\frac{1}{4\lambda^2}\sum_{i=1}^{2}\int_{M_{10}^{(i)}} \sqrt{-g_{10}}\,\left[\text{tr}F_{(i)}^2 - \frac{1}{2}\text{tr}R^2 + \text{fermions} + \mathcal{O}(\kappa^{2/3})\right]$$

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Some higher order (bulk) terms: $\sim \kappa^{2/3}\int_{M_{11}}\sqrt{-g}\left(t_8^2 - \frac{1}{2\sqrt{2}}e^{(11)}C\right)X_8$

X_8 quartic polynomial in R

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Branes: membrane and M 5-brane coupling to three-form C.

$$dG \sim \kappa^{2/3}\left(J^{(1)}\wedge\delta(x^{11}) + J^{(2)}\wedge\delta(x^{11}-\pi\rho) + \delta(M_5)\right)$$

Relation between d=11 strong and d=10 weak coupling

Witten '95

Small ρ limit of d=11 theory: no zero modes for odd fields g_{A11}, C_{ABC}

$$\begin{aligned} ds^2 &= e^{-\phi/6} ds_{10}^2 + e^{4\phi/3} dx_{11}^2 \\ B_{AB} &= C_{AB11} \end{aligned}$$

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Lalak, Lukas, Ovrut '97

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membrane supersymmetric along x^{11} -> d=10 string

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Lukas, Ovrut, Waldram '98

higher order terms: integrating out

$$G_{ABCD} \sim \kappa^{2/3} \left(f_1(x_{11}) J^{(1)} + f_2(x_{11}) J^{(2)} \right)$$

plus X_8 produces all d=10 terms in W_8

Calabi-Yau compactifications

Background solutions in d=10

Candelas et al. '85

For a supersymmetric background one need to satisfy Killing spinor eqs.

$$\delta\psi_A = D_A\eta + \mathcal{O}(H) = 0\,, \quad \delta\lambda \sim \partial\phi\eta + \mathcal{O}(H) = 0\,, \quad \delta\chi \sim F_{AB}\Gamma^{AB}\eta = 0$$

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Simplest choice: $\phi = \text{const}\,, \quad H = 0\,, \quad ds^2_{10} = dx^\mu dx^\nu \eta_{\mu\nu} + 2g_{a\bar{b}}dz^a d\bar{z}^b$

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Standard embedding: no 5-branes and $F = R$: H remains zero

In general: Expansion in α'/R^2_{CY} and g_S , corrections to CY metric

Background solutions in d=11

Witten '96

Background with N 5-branes at $y = x^{11} = y_1, \ldots, y_N$ wrapping holomorphic curves in X

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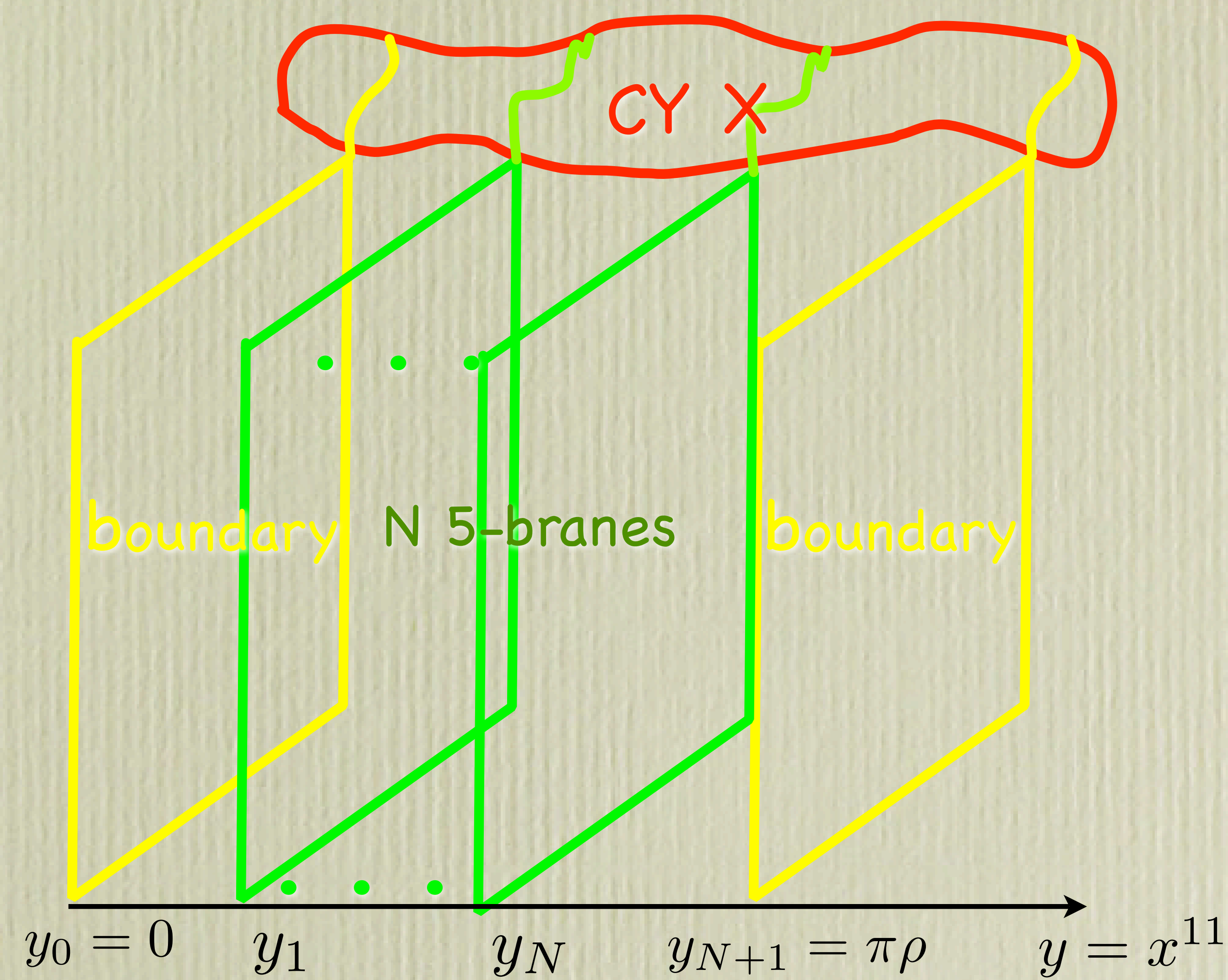

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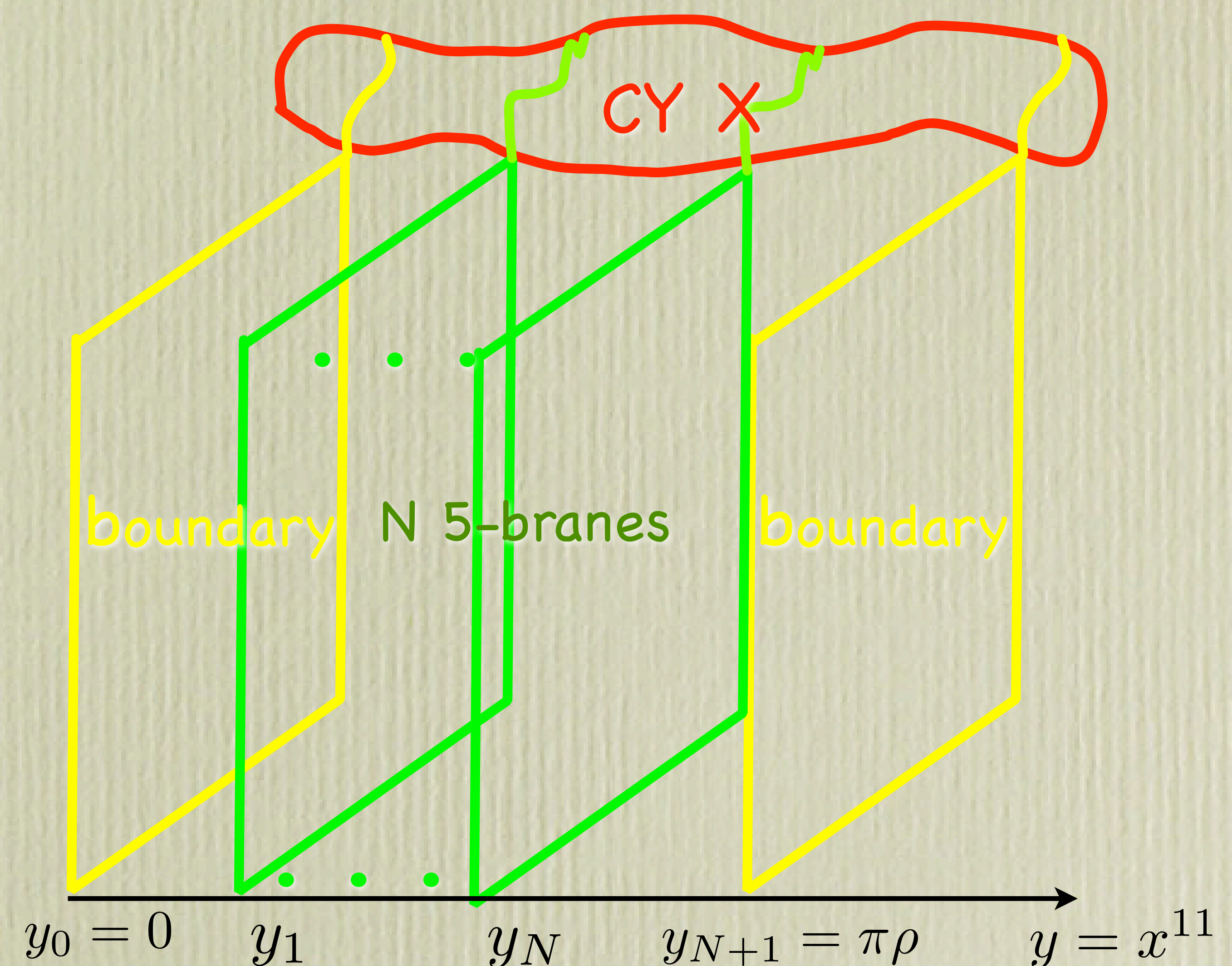

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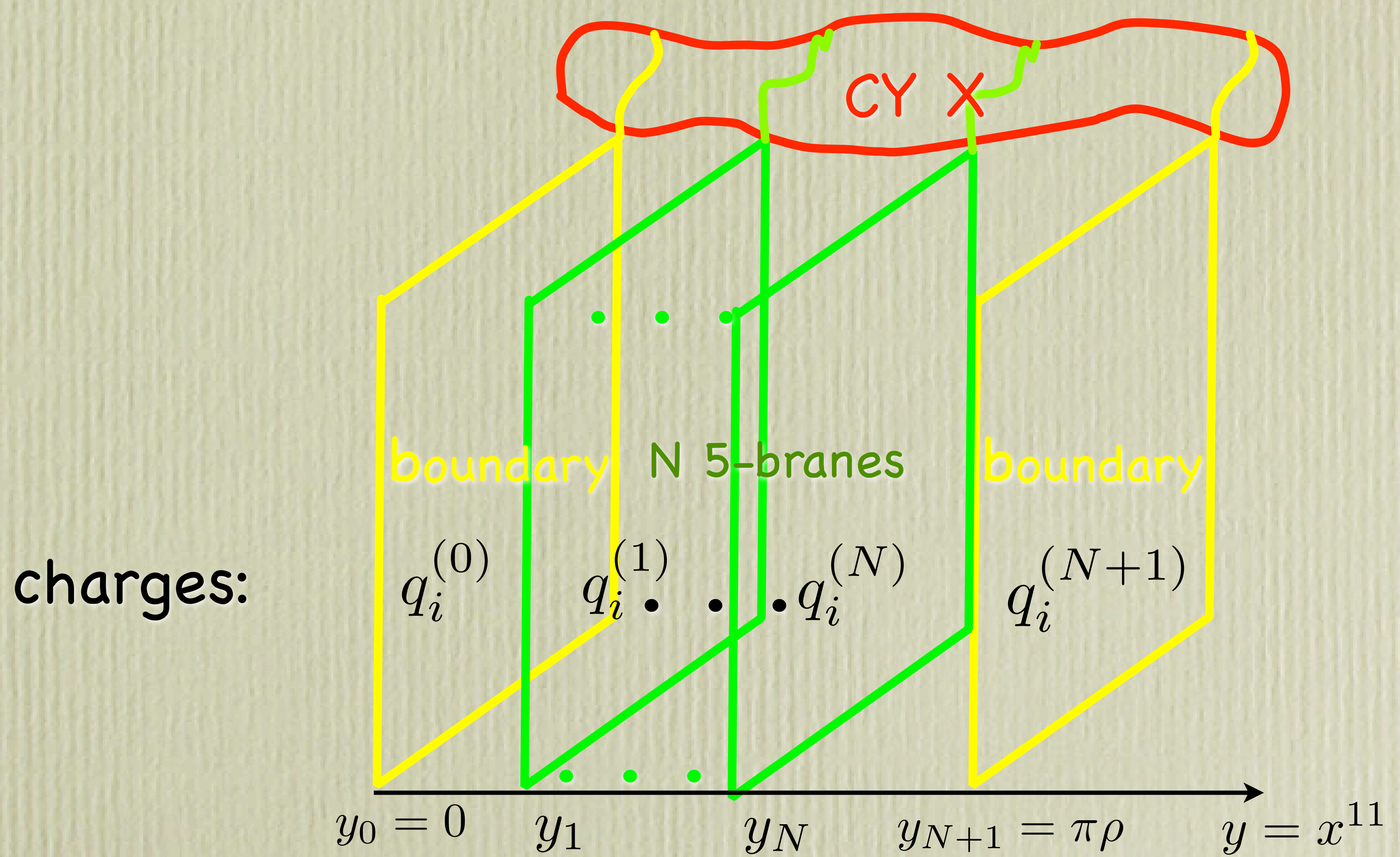

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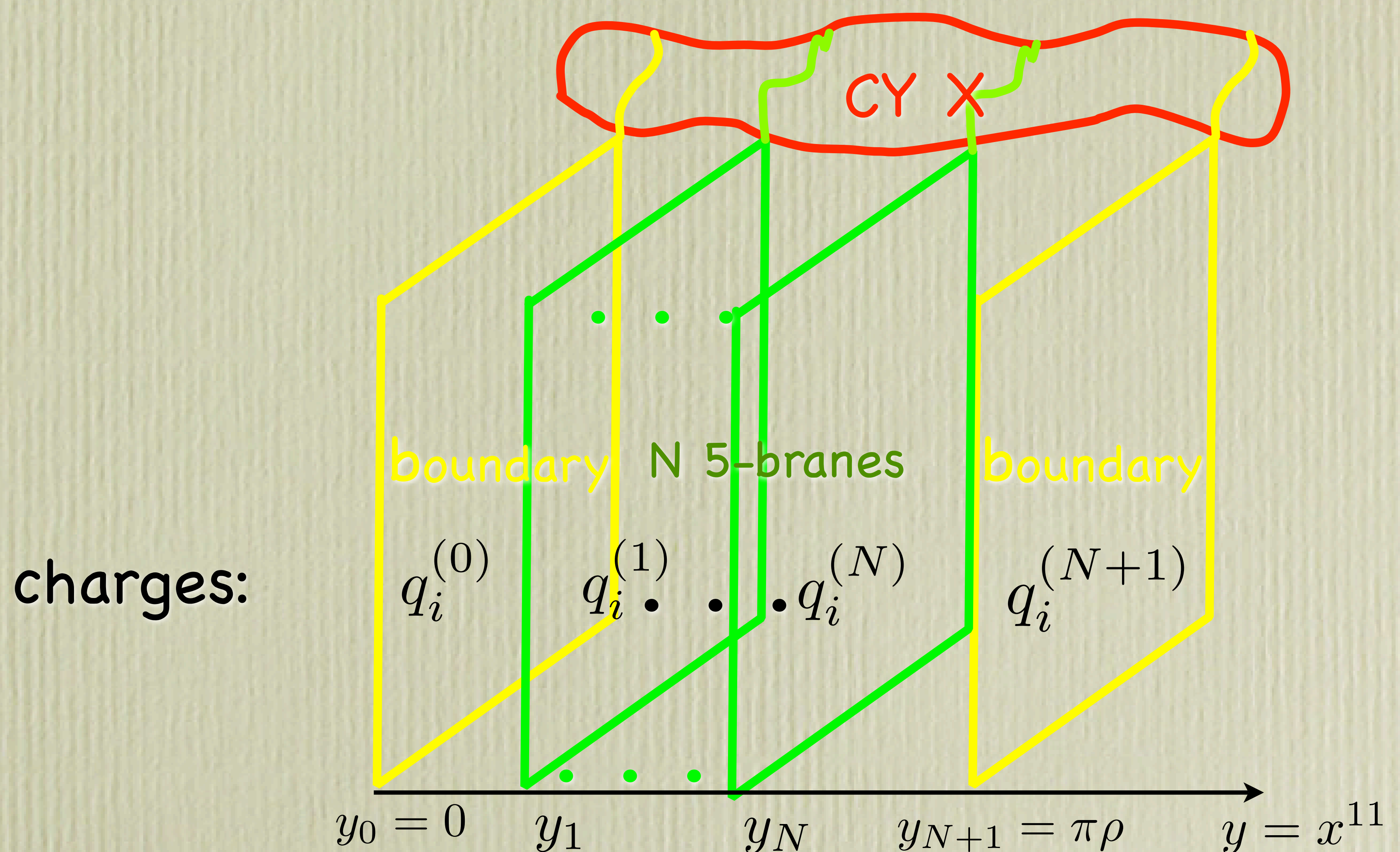

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So, main difference: inherent flux G_{ABCD} and warping along x^{11}

Solution can be written in terms of (1,1) form

Lukas, Ovrut, Waldram '98

$$\mathcal{B}_{a\bar{b}} = b_i(y, \mathbf{q})\,\omega^i_{a\bar{b}} + \text{massive modes} \qquad \mathcal{B} = 2\omega^{a\bar{b}}\mathcal{B}_{a\bar{b}} \qquad \omega_{a\bar{b}} = ig_{a\bar{b}}$$

The b_i are linear in y and depend on charges $q_i^{(n)}$

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Explicit solution:

$$ds^2 = (1 + \tfrac{\sqrt{2}}{6}\mathcal{B})dx^\mu dx^\nu \eta_{\mu\nu} + \left(g_{a\bar{b}} + \sqrt{2}i(\mathcal{B}_{a\bar{b}} - \tfrac{1}{3}\omega_{a\bar{b}}\mathcal{B})\right) dz^a d\bar{z}^{\bar{b}} + (1 - \tfrac{\sqrt{2}}{3}\mathcal{B})dx_{11}^2$$

$$G = \star_6 d\mathcal{B}$$

Solution can be written in terms of (1,1) form

Lukas, Ovrut, Waldram '98

$$\mathcal{B}_{a\bar{b}} = b_i(y, \mathbf{q})\, \omega^i_{\ a\bar{b}} + \text{massive modes} \qquad \mathcal{B} = 2\omega^{a\bar{b}}\mathcal{B}_{a\bar{b}} \qquad \omega_{a\bar{b}} = i g_{a\bar{b}}$$

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Solution is expansion in:

$$\epsilon_S \sim \kappa^{2/3}\frac{R_{11}}{V_{\mathrm{CY}}^{2/3}}$$

strong-coupling expansion parameter, controls warping $b_i = \mathcal{O}(\epsilon_S)$

$$\epsilon_R \sim \frac{V_{\mathrm{CY}}^{1/6}}{R_{11}}$$

controls massive modes

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For $\epsilon_S \to 1$ one looses control of supergravity ($\kappa^{2/3}$ expansion) and typically one E_8 becomes strongly coupled.

For $\epsilon_R \to 1$ the effect of massive modes becomes important.

A simple moduli space in $S_R = \mathrm{Re}(S)$ and $T_R = \mathrm{Re}(T)$:

$$S_R \sim V_{\mathrm{CY}} \qquad\qquad T_R \sim R_{11} V_{\mathrm{CY}}^{1/3}$$

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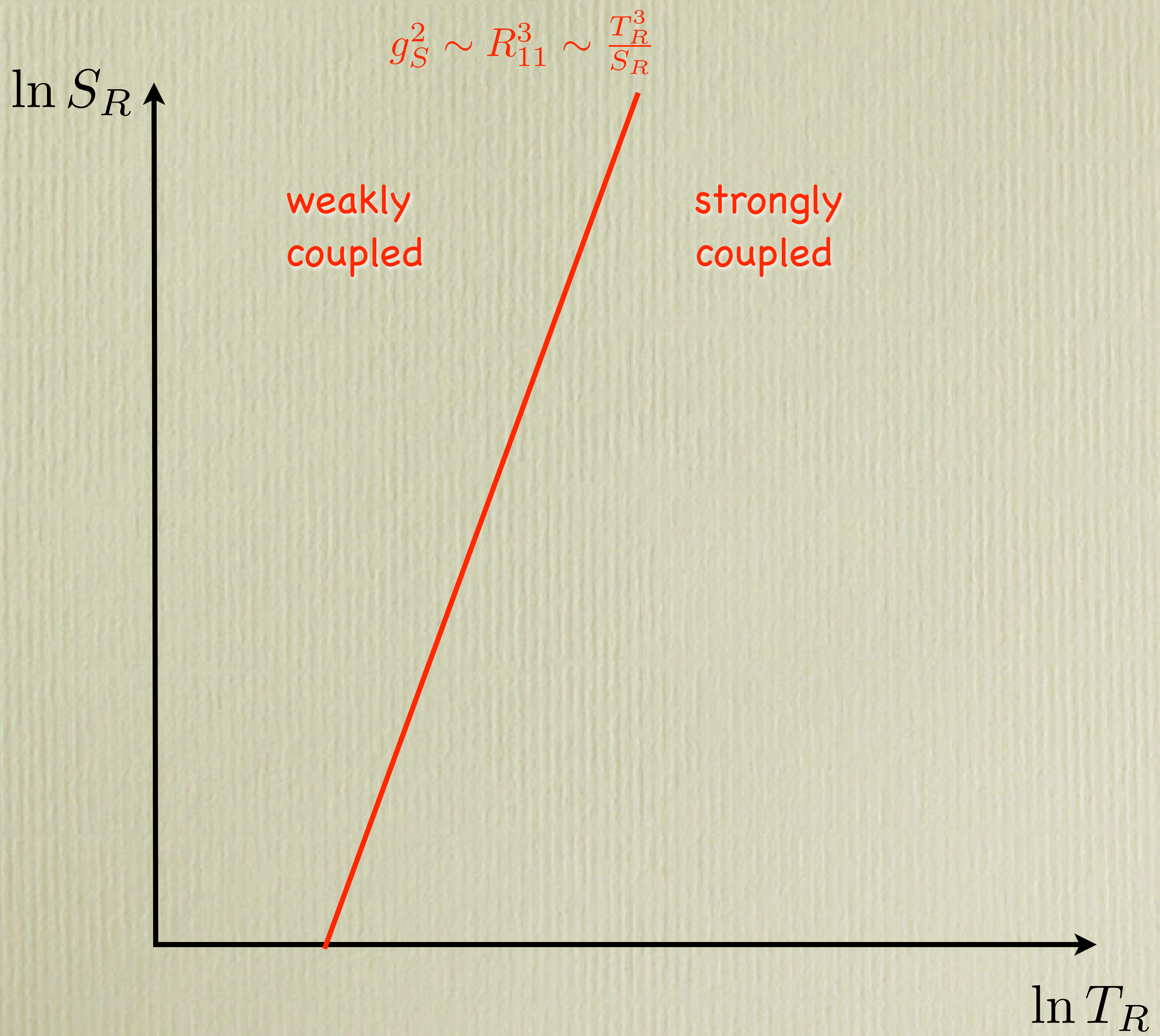

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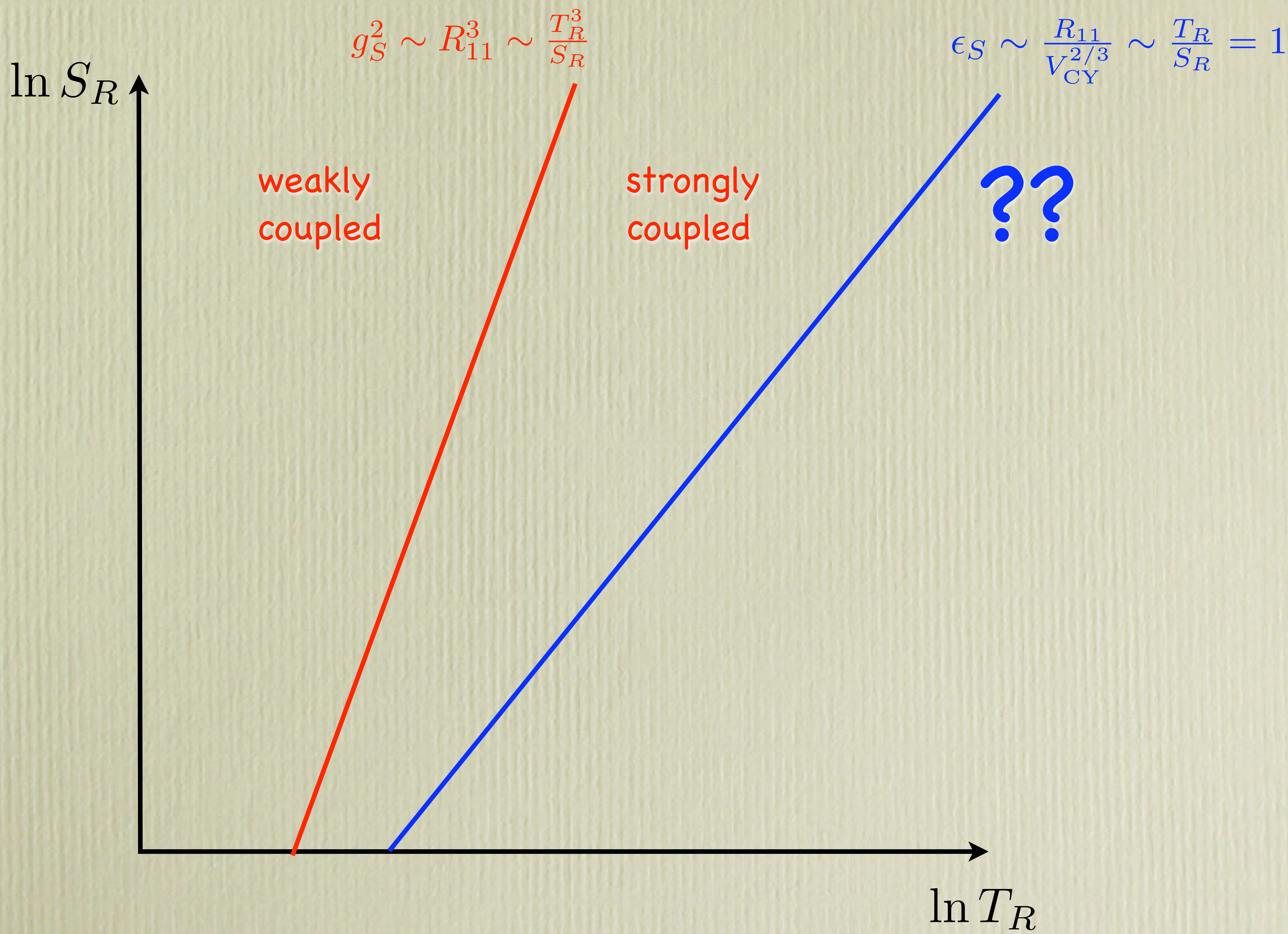

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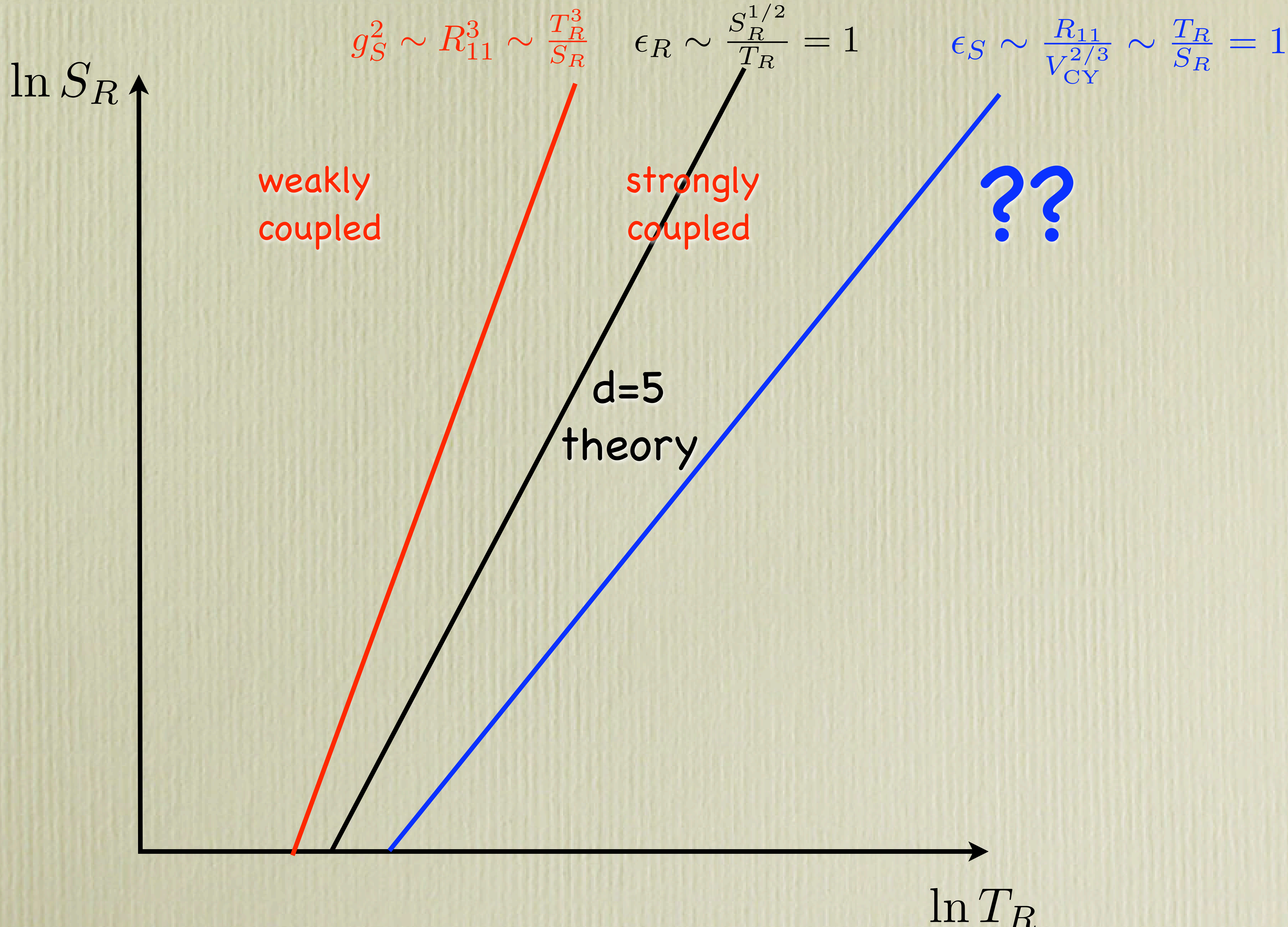

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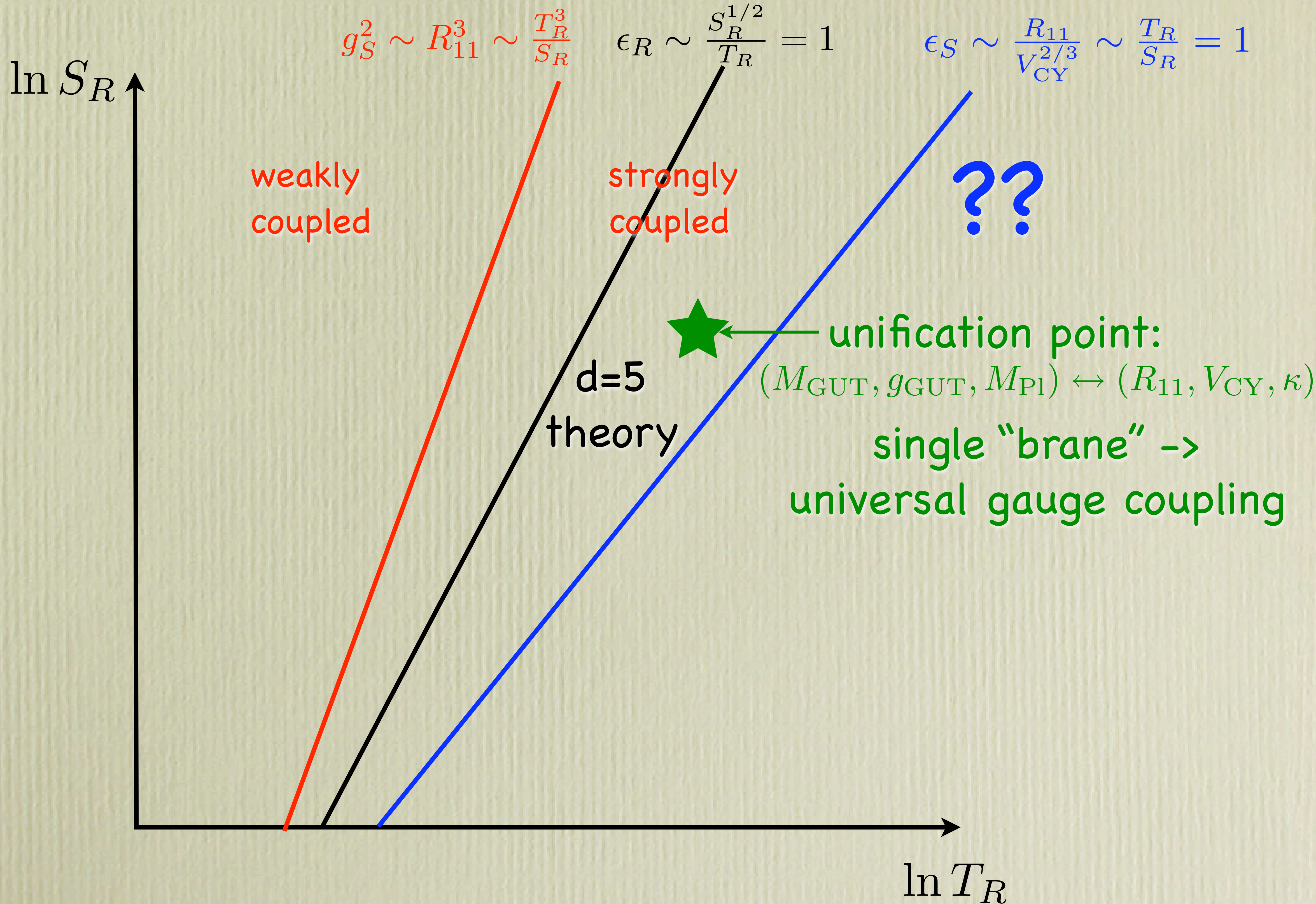
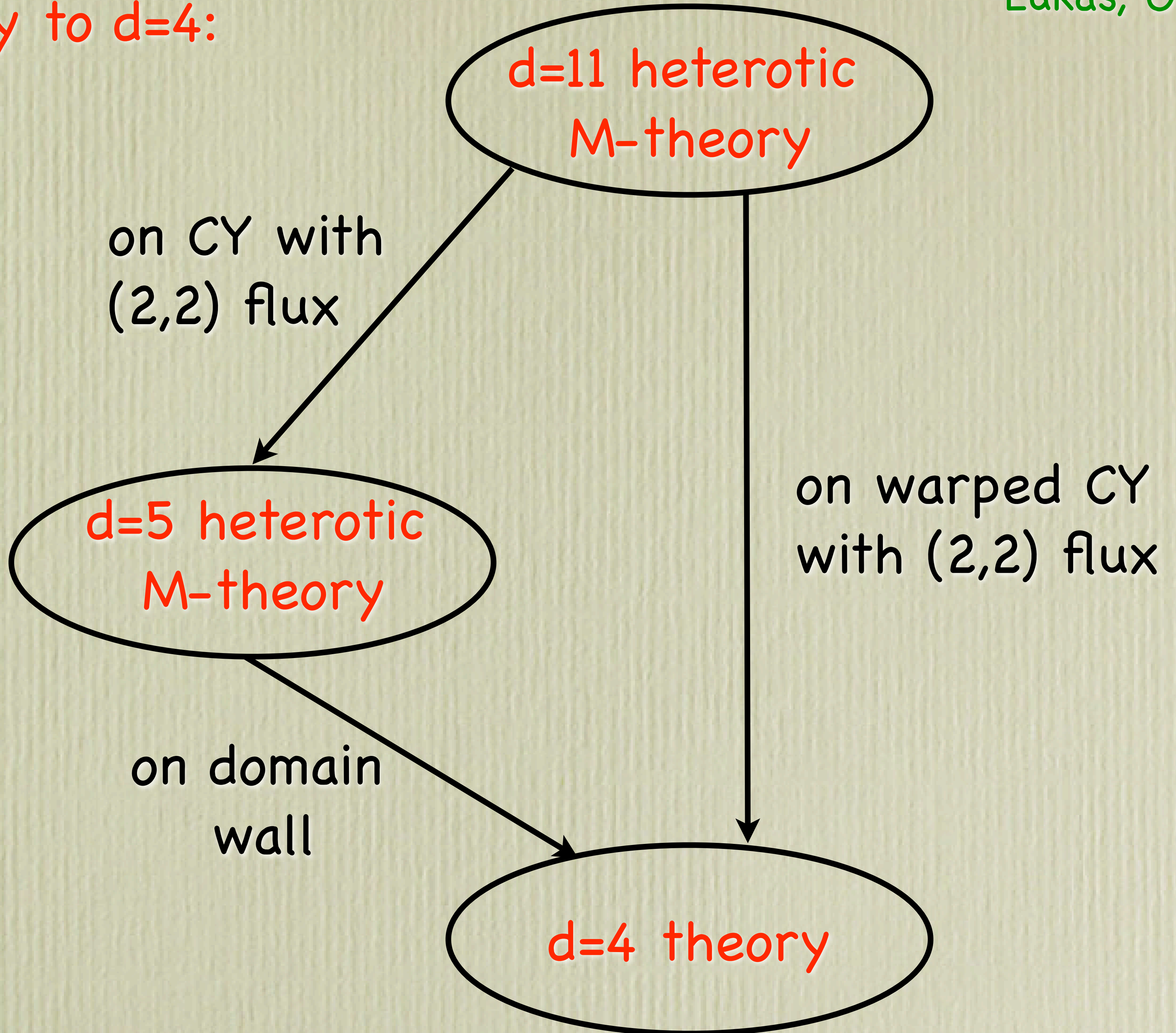


The way to d=4:

Lukas, Ovrut, Stelle, Waldram '98



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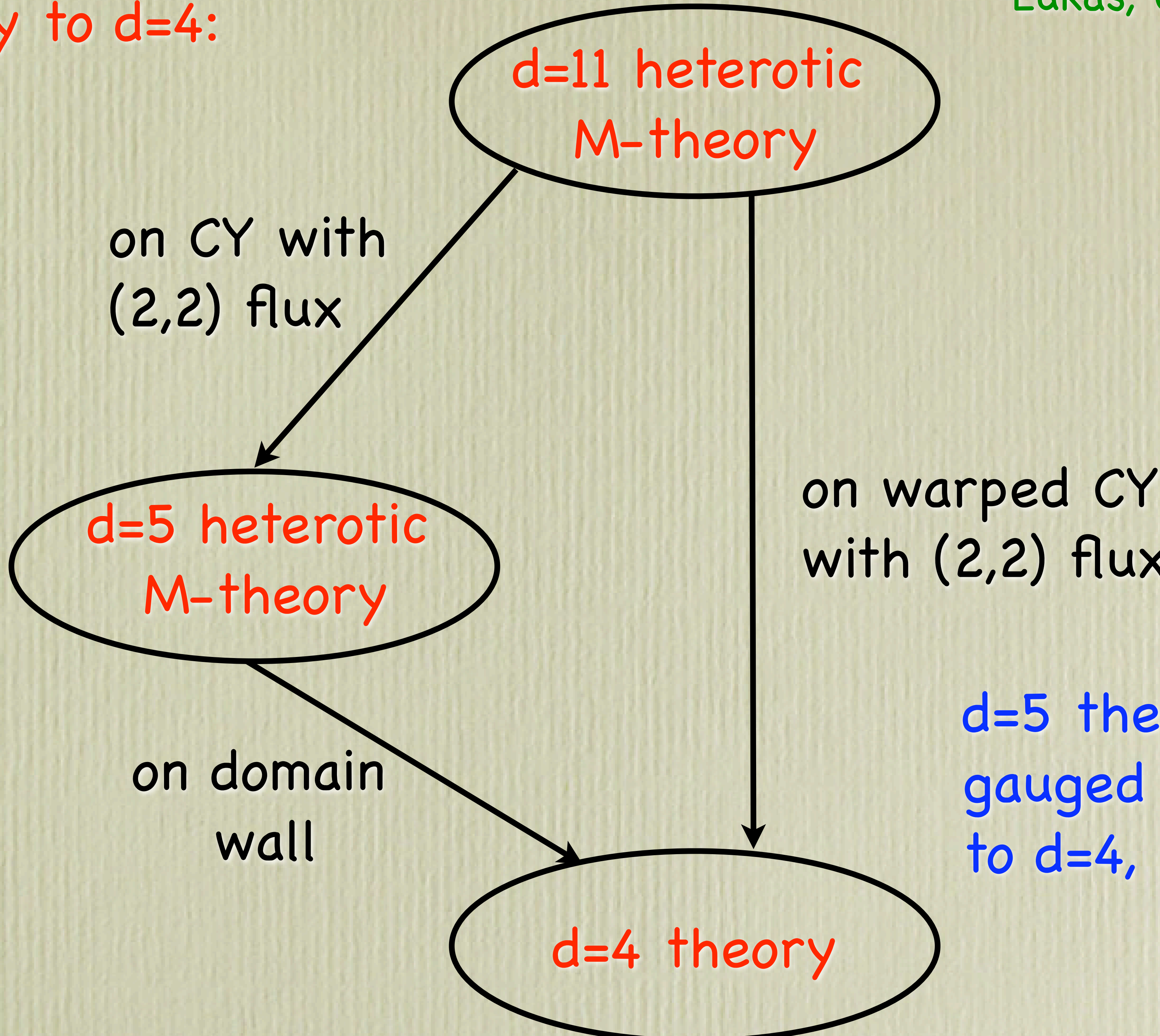

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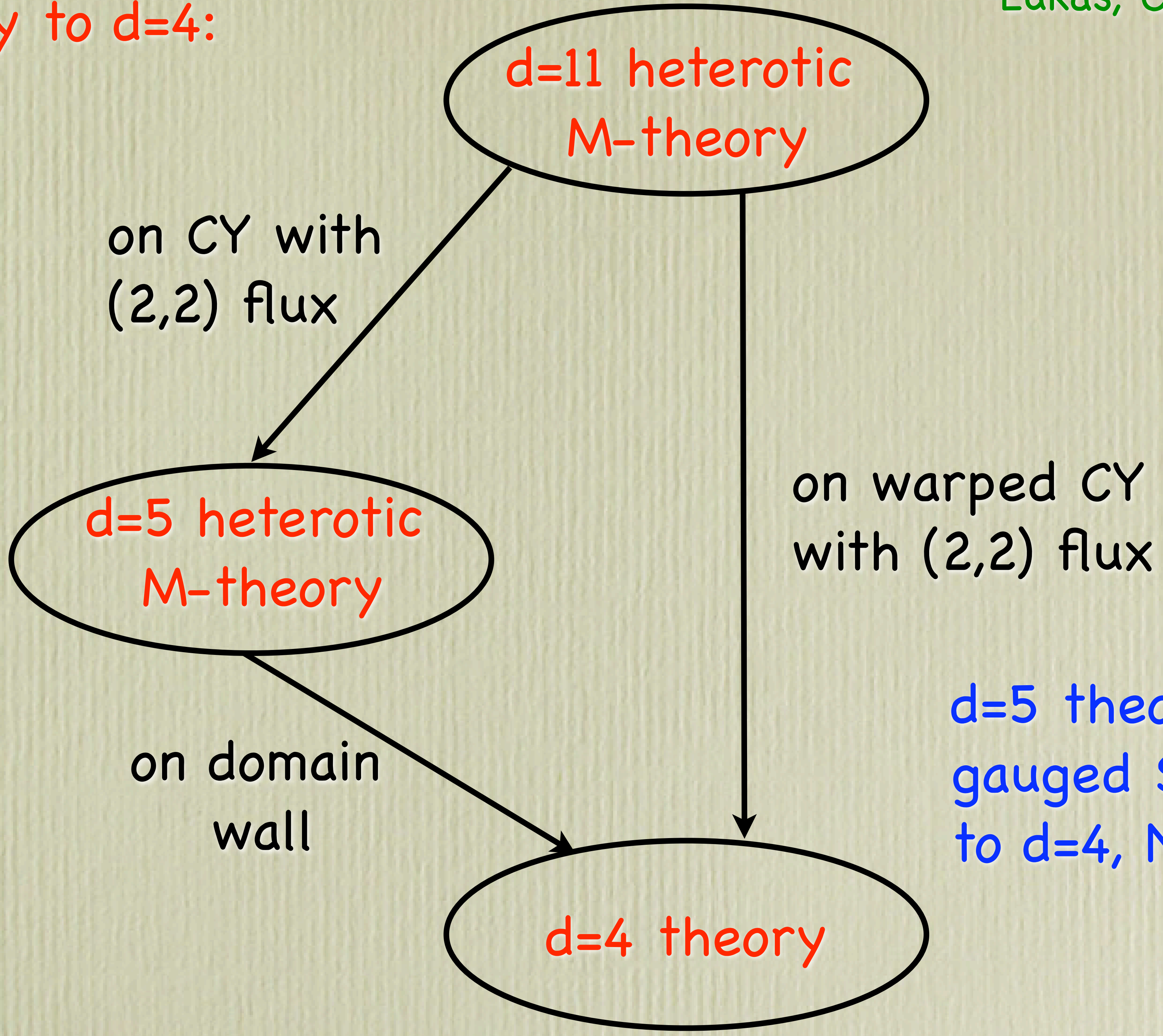

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Difference between d=10 and d=11 CY compactifications: accessible part of moduli space and possible existence of d=5 intermediate theory. Otherwise, the same!

Generic form of the d=4, N=1 effective theory:

vector multiplets: hidden and observable gauge multiplets with gauge groups $H_1 \subset E_8$ and $H_2 \subset E_8$

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plus fields localised on 5-branes

Kahler potential to leading order:

$$K = K_{\mathrm{dil}} + K_{(1,1)} + K_{(2,1)} + K_{\mathrm{bundle}} + Z_{x\bar{y}} C^x \bar{C}^{\bar{y}}$$

$$K_{\mathrm{dil}} = -\log\left(S + \bar{S} + \sum_{n=1}^{N} \frac{(Y^n + \bar{Y}^n)^2}{q_i^n (T^i + \bar{T}^i)}\right)$$

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everything calculable for explicit models, except

Calabi-Yau model building

General framework

Ricci-flat metric g on 6-dimensional manifold $\xleftrightarrow{\text{Yau's theorem}}$ Calabi-Yau three-fold X

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So, necessary data: CY 3-fold X, holomorphic vector bundles V_1, V_2 on X and $W \in H_2(X, \mathbb{Z})$ subject to the three conditions:

anomaly cancellation: $\mathrm{ch}_2(V_1) + \mathrm{ch}_2(V_2) - \mathrm{ch}_2(TX) = W$ from Bianchi identity

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What is stability?

Slope of a bundle (coherent sheaf) $\mathcal{F}$: $\mu(\mathcal{F}) = \frac{1}{\mathrm{rk}(\mathcal{F})} \int_X c_1(\mathcal{F}) \wedge J \wedge J$
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Stability of bundles is usually hard to prove!

Model-building basics

Choose "observable" bundle V with structure group $G = \mathrm{SU}(n) \subset E_8$, where $n = 3, 4, 5$ such that $c_2(TX) - c_2(V) \in \text{Mori cone of } X$

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$\mathrm{SU}(4) \times \mathrm{SO}(10)$	$\mathbf{248} \to (\mathbf{1}, \mathbf{45}) \oplus (\mathbf{4}, \mathbf{16}) \oplus (\overline{\mathbf{4}}, \overline{\mathbf{16}}) \oplus (\mathbf{6}, \mathbf{10}) \oplus (\mathbf{15}, \mathbf{1})$
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$\mathrm{SU}(4) \times \mathrm{SO}(10)$	$\mathbf{248} \to (\mathbf{1}, \mathbf{45}) \oplus (\mathbf{4}, \mathbf{16}) \oplus (\overline{\mathbf{4}}, \overline{\mathbf{16}}) \oplus (\mathbf{6}, \mathbf{10}) \oplus (\mathbf{15}, \mathbf{1})$
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low-energy gauge fields

families, anti-families and Higgs

Model-building basics

Choose "observable" bundle V with structure group $G = \mathrm{SU}(n) \subset E_8$, where $n = 3, 4, 5$ such that $c_2(TX) - c_2(V) \in \text{Mori cone of } X$

Then anomaly constraint can be satisfied by a suitable 5-brane curve (but hidden bundle or combination of hidden bundle and 5-branes may be possible).

families and anti-families

E_8 breaking and group structure

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singlets (bundle moduli)

computing the particle spectrum

Decomposition	Cohomologies
$SU(3) \times E_6$	$n_{27} = h^1(V), n_{\overline{27}} = h^1(V^*) = h^2(V), n_1 = h^1(V \otimes V^*)$
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Alternatively, use $\mathrm{U}(n)$ bundles. (Distler and Green '88, Blumenhagen et al. '06)

Which CYs and which bundles?

- Complete intersections in $\mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_m}$

  (Hubsch, Green, Lutken, Candelas... '87)

Monad bundles

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Focus on these! Looking for systematic, algorithmic approach to apply to large numbers.

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Complete classification of about 8000 spaces.

Intersections of polynomial zero-loci in ambient space $\mathcal{A} = \bigotimes_{r=1}^{m} \mathbb{P}^{n_r}$ with Kahler forms $J_1, \ldots, J_m$

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Using spectral sequences and tensor methods we can calculate the cohomology $h^q\left(X, \mathcal{O}_X(\mathbf{k})\right)$ of all line bundles!

Monad bundles

Definition: A monad bundle V on X defined by short exact sequence

$$0 \to V \to B \xrightarrow{f} C \to 0 \qquad \text{(hence } V = \mathrm{Ker}(f) \text{)}$$

where $B = \bigoplus_{i=1}^{r_B} \mathcal{O}_X(\mathbf{b}_i)\,, \quad C = \bigoplus_{a=1}^{r_C} \mathcal{O}_X(\mathbf{c}_a)$ and $\mathbf{c}_a > \mathbf{b}_i$.

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Properties: $n = \mathrm{rank}(V) = r_B - r_C \stackrel{!}{\in} \{3,4,5\}$

$$c_1^r(V) = \sum_i b_i^r - \sum_a c_a^r \stackrel{!}{=} 0$$

$$c_{2r}(V) = \frac{1}{2} d_{rst} \left(\sum_i b_i^s b_i^t - \sum_a c_a^s c_a^t\right) \stackrel{!}{\leq} c_{2r}(TX)$$

$$c_3(V) = \frac{1}{3} d_{rst} \left(\sum_i b_i^r b_i^s b_i^t - \sum_a c_a^r c_a^s c_a^t\right)$$

Some results for monad bundles on CICYs

(Anderson, He, Lukas '07)

- There is a finite number of positive monads (that is, $b_i^r > 0, c_a^r > 0$), about 7000 on 36 CICYs. We have found those explicitly.

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- The number of semi-positive monads $b_i^r \geq 0, c_a^r \geq 0$ is apparently infinite but may become finite after taking into account equivalences. They can be stable and the spectrum can be computed.

Calculating Yukawa couplings for monad bundles (Anderson, Gray, Grayson, He, Lukas)

Discuss E_6 but method works for other cases as well.

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$\mathbf{27}^3$ Yukawa couplings: $\lambda_{xyz} = \int_X \epsilon_{abc} u_x^a \wedge u_y^b \wedge u_z^c \wedge \Omega$

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Integral can be explicitly evaluated for standard embedding but it seems difficult to do this for more general bundles.

I would like to discuss an algebraic and practical way of computing λ_{xyz} for all monad bundles.

How does this work?

Positive monad defined by

$$0 \to V \to \bigoplus_{i=1}^{r_B} \mathcal{O}_X(\mathbf{b}_i) \xrightarrow{f} \bigoplus_{a=1}^{r_C} \mathcal{O}_X(\mathbf{c}_a) \to 0$$

Long exact sequence:

$$\begin{array}{lllllll} 0 & \to & H^0(X,V) & \to & H^0(X,B) & \to & H^0(X,C) \\ & \to & H^1(X,V) & \to & H^1(X,B) & \to & H^1(X,C) \\ & \to & H^2(X,V) & \to & H^2(X,B) & \to & H^2(X,C) \\ & \to & H^3(X,V) & \to & H^3(X,B) & \to & H^3(X,C) \quad \to 0 \end{array}$$

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Zero since B and C are positive.

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Zero since B and C are positive. Zero since V is stable.

Positive monad defined by

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$$\begin{array}{lllllllll} 0 & \to & 0 & \to & H^0(X,B) & \to & H^0(X,C) & & \\ & \to & H^1(X,V) & \to & 0 & \to & 0 & & \\ & \to & H^2(X,V) & \to & 0 & \to & 0 & & \\ & \to & 0 & \to & 0 & \to & 0 & \to & 0 \end{array}$$

Zero since B and C are positive. Zero since V is stable.

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$$Q_1 \cdot Q_2 \cdot Q_3 = \lambda(Q_1, Q_2, Q_3) P$$

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Yukawa coupling obtained by multiplying 3 quartics.

Moduli stabilisation

Moduli and moduli superpotential

Dine et al '85,.........

moduli: dilaton S

Kahler T^i

cmpl. str. Z^A

5-brane Y^n

bundle M

Even in "good" cases the number of these fields is O(10).

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string/membrane instantons

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If flux does not work how are complex structure moduli stabilised?

Beyond Calabi-Yau manifolds

Half-flat mirror manifolds

Gurrieri, Louis, Micu, Waldram '02

		2-forms	3-forms
mirror pair	X	ω_i	(α_A, β^B)
	$\tilde{X}$	$\tilde{\omega}_a$	$(\tilde{\alpha}_I, \tilde{\beta}^J)$

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$X_{\mathbf{e}}$ has SU(3) structure, half-flat, same 2- and 3-forms as X, and:

$$
\begin{aligned}
J &= t^i \omega_i \\
\Omega &= Z^A \alpha_A - \mathcal{F}_A \beta^A
\end{aligned}
$$

$$
\begin{aligned}
d\omega_i &= e_i \beta^0 \\
d\alpha_0 &= e_i \tilde{w}^i \\
d\alpha_0 &= d\beta^A = 0
\end{aligned}
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Heterotic on half-flat mirror manifolds

Gurrieri, Lukas, Micu '04, '07

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Still difficult to obtain $W_{\mathrm{flux}}|_{\mathrm{minimum}} \ll 1$

Generalised half-flat manifolds

D'Auria et al '04, Grana et al '06

$$\begin{aligned} d\omega_i &= P_{Ai}\beta^A - q_i^A \alpha_A \\ d\alpha_A &= p_{Ai}\tilde{\omega}^i \\ d\beta^A &= q_i^A \alpha_A \end{aligned}$$

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Small W_{flux} now possible with some effort...

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A value of Re(S) compatible with gauge unification arises for about 1 in 1000 flux choices...

Conclusion

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- Attractive features from a particle model building viewpoint, in particular: gauge unification, 16 of SO(10)

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- If we want truly realistic models (Yukawa couplings, masses,..) then, given the general lack of intuition for the finer properties, we need to be able to construct and analyse large numbers of models and filter out promising ones.

Heterotic Calabi-Yau compactifications

- Attractive features from a particle model building viewpoint, in particular: gauge unification, 16 of SO(10)

- Model building is mathematically involved due to presence of vector bundles, progress depends on (physicists) understanding the mathematics better.

- One can find models with a spectrum close to the MSSM.

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- To be able to analyse (physical) Yukawa couplings one needs to find a way to compute the matter field Kahler metric.

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Thanks!