

SUSY breaking in F-theory GUTs

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A Case for String Effective Field Theories

In the present times of the LHC, whilst experimentalists are still struggling with superconducting magnets and cooling systems, we still have some time to come up with **predictions for physics beyond the Standard Model**.

For the purpose of studying the regime accessible to the LHC ($O(10)$ TeV) **effective field theories** (EFT) or local models [Aldazabal, Ibanez, Quevedo, Uranga], [Gray, He, Jejjala, Nelson], [Verlinde, Weijnholt]... of some sort are sufficient.

String EFT = String-motivated SUSY GUTs \supset MSSM
with SUSY-breaking/mediation, depending on a cut-off scale

In this talk: constructed from **7-branes in F-theory**

Drawback: Not full-fledged string compactifications **yet**.

Parameters, need to be determined by UV completion, or experiment

Advantage: compared to other EFTs **well-defined how to UV complete**.

F-theory EFTs

In this talk we will explore the String EFT landscape of **F-theory 7-branes**.

F-theory [Vafa] = Type IIB [Green, Schwarz] vacua with varying axio-dilaton:

$$\tau = C_0 + ie^{-\phi}$$

F-theory on $\mathbb{R}^{1,3} \times X_4$ where X_4 = elliptically fibered Calabi-Yau fourfold with three-fold base B_3 :

$$\begin{array}{ccc} \mathbb{E}_\tau & \rightarrow & X_4 \\ & & \downarrow \\ & & B_3 \supset S \end{array}$$

Loci $S \subset B$ where fibres degenerate	\Leftrightarrow	Surfaces S wrapped by 7-branes in B
ADE singularity type of degeneration	\Leftrightarrow	Gauge group G_S of 7-branes on S

Locally in the vicinity of the 7-brane extending along $\mathbb{R}^{1,3} \times S$ the CY4 is modeled by a local K3-fibration

$$\begin{array}{ccc} K3 & \rightarrow & X_4 \\ & & \downarrow \\ & & S \end{array}$$

Singularity type of the K3-surface: ADE

G_S : Perturbative type

A_n : IIB with D7-branes

D_n : IIB orientifolded with D7-branes and O-planes

E_n : no perturbative IIB picture, "exceptional 7-branes"

F-theory GUT models [Donagi,Wijnholt] [Beasley et al]

Idea: Use this framework of 7-branes to engineer SUSY GUT models.

Gauge fields: surface S wrapped by 7-branes

GUT gauge group G_S

Bifundamental Chiral Matter: lives at intersection of surfaces: Σ

Interactions: Triple-intersection of three curves in a point p .

1.) S =del Pezzo dP_8 : decoupling of M_{GUT} and M_{Pl}

2.) Cut-off scale: $M_{GUT} \sim 10^{16}$ GeV set by $Vol(S) \sim \frac{1}{M_{GUT}^4}$.

GUT breaking flux on S sets mass-scale $F_S \sim M_{GUT}^2$, which is measured in $1/\text{Length}^2$ on S .

Gauge coupling $\alpha_{GUT} \sim \frac{1}{Vol(S)M_*^4}$, $M_* = 7\text{-brane-tension}$.

String EFT/local model building philosophy: first construct EFT in local string setup. This will provide a class of EFT that can then potentially be embedded into full-fledged compact string models.

1-page summary: Complete Local Models

Ingredient 1: BHV $SU(5)$ GUT model without exotics. This is our starting point of investigation.

A complete model requires: SUSY-breaking sector and mediation.

Ingredient 2: Construct SUSY breaking sector

⇒ Introduce scales by non-perturbative D3-instanton effects [HMSS-NV]

⇒ Generate Polonyi model $W = e^{-S_{inst}} X$

Ingredient 3: Gauge mediation in F-theory GUTs [MSS-N]

Final result: Complete model of GUT, SUSY-breaking and mediation, which has many phenomenologically interesting features ("Sweet-spot" of Ibe-Kitano) [MSS-N]

Future: Embed into compact model.

Plan

1. F-theory GUTs: Review
2. Supersymmetry breaking and D3-instanton effects
3. Gauge-mediation in F-theory GUTs: the Sweet-Spot
4. Conclusions and Outlook

1. F-theory GUTs Review of [DW], [BHV]

Effective theory of 7-branes wrapped on $\mathbb{R}^{1,3} \times S$ with $S = \text{del Pezzo}$:

S Kähler \Rightarrow unique **partially twisted** $\mathcal{N} = 1$ G_S SYM in $\mathbb{R}^{1,3} \times S$

Twist combines $U(1)_R$ with $U(1) \subset U(2)$ of

$$SO(1,7) \times U(1)_R \rightarrow SO(1,3) \times U(2) \times U(1)_R$$

After twist: bulk superpotential for adjoint fields

$$W_S = \int_S \text{Tr} (\bar{\partial} A + A \wedge A) \wedge \Phi$$

$A = (0,1)$ -form (bulk gauge field)

$\Phi = (2,0)$ -form (chiral superfields) \Rightarrow **zero-modes vanish on dP_n**

Resulting theory: BPS equations for gauge field restricted to S :

$$F_S^{(0,2)} = F_S^{(2,0)} = 0, \quad J \wedge F_S = 0$$

$J =$ Kähler form on S .

Gauge-bundles and chiral spectrum

Fields on S transform in adjoint of G_S .

Switch on **susy gauge bundles \mathcal{E} with structure group H_S** , breaks adjoint

$$\mathfrak{g}_S \rightarrow \mathfrak{h}_S \oplus \mathfrak{g} \oplus \bigoplus_i \rho_i \otimes \sigma_i$$

Chiral spectrum in representation σ_i of \mathfrak{g} is determined by

$$\#_{\sigma_i} = -\chi_S(\mathcal{R}_i), \quad \mathcal{R}_i = \text{bundle transforming in } \rho_i$$

Vanishing theorems on del Pezzo imply: **Yukawa couplings between three fields on S vanish**. Seek other source of matter!

Example: $SU(5)$ GUT [DW], [BHV]

$G_S = SU(5)$ with $U(1)_Y$ hypercharge gauge bundle

$$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$$

$$\mathbf{24} \rightarrow (\mathbf{8}, \mathbf{1})_0 \oplus (\mathbf{1}, \mathbf{3})_0 \oplus (\mathbf{1}, \mathbf{1})_0 \oplus (\mathbf{3}, \bar{\mathbf{2}})_{-5} \oplus (\bar{\mathbf{3}}, \mathbf{2})_{+5}$$

Gauge Fields

Exotics

Choose $U(1)$ gauge bundle such that

$$\chi_S(\mathcal{L}^{\pm 5}) = 0$$

Denote generators of

$$H_2(dP_8, \mathbb{Z}) : \quad \begin{cases} H & = \text{hyperplane class} \\ E_i & = \text{exceptional classes} \end{cases}$$

Then following choice will remove exotics: $\mathcal{L}^5 = \mathcal{O}(E_i - E_j)$, $i \neq j$.

$$\left[\chi_S(\mathcal{L}) = 1 - \frac{1}{2}c_1(\mathcal{L}) \cdot \mathcal{K}_S + \frac{1}{2}(c_1(\mathcal{L})^2 - 2c_2(\mathcal{L})) \quad \text{and} \quad \mathcal{K}_S = -3H + \sum_i E_i \right]$$

Chiral matter from curves

Consider: two del Pezzo surfaces S_1 and S_2 with G_i gauge group.

$S_1 \cap S_2 = \Sigma =$ Riemann surface

\Rightarrow Bifundamental matter localized on Σ

\Rightarrow Gauge group enhances to G_Σ

$$\mathfrak{g}_\Sigma \rightarrow \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \bigoplus_i (\rho_i^1, \rho_i^2)$$

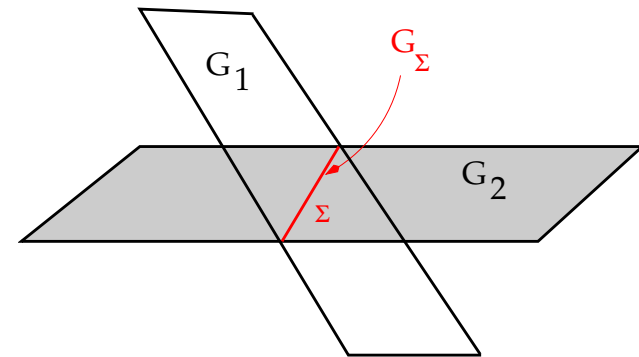
E.g. $G_1 = SU(5)$, $G_2 = U(1)$:

$$G_\Sigma = SU(6) \rightarrow SU(5) \times U(1)$$

$$35 \rightarrow 24_0 \oplus 1_0 \oplus 5_6 \oplus \bar{5}_{-6}$$

Adjoints Bifundamentals

$5 \oplus \bar{5}$ are the bifundamental matter fields localized at Σ . Group theoretic analysis has direct reflection in colliding geometric singularities. [Johansen]



Chiral matter from Curves (continued)

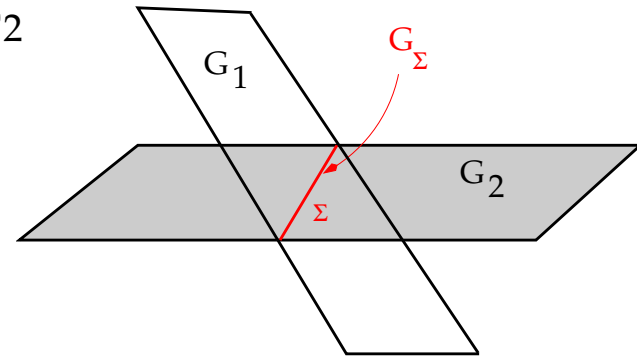
$U(1)$ -bundles $\mathcal{L}_{1,2}$ on S_1 and S_2 :

$$G_i \rightarrow H_i \times U(1)_i \Rightarrow G_\Sigma \rightarrow H_1 \times H_2 \times U(1)_1 \times U(1)_2$$

Decompose **bifundamentals**:

$$G_1 \times G_2 \rightarrow H_1 \times H_2 \times U(1)_1 \times U(1)_2$$

$$(\rho^1, \rho^2) \rightarrow \bigoplus_j (r_j^1, r_j^2)_{\alpha_j, \beta_j}$$



Zero modes in representation (r_j^1, r_j^2) of $H_1 \times H_2$ with $U(1)$ charges α_j, β_j is

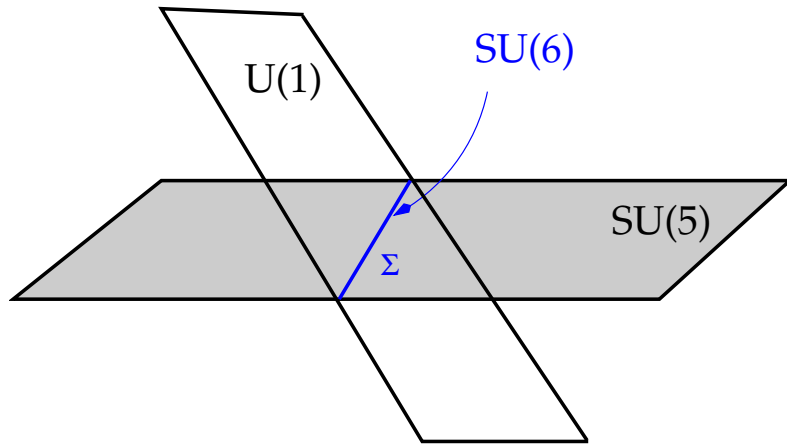
$$N_{(r_j^1, r_j^2)_{\alpha_j, \beta_j}} = h^0(\Sigma, K_\Sigma^{1/2} \otimes \mathcal{L}_1^{\alpha_j}|_\Sigma \otimes \mathcal{L}_2^{\beta_j}|_\Sigma)$$

$K_\Sigma^{1/2}$ = spin-bundle on Σ

\mathcal{L} susy bundles: $J \wedge \mathcal{L} = 0$. Large volume $J = AH + \sum B_i E_i$, $A \gg |B_i|$

Example: $SU(6)$ and $SO(10)$ Enhancements

$G_S = SU(5)$. Consider $\Sigma = \mathbb{P}^1$, and generate matter $3 \times \bar{\mathbf{5}}$ and $3 \times \mathbf{10}$. In particular we want complete $SU(5)$ multiplets, so $\mathcal{L}_Y|_\Sigma = 0$



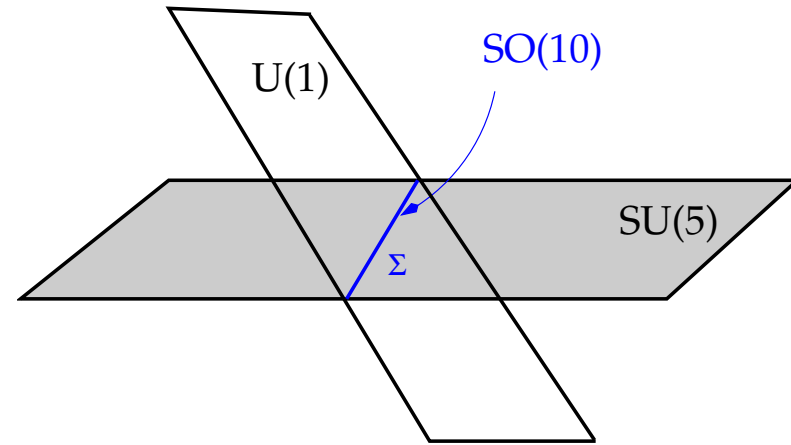
$$SU(6) \rightarrow SU(5) \times U(1)_a$$

$$\mathbf{35} \rightarrow \mathbf{24}_0 \oplus \mathbf{1}_0 \oplus \mathbf{5}_{+6} \oplus \bar{\mathbf{5}}_{-6}$$

$$h^0(\mathbb{P}^1, \mathcal{O}(-1) \otimes \mathcal{L}_a^{+6}|_\Sigma) = 0$$

$$h^0(\mathbb{P}^1, \mathcal{O}(-1) \otimes \mathcal{L}_a^{-6}|_\Sigma) = 3$$

$$\Rightarrow \mathcal{L}_a^{-6}|_\Sigma = \mathcal{O}(3)$$



$$SO(10) \rightarrow SU(5) \times U(1)_b$$

$$\mathbf{45} \rightarrow \mathbf{24}_0 \oplus \mathbf{1}_0 \oplus \mathbf{10}_{+4} \oplus \bar{\mathbf{10}}_{-4}$$

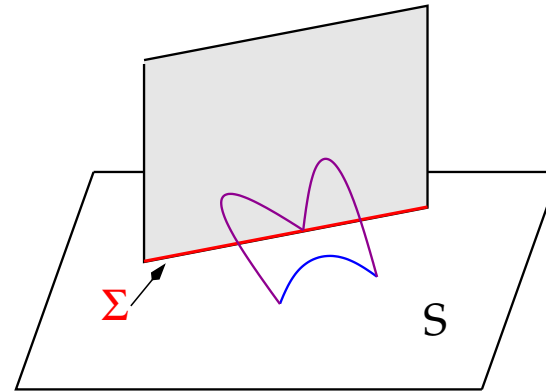
$$h^0(\mathbb{P}^1, \mathcal{O}(-1) \otimes \mathcal{L}_b^{+4}|_\Sigma) = 3$$

$$h^0(\mathbb{P}^1, \mathcal{O}(-1) \otimes \mathcal{L}_b^{-4}|_\Sigma) = 0$$

$$\Rightarrow \mathcal{L}_b^4|_\Sigma = \mathcal{O}(3)$$

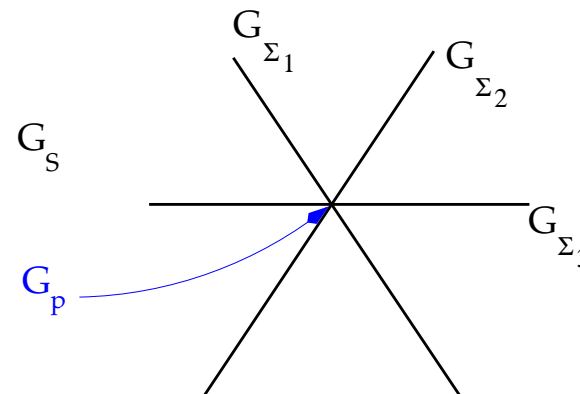
Yukawa couplings from Triple-Intersections

Yukawa couplings from $\Sigma\Sigma S$:
However, $SU(5)$ GUT: no chiral
matter in the bulk



Yukawa couplings from
 $\Sigma_1 \cap \Sigma_2 \cap \Sigma_3$ triple intersection
points p . Double-enhancement
of G_S to

$$G_p \rightarrow G_S \times U(1)_1 \times U(1)_2$$



Coupling from $SU(7)$ Enhancements

$$SU(7) \rightarrow SU(5) \times U(1)_1 \times U(1)_2$$

$$48 \rightarrow (\mathbf{24}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{1}_{0,0})$$

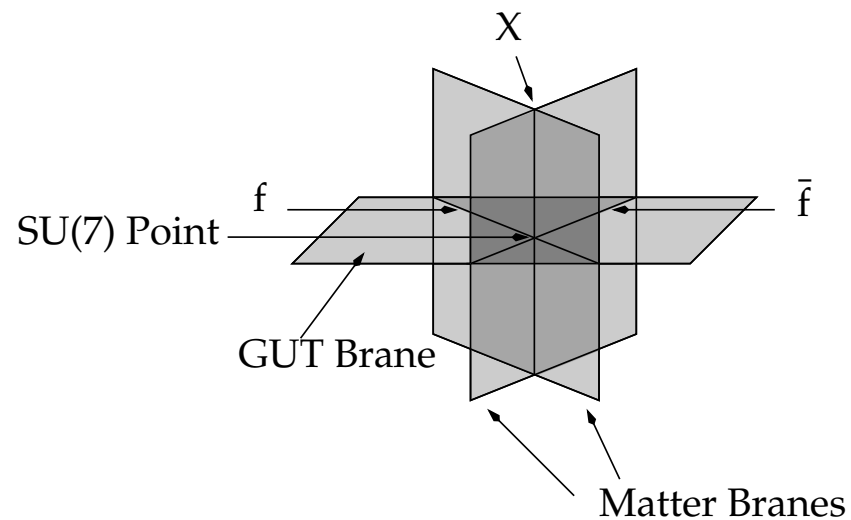
$$\oplus (\mathbf{5}_{0,+6} \oplus \bar{\mathbf{5}}_{0,-6})$$

$$\oplus (\mathbf{5}_{-6,0} \oplus \bar{\mathbf{5}}_{+6,0})$$

$$\oplus (\mathbf{1}_{+6,+6} \oplus \mathbf{1}_{-6,-6})$$

$$f = \mathbf{5}_{0,+6}, \bar{f} = \bar{\mathbf{5}}_{+6,0}, X = \mathbf{1}_{-6,-6}$$

$$\Rightarrow W \sim X f \bar{f}$$



GUT couplings from $SO(12)$ and E_6 Enhancements

Higgs-Matter couplings that are essential for $SU(5)$ GUTs can be generated from two types of enhancements:

$$SO(12) \rightarrow SU(5) \times U(1)_1 \times U(1)_2$$

$$66 \rightarrow (\mathbf{24}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{1}_{0,0})$$

$$\oplus (\mathbf{5}_{2,2} \oplus \bar{\mathbf{5}}_{-2,-2})$$

$$\oplus (\mathbf{5}_{-2,2} \oplus \bar{\mathbf{5}}_{2,-2})$$

$$\oplus (\mathbf{10}_{0,4} \oplus \bar{\mathbf{10}}_{0,-4})$$

$$W \sim \mathbf{5} \times \mathbf{5} \times \bar{\mathbf{10}} + \bar{\mathbf{5}} \times \bar{\mathbf{5}} \times \mathbf{10}$$

$$\Rightarrow W \sim \bar{H}_{\bar{\mathbf{5}}} \Phi_{\bar{\mathbf{5}}} \Phi_{\mathbf{10}}$$

$$E_6 \rightarrow SU(5) \times U(1)_1 \times U(1)_2$$

$$78 \rightarrow (\mathbf{24}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{1}_{0,0})$$

$$\oplus (\mathbf{5}_{-3,3} \oplus \bar{\mathbf{5}}_{3,-3})$$

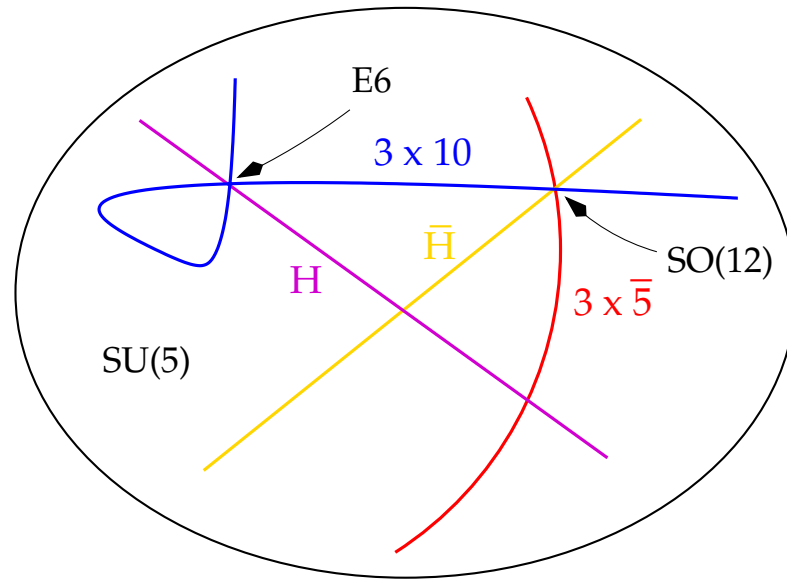
$$\oplus (\mathbf{10}_{-1,-3} \oplus \bar{\mathbf{10}}_{1,3})$$

$$\oplus (\mathbf{10}_{4,0} \oplus \bar{\mathbf{10}}_{-4,0}) \oplus (\mathbf{1}_{-5,-3} \oplus \mathbf{1}_{5,3})$$

$$W \sim \mathbf{5} \times \mathbf{10} \times \mathbf{10} + \bar{\mathbf{5}} \times \bar{\mathbf{10}} \times \bar{\mathbf{10}}$$

$$\Rightarrow W \sim H_{\mathbf{5}} \Phi_{\mathbf{10}} \Phi_{\mathbf{10}}$$

Ingredient 1: An $SU(5)$ GUT à la [BHV]



$$W \sim \bar{5} \times \bar{5} \times \mathbf{10} + \mathbf{5} \times \mathbf{10} \times \mathbf{10}$$

$$\sim \lambda_{ij}^d \bar{H}_{\bar{5}} \Phi_{\bar{5}}^i \Phi_{\mathbf{10}}^j + \lambda_{ij}^u H_{\mathbf{5}} \Phi_{\mathbf{10}}^i \Phi_{\mathbf{10}}^j \Rightarrow QDH_d + LEH_d + QUH_u$$

Intersections realized within one del Pezzo.

Comments

1. Note: we do not want to generate $\mu H\bar{H}$, and neither right-handed neutrino masses here. There will be better ways of doing this.
2. **Proton decay**: H and \bar{H} on different matter curves: by missing partner mechanism, triplets are paired up and become massive. No $QQQL$ operators can be generated from QQH , $QL\bar{H}$ and $H\bar{H}$.

2. SUSY breaking and scales via D3-instantons

Successful EFT requires not only GUT, but also

- Mechanism of **SUSY-breaking**, in particular generation **scales** (intermediate, not M_{GUT}, M_{Pl})
- **Mediation** of SUSY-breaking

To generate scales: use **D-instanton effects**: in other context studied in [Billo et al], [Blumenhagen et al], [Florea et al], [Ibanez, Uranga],...

→ Talks by Lerda and Billo

Retrofitting simplest SUSY-breaking models: [Aharony, Kachru, Silverstein] **Polonyi** for chiral superfield X

$$W = F_X X, \quad F_X \sim e^{-S_{inst}}$$

⇒ How to construct SUSY-breaking sector in F-theory?

Instantons: general considerations

Consider D3-instantons: can these contribute to the superpotential?

Setup: 7-brane on S , and wrap in addition D3-instanton on S .

Types of strings: "3-3", "3-7", "7-7".

To contribute: saturate "3-3" fermionic zero-mode integrals!

D3-D3: 4 universal fermionic zero-modes

$$\theta_\alpha \in S_+ \otimes H^2(S, K_S), \mu_{\dot{\alpha}} \in S_- \otimes H^0(S, \mathcal{O})$$

"Goldstinos from 4 broken SUSY that are preserved by background but broken by instanton"

D3-D7: bosonic zero-modes: $b_{\dot{\alpha}}, \bar{b}_{\dot{\alpha}} \in S_- \otimes H^0(S, \mathcal{O})$

fermionic zero-modes: $f, \bar{f} \in H^2(S, K_S)$

Instanton-contribution

- **Without world-volume flux:** 2 modes $\mu_{\dot{\alpha}}$ should be lifted: D7-brane breaks already half of the modes, so only 4 SUSY left. D3 breaks 2. Indeed, coupling to "3-7" strings saturates integral: cf. [Billo, Frau, Fucito, Lerda, Liccardo, Pesando], [Akerblom, Blumenhagen, Lüst, Plauschinn, Schmidt-Sommerfeld], [Petersen]:

$$S_{\text{inst}} \supset \mu_{\dot{\alpha}}^{3-3} (b^{\dot{\alpha}} \bar{f} + \bar{b}^{\dot{\alpha}} f)^{3-7}$$

- **Including world-volume flux:** "3-7" zero-modes are generically lifted!
No D3-effect?

However, same coupling exists to KK-modes of "3-7" strings exist from 8d reduction: [HMSS-NV]

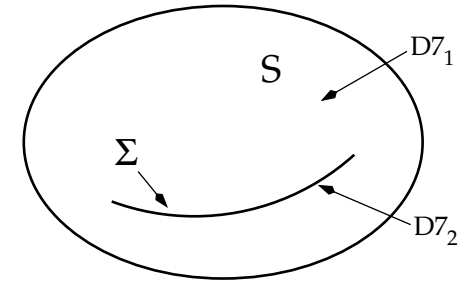
$$S_{\text{inst}} \supset \mu_{\dot{\alpha}} \left(\bar{b}_{KK}^{\dot{\alpha}} f_{KK} + b_{KK}^{\dot{\alpha}} \bar{f}_{KK} \right) + M_{b, KK} \bar{b}_{KK} b_{KK} + M_{f, KK} \bar{f}_{KK} f_{KK}$$

Polonyi from D3-instantons [Heckman, Marsano, Saulina, SSN, Vafa]

D3-instanton should contribute. Produce $W = F_X X!$

$S_1 \cap S_2 = \Sigma$, then $U(1)_1 \times U(1)_2$ charged chiral matter:

$$7_1-7_2: \begin{cases} n_{+-} = h^0(\Sigma, K_\Sigma^{1/2} \otimes \mathcal{L}_1|_\Sigma \otimes \mathcal{L}_2^{-1}|_\Sigma) = 1 \Rightarrow X_{+-0} \\ n_{-+} = h^0(\Sigma, K_\Sigma^{1/2} \otimes \mathcal{L}_1^{-1}|_\Sigma \otimes \mathcal{L}_2|_\Sigma) = 0 \end{cases}$$



Include D3-instanton with gauge-bundle $\mathcal{L}_{\text{inst}}$ to generate coupling to S

\Rightarrow fermi-zero-modes, charged under $U(1)_1 \times U(1)_2 \times U(1)_{\text{inst}}$:

$$\text{D3-7}_1: \begin{cases} n_{+0-} = h^1(S_1, \mathcal{L}_1 \otimes \mathcal{L}_{\text{inst}}^{-1}) = 0 \\ n_{-0+} = h^1(S_1, \mathcal{L}_1^{-1} \otimes \mathcal{L}_{\text{inst}}) = 1 \Rightarrow \beta_{-0+} \end{cases}$$

$$7_2\text{-D3}: \begin{cases} n_{0+-} = h^0(\Sigma, \mathcal{L}_2|_\Sigma \otimes \mathcal{L}_{\text{inst}}^{-1}|_\Sigma) = 1 \Rightarrow \alpha_{0+-} \\ n_{0-+} = h^0(S_1, \mathcal{L}_2^{-1}|_\Sigma \otimes \mathcal{L}_{\text{inst}}|_\Sigma) = 0 \end{cases}$$

\Rightarrow Coupling $\alpha_{0+-} \beta_{-0+} X_{+-0}$

Supersymmetric Instanton Bundles

$$\alpha\beta X : \left\{ \begin{array}{l} h^0(\Sigma, K_\Sigma^{1/2} \otimes \mathcal{L}_1|_\Sigma \otimes \mathcal{L}_2^{-1}|_\Sigma) = 1 \Rightarrow X_{+-0} \\ h^0(\Sigma, K_\Sigma^{1/2} \otimes \mathcal{L}_1^{-1}|_\Sigma \otimes \mathcal{L}_2|_\Sigma) = 0 \\ h^1(S_1, \mathcal{L}_1 \otimes \mathcal{L}_{\text{inst}}^{-1}) = 0 \\ h^1(S_1, \mathcal{L}_1^{-1} \otimes \mathcal{L}_{\text{inst}}) = 1 \Rightarrow \beta_{-0+} \\ h^0(\Sigma, \mathcal{L}_2|_\Sigma \otimes \mathcal{L}_{\text{inst}}^{-1}|_\Sigma) = 1 \Rightarrow \alpha_{0+-} \\ h^0(S_1, \mathcal{L}_2^{-1}|_\Sigma \otimes \mathcal{L}_{\text{inst}}|_\Sigma) = 0 \end{array} \right.$$

$$\Sigma = \mathbb{P}^1 \Rightarrow \mathcal{L}_2|_\Sigma = \mathcal{L}_1|_\Sigma \otimes \mathcal{O}(-1)$$

E.g. $[\Sigma] = H - E_i - E_j \in H_2(S_1)$:

$$\mathcal{L}_{\text{inst}} = \mathcal{O}(E_p - E_i - E_j), \quad i \neq p \neq j$$

\mathcal{L} susy bundles: $J \wedge \mathcal{L} = 0$. Large vol: $J = AH - \sum B_i E_i$, $A \gg B_i > 0$.

For given such $\mathcal{L}_{1,2}$ and Σ , need to **sum over all supersymmetric $\mathcal{L}_{\text{inst}}$** !

D3-instanton generated superpotential

Contribution to superpotential $\langle \psi_1^\dagger \psi_2^\dagger \rangle \sim \partial_t^2 W_{\text{inst}}$:

$$W_{\text{inst}} \sim e^{-t_{S_1}} \int d\alpha d\beta d\mu df_{KK} db_{KK} d\bar{f}_{KK} d\bar{b}_{KK} e^{-\alpha\beta X - \mu_{\dot{\alpha}} \left(\bar{b}_{KK}^{\dot{\alpha}} f_{KK} + b_{KK}^{\dot{\alpha}} \bar{f}_{KK} \right) - M_{b, KK} \bar{b}_{KK} b_{KK} - M_{f, KK} \bar{f}_{KK} f_{KK}} .$$

where $t_{S_1} \sim \int_{S_1} (J + c_1(\mathcal{L}_{\text{inst}}))^2 + S_{\text{WZW}}$.

Integrating over fermi-zero modes yields

$$W_{\text{inst}} = F_X X, \quad F_X \sim M_{S_1}^2 e^{-t_{S_1}}$$

$M_{S_1}^4 \sim 1/\text{Vol}(S_1)$ is characteristic mass scale. E.g. for GUT-cycle M_{GUT} .

Exponential suppression of F_X by Kähler parameter on S_1 .

Will discuss SUSY breaking later in complete model.

Ingredient 2: Instantons at the $SU(7)$ point [Marsano, Saulina, SSN]

In the $SU(5)$ GUT generate Polonyi + coupling to gauge messengers in the following simple way at an $SU(7)$ point:

GUT singlet X couples to $\mathbf{5}$ and $\bar{\mathbf{5}}$ messengers by

$$W = \lambda_X X f \bar{f}$$

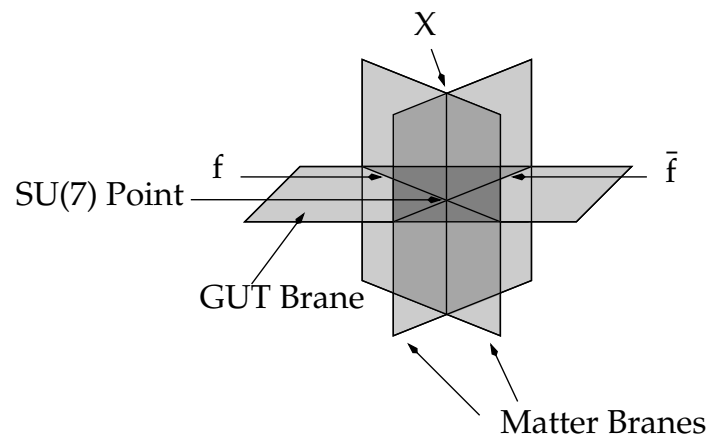
Linear term for X by instantons: X localized on $S_f \cap S_{\bar{f}} = \Sigma_{Pol}$

$$W_{inst} = F_X X$$

Ensure: no extra fermi zero modes are generated. In particular \mathcal{L}_Y restricts trivially here!

Ingredient 2: $W \supset F_X X + \lambda_X X f \bar{f}$

Remains to couple this to the $SU(5)$ GUT and mediate SUSY breaking.



Instantons for other scales 1: μ -term?

$$\mathcal{L} \supset \int d^2\theta \mu H \bar{H}$$

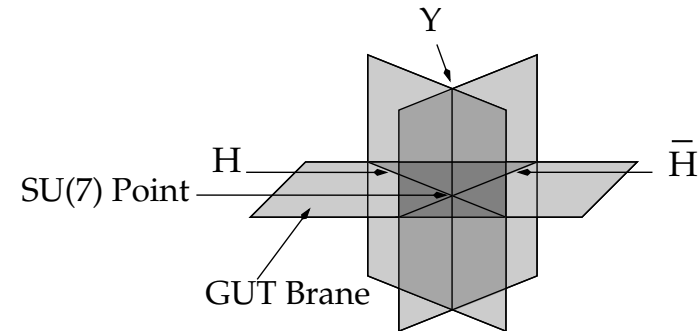
Electro-weak symmetry breaking requires $\mu \sim O(100)$ GeV

" μ -problem": Why is μ so small?

Naively one could anticipate generating by analogy: Y GUT-singlet

$$W \sim Y H \bar{H}$$

$\langle Y \rangle \neq 0 \Rightarrow$ would generate small μ -term.



Doublet-triplet splitting requires: $U(1)_Y$ flux restricts non-trivially

\Rightarrow Generates extra zero-modes which cannot be cancelled!

\Rightarrow μ -term CANNOT be generated in these models directly by instantons

\Rightarrow Will see: μ will be related to other scale, namely SUSY breaking scale, and yield very elegant solution to μ -problem

Instantons for other scales 2: Right-handed neutrino-mass

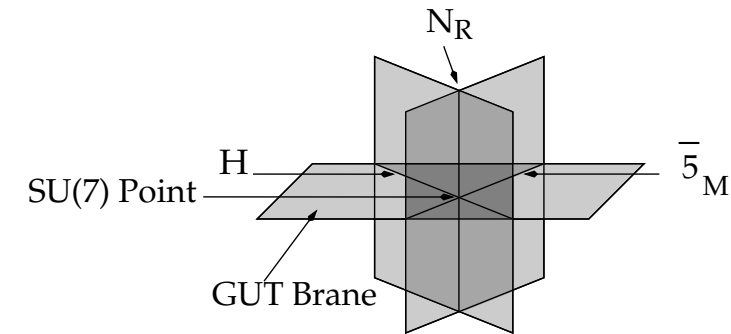
We can use instantons to generate small mass for right-handed neutrinos:

N_R = GUT-singlet at $SU(7)$ point. Coupling:

$$W \sim N_R H \Phi_{\bar{5}}$$

Instantons generated $W_{inst} = \mu_N N_R^2$, $\mu_N \sim e^{-S_{inst}}$.

If N_R = Right-handed neutrino, masses using see-saw mechanism.



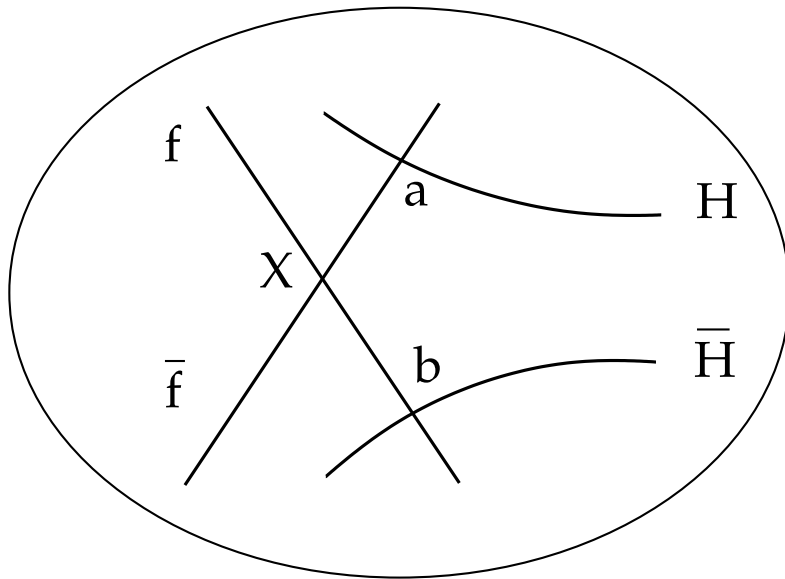
General lesson: D3-instantons

- Introduce **scales**
- Can construct Polonyi (also O'R and Fayet) model of **SUSY breaking**
- Right-handed neutrino-masses can be generated
- **μ -term CANNOT be generated** in $SU(5)$ GUT due to $U(1)_Y$ flux

3. Gauge Mediation

Coupling to Higgs sector [Marsano, Saulina, SSN]

Gauge-mediation of SUSY-breaking most natural in local F-GUTs:

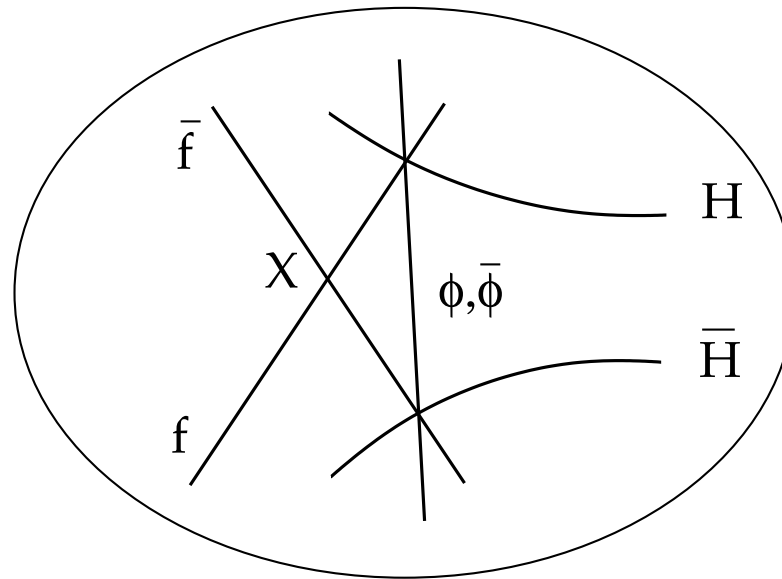


$SU(7)$ point

$$W \sim \mathbf{1} \times \mathbf{5} \times \bar{\mathbf{5}}$$

a, b GUT singlets:

$$W \sim Xf\bar{f} + aH\bar{f} + b\bar{H}f$$



$SO(12)$ point

$$W \sim \mathbf{5} \times \mathbf{5} \times \bar{\mathbf{10}} + \bar{\mathbf{5}} \times \bar{\mathbf{5}} \times \mathbf{10}$$

$\mathbf{10}$'s KK-mode: $M \sim M_{GUT}$

$$W \sim Xf\bar{f} + Hf\bar{\phi} + \bar{H}\bar{f}\phi + M\phi\bar{\phi}$$

Coupling to Higgs sector: $U(1)_{PQ}$

$U(1)$ Peccei-Quinn symmetry under which X is charged $+2$.

$SO(12)$ point : $W \sim Xf\bar{f} + Hf\bar{\phi} + \bar{H}\bar{f}\phi + M\phi\bar{\phi}$

respects

	X	f	\bar{f}	ϕ	$\bar{\phi}$	H	\bar{H}
PQ	2	-1	-1	0	0	1	1

D3-instanton-generated Polonyi term $W_{\text{inst}} = F_X X$ breaks $U(1)_{PQ}$

$U(1)_{PQ}$ forbids μ -term $\mu H\bar{H}$

How do we generate μ ?

Generation of μ -term

$U(1)_{PQ}$ prohibits $\mu H\bar{H}$ because $PQ(H) = PQ(\bar{H}) = 1$.

In $SO(12)$ model:

we obtain μ by integrating out KK-modes and generating

$$\frac{1}{M_{GUT}} \int d^4\theta X^\dagger H\bar{H} \quad \Rightarrow \quad \mu \sim \frac{F_X}{M_{GUT}}$$

B_μ term cannot be generated as $X^\dagger X H\bar{H}$ is forbidden by $U(1)_{PQ}$

$\Rightarrow B_\mu = 0$ at messenger scale.

SUSY-breaking vacuum

Polonyi model by itself does not yield SUSY-breaking vacuum. Need to lift flat directions and find non-zero M for $\langle X \rangle = M + \theta^2 F_X$

- Anomalous $U(1)$:

integrating out massive gauge boson [Arkani-Hamed, Dine, Martin]

$$\delta K_{AHDM} \sim -\frac{c(X^\dagger X)^2}{M_{KK,Polonyi}^2}$$

$c > 0 \Rightarrow$ Favours $M = 0$

($c < 0$: instability, roll out of regime of validity)

- Integrating out KK-modes: Leading KK yield O’Raifeartaigh model

$$W = F_X X + \lambda X \tilde{X}_{KK} \Phi_{KK} + M_{KK} X_{KK} \tilde{X}_{KK} + M_{KK} \Phi_{KK}^2$$

Coleman-Weinberg potential lifts flat directions, captured by [Shih]

$$\delta K_{KK} \sim -\frac{a|X|^4}{M_{GUT}^2} + \frac{b|X|^6}{M_{GUT}^4} + \dots$$

$a > 0 \Rightarrow$ Favours $M = 0$

SUSY-breaking vacuum (cont.)

But M gives mass to messengers via $\lambda_X X f \bar{f}$! Disaster?

In order to generate $M \neq 0$ we have to include coupling to supergravity.

Leading corrections to M turns out to be not small, namely

$O(M_{GUT}^2/M_{Pl})$. [Kitano]

Additional term from energy density in bulk: $W = F_X X + W_0$

$$\Rightarrow V_{Sugra} \sim \frac{1}{M_{Pl}^2} W_0 F_X X + c.c. + \frac{a |F_X|^2 |X|^2}{M_{GUT}^2}$$

This shifts

$$M = 0 \quad \Rightarrow \quad M \sim \frac{|W_0| M_{GUT}^2}{|F_X| M_{Pl}^2}$$

Imposing vanishing vacuum energy, yields in fact $W_0 = F_X M_{Pl}$, so that

$$M \sim \frac{M_{GUT}^2}{M_{Pl}} = O(10^{14}) \text{ GeV} \quad \text{Clearly not negligible!}$$

Complete Model: Sweet-spot SUSY

Ingredient 1: $SU(5)$ GUT, with hypercharge flux to eliminate Higgs 3's

Ingredient 2: SUSY-breaking by D3-instanton generated Polonyi model

$$W = F_X X + W_0 \text{ with } \langle X \rangle = M + \theta^2 F_X,$$

$$M \sim \frac{M_{GUT}^2}{M_{Pl}}, \quad F_X \sim M_{GUT}^2 e^{-t_{inst}}$$

Ingredient 3: Gauge-mediation $SO(12)$ model, $X f \bar{f}$ and $U(1)_{PQ}$ symmetry, which allows for following couplings (after integrating out KK-modes)

$$\mathcal{L}_{sweet} \sim \int d^4\theta \left(X^\dagger X - \frac{a(X^\dagger X)^2}{M_{GUT}^2} + \frac{c_\mu X^\dagger H \bar{H}}{M_{GUT}} + \frac{c_H X^\dagger X (H H^\dagger + \bar{H} \bar{H}^\dagger)}{M_{GUT}^2} + \dots \right) \\ + \int d^2\theta (F_X X + \lambda_X X f \bar{f} + \dots) + \mathcal{L}_{MSSM}$$

This is essentially the effective model of [Ibe, Kitano]'s Sweet-spot SUSY.
Complete realization in local F-theory GUT.

Sweet-spot SUSY phenomenology

The Ibe-Kitano's sweet-spot SUSY model has various appealing features

Input into EFT:

- Order 1 coefficients c_μ, c_H
- $\lambda_X \sim 10^{-2}$
- $F_X \sim 10^{19} \text{GeV}^2$
- Gravitino mass $m_{3/2} = \frac{F_X}{M_{Pl}} = 1 \text{ GeV}$

Implies

$$\mu \sim \frac{F_X}{M_{GUT}} = 10^3 \text{ GeV}$$

Solves μ -problem.

Features of F-theory Sweet-spot SUSY

More detailed look at parameters and scales from F-theory point of view:

Scales: $M_{GUT} \sim 10^{16}$ GeV, $M_{Pl} = 10^{19}$ GeV

Couplings: Tension of branes M_* and τ

GUT coupling constant $\alpha_{GUT} \sim \frac{M_{GUT}^4}{M_*^4}$

Scales in the model: $F_X \sim M_{GUT}^2 e^{-t}$

$$M \sim \frac{M_{GUT}^2}{M_{Pl}}$$

$$\mu \sim \frac{F_X}{M_{GUT}}, \quad B_\mu = 0 \text{ at messenger scale}$$

Important feature as in Ibe-Kitano:

- μ -term is related to SUSY breaking scale.
- Geometrically realized $U(1)_{PQ}$ dictates which couplings are possible.
- $B_\mu = 0$ and gets generated by MSSM RG flow below messenger scale

[Babu, Kolda, Wilczek]

To see, whether Sweet-spot model is realizable in this context, assume that these are $O(1)$ constants.

For $m_{3/2} = 1 \text{ GeV}$, we obtain from

$$m_{3/2} \sim \frac{F_X}{M_{Pl}} \quad \Rightarrow \quad F_X \sim 10^{19} \text{ GeV}^2$$

In our setup: F_X was related to size of cycle S that is wrapped by D3-instanton: $M_{Pol}^4 = 1/Vol(S)$. Up to $O(1)$ parameter η :

$$M_{Pol} = \eta M_{GUT}$$

and

$$F_X \sim M_{Pol}^2 e^{-\frac{M_*^4}{M_{Pol}^4}}$$

which gives correct order for $\eta \sim 0.68$. Recall: $\alpha_{GUT} \sim M_{GUT}^4 / M_*^4$.

\Rightarrow there is Sweet-spot in F-theory EFT landscape

4. Summary and Outlook

F-theory GUT models are interesting String EFT with realistic GUTs

Our findings:

- D3-instanton effects introduce **scales**
- **SUSY breaking by D3-instanton effects**
- **Gauge-mediation** natural
- **μ -term** has to be **related SUSY breaking scale**, cannot be generated by other effects, e.g. instantons; solution to μ -problem
- **Complete local model for SU(5) GUT with SUSY breaking**

Future directions:

- Compute **Yukawa couplings**, check hypothesis of $O(1)$ coefficients
- For this: need to find **compact models**, and conditions on compact geometries
- **Moduli stabilization**

Thank
You