D-brane model building
and non-perturbative effects

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Outline

- Motivation/Introduction
- Compactifications with D-branes
- Model building
- Fluxes
- Euclidean D-brane instantons
- Conclusions

String Phenomenology

- String theory describes gravitational and gauge interactions in a unified framework, consistent at the quantum level

- If string theory is realized in Nature, it should be able to describe a very specific gauge sector: Standard Model

- Aim of String Phenomenology:

  - Determine classes of constructions with a chance to lead to SM
    Non abelian gauge interactions, replicated charged fermions, Higgs scalars with appropriate Yukawa couplings, ...

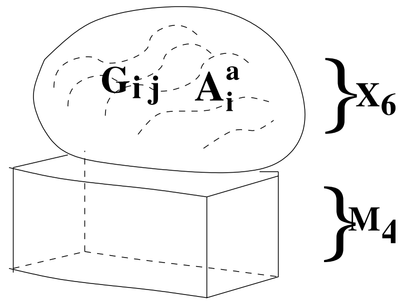
  - Within each class, obtain explicit models as close to SM as possible with the hope of learning more about the microscopics of SM in string theory

- Old program, yet continuous progress
  Moduli stabilization, non-perturbative effects, ...

Prototypical example: Heterotic string models

[Candelas, Horowitz, Strominger, Witten, '85]

- The 10d heterotic string has as effective theory 10d N=1 sugra coupled to E8xE8 (or SO(32)) gauge multiplets
- Compatification: six extra dimensions parametrize small Calabi-Yau space, on which we also turn on a non-trivial gauge field background



- Gauge group is reduced to transformations leaving bckgnd invariant
Possible to break down to something close to SM gauge group

- 4d charged chiral fermions arise from zero modes of 10d gauginos, in the Kaluza-Klein reduction of the spectrum

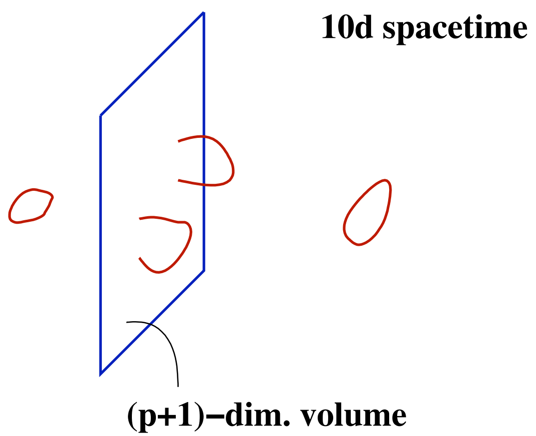
- Within this general class, very explicit models close to (MS)SM

[reviews by A.Lukas & V. Braun]

D-branes [Polchinski, '95]

- In this talk, focus on D-brane models in type II string compactifications
- Interested in weak coupling and small numbers, $g_s N << 1$, treat in the probe approximation

- Described as subspaces of 10d space on which open strings end
- Models may be followed to strong coupling (if susy protects)
- lifting relates to other setups: M-theory on G2 or F-theory on CY4



Supersymmetric D-branes

- In a CY compactification, supersymmetric D-branes are already present in the topological version of the model

- A-branes: Appear in Type IIA compactifications

- D6-branes wrapped on Special Lagrangian 3-cycles with flat gauge field

[Also coisotropic branes [Kapustin, Orlov], model building by [Font, Ibanez, Marchesano]]

- Calibrated by ReΩ, up to a phase uncorrected by α'

- B-branes: Appear in type IIB compactifications

- At large volume, D-branes on holomorphic cycles with holomorphic stable gauge bundles

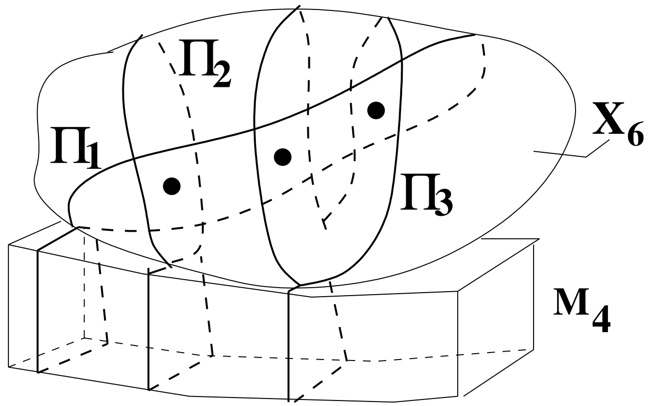
- Calibrated by Re exp(J+i(B+F)), up to phase with α' corrections

- At general points in Kahler moduli space, described as matrix factorizations of linear sigma model

Exchanged by open string mirror symmetry [review by Herbst]

Intersecting brane worlds

[Blumenhagen, Gorlich, Kors, Lust;
Aldazabal, Franco, Ibanez, Rabadan, AU]



- Focus on models of A-branes

(actually, orientifold version: Z2 invariance
under antiholomorphic involution
Preferred 4d N=1 susy and calibrating phase)

General class of string compactifications with non-abelian gauge symmetry and replicated charged chiral fermions

Mathematical construction of Special Lagrangian 3-cycles on compact CY manifolds...?

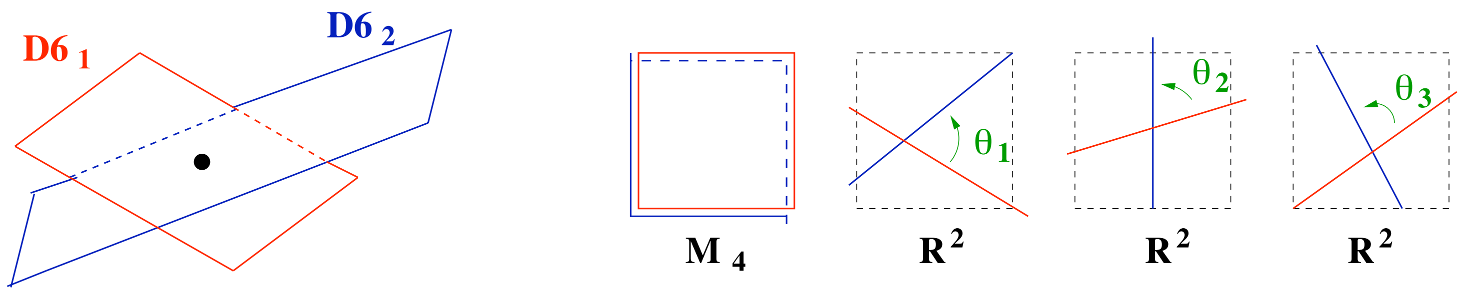
Even simpler questions: For which holomology classes there is irreducible special lagrangian representative, ...

Physics can be translated to math concepts for (category of) A-branes
But explicit computations only for tori and quotients thereof

Structure of a local intersection

[Berkooz, Douglas, Leigh,'96]

- Consider type IIA string theory with two stacks of D6-branes (hence 7d subspaces) intersecting over a 4d subspace of their volumes



- Three sectors of open strings
  - $D6_1$-$D6_1$: $U(N_1)$ on 7d plane 1
  - $D6_2$-$D6_2$: $U(N_2)$ on 7d plane 2
  - $D6_1$-$D6_2$: 4d chiral fermion in $(N_1,\overline{N}_2)$ on 4d intersection

- Chirality is a consequence of the geometry of the intersection
  e.g. two D5's intersecting over 4d leads to non-chiral fermions

Spectrum of light 4d fields

- Closed string sector 4d N=1 supergravity multiplet plus moduli chiral multiplets (dilaton, Kahler, complex structure)

  see later for moduli stabilization

- Open string spectrum (morphisms):
  - Chiral part can be determined with just the above topological data

Gauge group $\prod_a U(N_a)$

Chiral fermions $\sum_{a,b} I_{ab}\,(N_a, \overline{N}_b)$

$$I_{ab} = (n_a^1 m_b^1 - n_b^1 m_a^1) \times (n_a^2 m_b^2 - n_b^2 m_a^2) \times (n_a^3 m_b^3 - n_b^3 m_a^3)$$

Intersection number = geometric origin of family replication!

- Non-chiral features of the spectrum (susy, scalars,...) depend on less robust data of the configuration

Supersymmetry and BPS phase

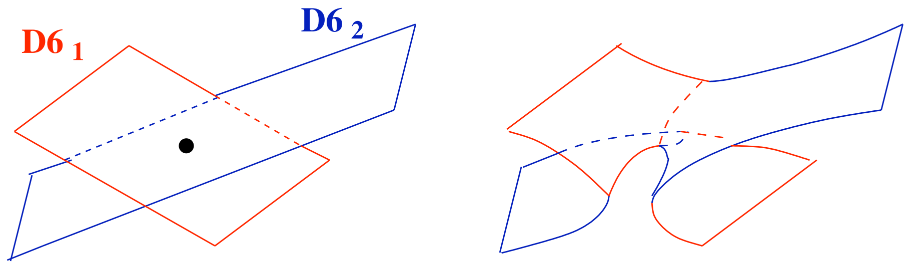
- D6-branes with misaligned BPS/calibrating phase lead to non-susy open string sector

  Light scalar spectrum in flat space intersection

$$M^2 = \frac{1}{2}(\,\theta_1 \pm \theta_2 \pm \theta_3\,)\,M_s^2$$

- 4d physical description: Complex structure modulus controling the phase misalignment couples as FI term

- Marginal stability walls $\qquad V_D = (\,|\phi|^2 - \xi\,)^2$



Brane recombination
= Higgs mechanism

- Nice geometric interpretation in terms of volume minimization using "angle theorem" by Lawlor [Douglas, '01]

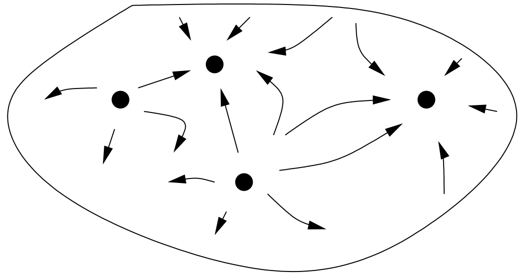
RR tadpoles and anomalies

- In a compact space, the total D6-brane charge* must be zero

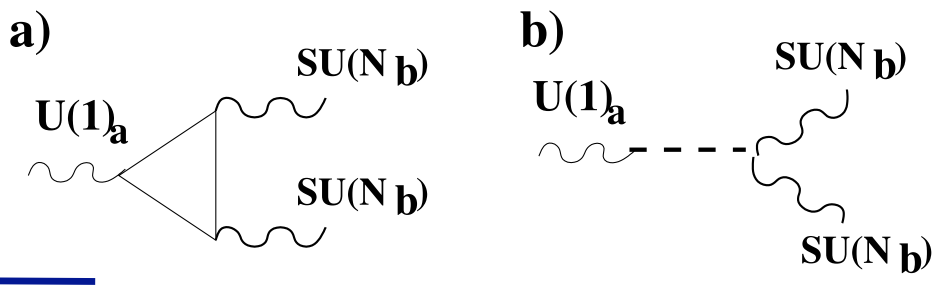
  Homology

$$\sum_{D6_a, D6_{a'}} N_a\,[\Pi_a] - 4[\Pi_{06}] \;=\; 0$$

Gauss' law



- Deeply related to consistency of 4d theory: anomaly cancellation
  - $SU(N_a)^3$ non-abelian vanish identically
  - $U(1)_a$-$SU(N_b)^2$ mixed cancel via Green-Schwarz mechanish involving the D6-brane couplings $\sum_{k,a}\int_{4d} B_k \wedge \mathrm{tr}\, F_a$ & $\sum_{k,a}\int_{4d} a_k\, \mathrm{tr}\,(F_b \wedge F_b)$



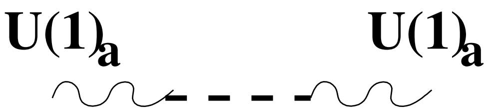
* Actually cancel K-theory charge: cancell. of global gauge anomalies [AU]

U(1)'s

- Due to BF couplings, all 'anomalous' and some 'non-anomalous' U(1)'s become massive, with mass of order the string scale

[see Grimm's talk]

$$\sum_{k,a} \int_{4d} B_k \wedge \mathrm{tr}\, F_a = - \sum_{k,a} \int_{4d} \partial_\mu a_k \, A^a_\mu$$



$$= m^2 A_\mu^2$$

- Consequences

  - Impose that hypercharge generator remains massless
  - Additional U(1)'s removed
    remain as global symmetries exact in perturbation theory
  - Operators violating the latter can appear non-perturbatively
    D-brane instantons, see later

[other talks today]

Towards the SM

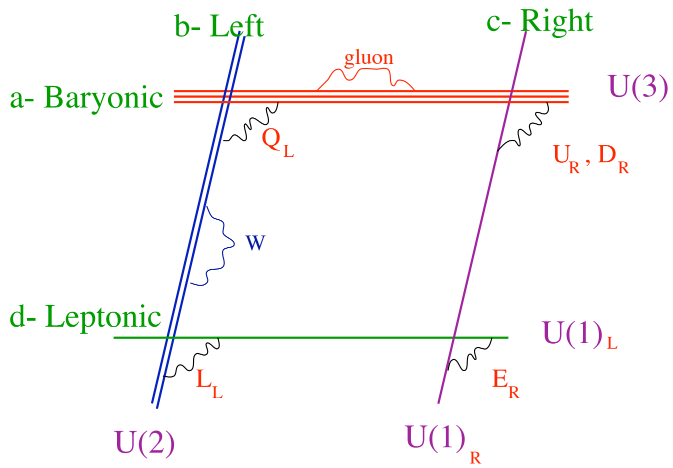
- A simple road to SM

[Ibanez, Marchesano, Rabadan; Cremades, Ibanez, Marchesano;'01]

Introduce four stacks of D6's a,b,c,d with

$$U(3)_a \times USp(2)_b \times U(1)_c \times U(1)_d$$

$$I_{ab} = 3 \rightarrow \quad Q_L$$
$$I_{ac} = -3,\ I_{ac'} = 3 \rightarrow \ U_R,\ D_R$$
$$I_{db} = 3 \rightarrow \quad L$$
$$I_{dc} = -3,\ I_{dc'} = -3 \rightarrow \ E_R,\ \nu_R$$



Spectrum of SM with hypercharge

$$Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d$$

- Explicit realization of this structure in several toroidal models

Phenomenological properties

- Gauge couplings

  Related to the volume of the 3-cycles

  integral of calibrating form

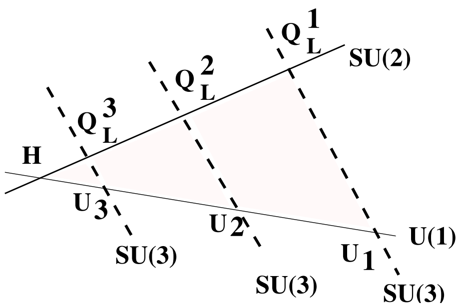
$$\frac{1}{g_a^2} = \frac{V_{\Pi a}}{g_s}$$

- Superpotential couplings

  Depend only on Kahler moduli

  Mediated by open string worldsheet instantons $Y_{jk} \simeq e^{-A_{Hjk}+i\phi_{jk}}$

  Exponential hierarchies e.g. in fermion masses



Down-to-earth computations of general formula on tori

[Cremades, Marchesano, Ibanez; Cvetic, Papdimitriou;'02]

But related to products in Fukaya category (of A-branes)

[Herbst, Lerche, Nemechansky]

(Meaning of e.g. quantum A-infinity relations?)

Complicated function of moduli. See later for moduli stabilization

The B-picture

- Same kind of construction can be carried out in mirror B-side
- In large volume language,
  - D-branes on holomorphic cycles with holomorphic bundles (or D9's on CY with coherent sheaves)

  D7-branes, similar to F-theory picture [reviews by Beasley, Wijnholt]
  - Chiral matter from index of Dirac op. coupled to difference bundle
  - Couplings (e.g. Yukawa) from overlap of Gaussian wavefunctions
  - Susy, BPS phase, wall crossing, controlled by slope stability
- Efficient tools for construction of bundles could prove useful:

  Spectral cover in elliptic fibrations

  Monad constructions

  How to apply to lower dimensional subspaces?

  How to mirror-map efficiently to A-side these efficient techniques?
- In linear sigma model language, matrix factorizations:

  Not much application to model building [Omer]
- Some explicit well studied stringy regimes on B-side

The B-picture (cont.)

- Gepner models: [see Bianchi's talk]

  Pure CFT description at a special point in Kahler moduli space

  - Very rich [Dijkstraa, Huiszoon, Schellekens; Anastasopoulos, Dijkstraa, Kiritsis, Schellekens; Gato, Schellekens]
  - No available formulae to compute couplings
  - Cannot move in moduli space

  Translation to matrix factorizations & linear sigma model?

  Extrapolation to large volume?

- Non-compact singular CYs [Aldazabal, Ibanez, Quevedo, AU; Berenstein, Jejjala, Leigh; Verlinde, Wijnholt]

  Techniques to study holomorphic information:

  Quiver diagrams [Douglas, Moore]

  - Dimer diagrams for toric singularities [Kennaway, Hanany]
  - Exceptional collections in large volume [Herzog; ...]

  Stability is King's θ-stability [Douglas, Fiol]

  Constraints on local models to be embeddable in global??

Flux compactifications and moduli stabilization

- Compactifications beyond the usual CY ansatz have less moduli
- Prototype: type IIB on CY with NSNS and RR 3-form fluxes

Turn on field strength fluxes F_3, H_3 in H^3(CY,Z) cohomology

- Microscopic 10d picture [Dasgupta, Rajesh, Sethi; Giddings, Kachru, Polchinski]

Warped compactification, warping sourced by fluxes

$$ds^2 = Z(y)^{-\frac{1}{2}} \, ds^2_{4d} + Z(y)^{\frac{1}{2}} g^{CY}_{mn}(y) \, dy^m \, dy^n$$

$$\nabla^2 Z = g_s N \, |F_3 - \tau H_3|^2$$

- Macroscopic effective 4d field theory picture

At large radius, flux scale α'/R^3 much smaller than KK scale $1/R$

Flux superpotential [Gukov, Vafa, Witten]

$$W = \int_{X_6} (F_3 - \tau H_3) \wedge \Omega$$

Effects of warping in KK reduction? [Douglas, Shelton, Torroba; Shiu, Torroba, Underwood, Douglas; Douglas, Torroba]

Type IIB with 3-form fluxes

- Generically stabilization of all complex structure moduli and dilaton
- Description is explicit enough to allow statistical study of distribution of vacua and their properties [Douglas; Ashok, Douglas; Denef, Douglas; ...]

E.g. density of vacua in complex structure moduli space

$$\rho \simeq \det(-R-\omega)$$

- Kahler moduli stabilized by non-perturbative effects*, see later [Kachru, Kallosh, Linde, Trivedi]

Would be desirable to describe them globally in moduli space

(*possible contributions from α' corrections [Balasubramanian, Berglund, Conlon, Quevedo]

and from g_s corrections [Berg, Haack, Kors; Cicoli, Conlon, Quevedo,...])

Other generalizations, in IIA, IIB [talks on Friday]

- Classifications of N=1 supersymmetric backgrounds

  Configurations SU(3) or SU(3)xSU(3) structure

  Generalized complex geometry [Graña, Minasian, Petrini, Tomasiello; ...]

  (Recent results for N=0 vacua [Camara, Graña; Lust, Marchesano, Martucci, Tsimpis])

  ⇒ Microscopic local picture, but few compact examples

- Description in effective field theory

  Consider as deformations of underlying CY compactification

  Superpotential for dilaton, complex structure and Kahler moduli

$$W(\tau, z_i, t_a)$$

- Geometric fluxes: torsion classes dΩ, dJ

  Non-geometric fluxes: stringy monodromies along 1-cycles

- Systematically included in gauged supergravity formalism

  Fluxes as parameters of the gauging of an isometry of moduli space

  Embedding formalism $\Theta_M{}^A$

Better global microscopic descriptions? (T-folds) [see Hull's talk]

Fluxes and D-branes

- Interplay of fluxes and D-branes, at different levels

  - Mutual consistency conditions: Freed Witten anomalies

    Bianchi identity for worldvolume gauge field dF=H3

    Related to twisted K-theory in presence of H3

    4d flux superpotential invariant under D-brane U(1) [Font, Camara, Ibanez]

  - Change of supersymmetry conditions for D-branes

    Generalized calibrations: minimize action rather than volume

    Flux dependence of open string superpotential

    Need to consider open-closed moduli space: N=1 special geometry

    [see Jockers' talk]

  ⇒ Stabilization of D-brane moduli

  - Supersymmetric D-branes perturbed by supersymmetry-breaking fluxes

  ⇒ Soft terms

Fluxes, susy breaking and soft terms

- An appealing scenario: Susy MSSM D-brane sector and non-susy flux

- Soft terms arise from effect of non-susy flux on susy D-branes

  Explicitly computable using D-brane world-volume action in general supergravity background, or using 4d effective theory approach

  [Grana; Camara, Ibanez, AU; Lust, Mayr, Reffert, Stieberger; '03-'04]

- Flux components work as vevs for auxiliary fields of chiral multiplets of (complex structure) moduli

  ⇒ Realization of gravity-mediated susy breaking

  - Flavour problem: Decoupling of flavor physics and soft terms
    Geometrization squark masses determined by intersection angles

  - μ-problem: susy components of flux induce it on the branes

- Very explicit discussion of susy spectrum etc is possible in specific models

  e.g. in 'large volume compactifications' [Quevedo et al '06-'07]

Instanton effects

[Becker's, Strominger; Witten; Harvey, Moore; ... talks by Lerda, Billo]

D-brane instantons from A- or B-branes pointlike in (euclidean) 4d
Violate certain perturbatively exact U(1) global symmetries

Consider IIA CY orientifold compactification, and complex structure moduli associated to a 3-cycle C

$$T = t + i\,a = \int_C \mathrm{Re}\,\Omega + i\int_C C_3$$

Peccei-Quinn symmetry $a \to a + \lambda$

Violated by euclidean D2-brane instanton wrapped on C $\simeq e^{-T}$

Can contribute to stabilization of Kahler moduli in IIB models via non-perturbative superpotential [Kachru, Kallosh, Linde, Trivedi]

Models with enough instanton generated non-perturbative superpotentials [Denef, Douglas, Florea; + Grassi, Kachru]

Start with compactifications without fluxes, effects studied later on

D-brane instantons and effective operators

[Blumenhagen, Cvetic, Weigand; Ibanez, AU]

- In models with D-branes, gauging of PQ by U(1) in U(N)

Consider N D6-branes on C', there is a 4d world-volume coupling

$$\int_{C'\times M_4} C_5 \wedge \mathrm{tr}\, F \;\to\; \int_{M_4} B_2 \wedge \mathrm{tr}\, F \;\to\; \int d^4x\, (\,\partial_\mu a \,+\, A_\mu\,)^2$$

⇒ Instanton generates terms such that phase rotations compensate

$e^{-T}\,\Phi_1 \ldots \Phi_n$ allows couplings forbidden in pert.th.

- insersions from fermion mode couplings $\int d\lambda\, d\tilde{\lambda}\, e^{-T+\lambda\Phi\tilde{\lambda}} \,=\, e^{-T}\,\Phi$

- number of insertions is number of charged chiral fermion modes (instanton intersection numbers)

- Can generate interesting SM operators forbidden by U(1) symmetries in perturbation theory e.g. Majorana mass for ν_R (singlet in cd' sector)
- Important role in model building
  μ-term, certain GUT Yukawas, etc

[much explored in last two years]

Instanton effects

According to interplay with 4d D-branes

- Gauge instantons

Instanton D-brane wraps same cycle as 4d gauge D-brane

Ex. ADS "fractional" instantons

$$W = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}}$$

[..., many authors]

- Non-gauge instantons

General D-brane instantons

In perturbative models, need O(1) Chan-Paton group

The latter provide new sources of interesting 4d operators
violating certain perturbatively exact global symmetries
Application to neutrino masses, mu-term, GUT yukawas, ...

[Argurio, Bertolini, Bianchi,Billo,Blumenhagen, Cvetic, Ferretti, Frau,Ibanez,
Kiritsis,Lerda, Marotta,Petersson,Richter, Schellekens,Weigand,A.U....]

Instanton effects

Unlifted fermion zero modes ⇒ kind of 4d superspace interaction

- Instantons contributing to superpotential

  BPS D-branes with exactly 2 fermion zero modes (goldstinos)

  Generate 4d superpotentials $$\int d^4x\, d^2\theta\, e^{-T}\, \Phi_1 \ldots \Phi_n$$

- Beasley-Witten instantons

  BPS D-branes with more than 2 decoupled fermion zero modes

  Generate multi-fermion F-term, sketchily

$$\int d^4x\, d^2\theta\, w_{\bar{i}_1\bar{j}_1\ldots\bar{i}_n\bar{j}_n}(\Phi)\, \bar{D}\bar{\Phi}^{\bar{i}_1}\, \bar{D}\bar{\Phi}^{\bar{j}_1} \ldots \bar{D}\bar{\Phi}^{\bar{i}_n}\, \bar{D}\bar{\Phi}^{\bar{j}_n}$$

  Can be written as D-terms locally, but not globally: BW cohomology

- Non-BPS instantons

  Have at least 4 fermion zero modes (goldstinos of 4 broken susys)

  Generate 4d D-terms $$\int d^4x\, d^2\theta\, d^2\bar{\theta}\, f(T, \bar{T}, \Phi, \bar{\Phi})$$

Non-perturbative effects globally in moduli space

[Garcia-Etxebarria, AU;+Marchesano]

- For some applications, convenient to have a global picture of non-perturbative effects as a function of closed string moduli space

  e.g. moduli stabilization

  Wall crossing phenomena: Spectrum of BPS instantons changes
  But BPS=holomorphic non-perturbative contributions must be continuous across real codimension one

- Continuity is restored by including multi-instanton processes

  New physics of multi-instantons [talk by Schmidt-Sommerfeld]

- Physical explanation of mathematical wall-crossing formulas

  Explicitly illustrated by Gaiotto, Moore, Neitzke relating jumps in BPS spectrum of 4d N=2 supersymmetric gauge theory (BPS instantons in 3d N=4 gauge theory) to Kontsevich-Soibelmann wall crossing formula for certain generalized Donaldson-Thomas invariants.

- Suggests deep role of algebra of BPS objects

Some general considerations (in 4d N=1)

Distinguish: [Denef]

- Marginal stability: BPS brane splits, decay products misalign

  U(1)xU(1) theory with boson with charges (+1,-1)

$$V_D = (|\phi|^2 - \xi)^2$$

- Threshold stability: BPS brane splits, pieces recombine to new BPS

  U(1)xU(1) theory with bosons with charges (+1,-1), (-1,1)

$$V_D = (|\phi_1|^2 - |\phi_2|^2 - \xi)^2$$

- No-split BPS stability: BPS brane becomes non-BPS, with no splitting

  U(1) theory with no boson

$$V_D = \xi^2$$

Goldstinos:

BPS instantons must have at least 2 unlifted fermion zero modes

Non-BPS instantons must have at least 4 unlifted fermion zero modes

Wall crossings and non-perturbative terms

The structure of fermion zero modes already determines the BPS stability properties of the instantons

- Instanton contributing to the superpotential (2fzm) cannot cross genuine lines of marginal stability: cannot become non-BPS

Not enough fermion zero modes to account for the 4 goldstinos

- Can reach a line of threshold stability, and split into mutually BPS decay products

Multi-instanton process ensures globally defined holomorphic superp.

- BPS instanton with additional fermion zero modes (thus contributing higher F-terms)to the superpotential can cross genuine lines of marginal stability and become non-BPS

Two of the extra fermion zero modes become extra goldstinos

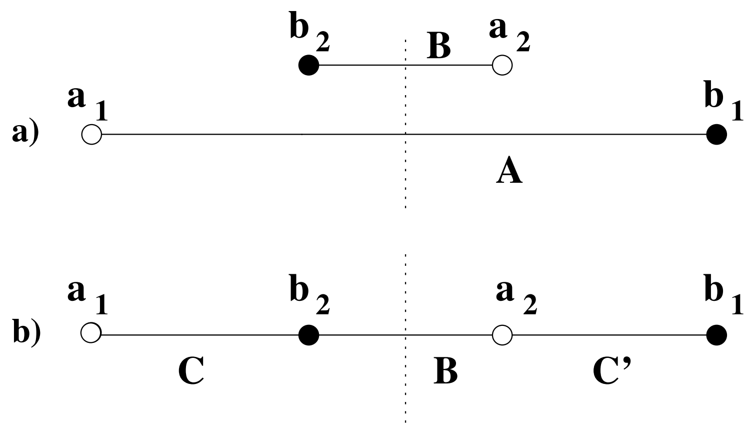
Possible multi-instanton process generates the amplitude on other side
Instanton amplitude is globally in non-trivial BW cohomology class

Away from BPS locus can be written locally as D-term

Example of continuity involving multi-instantons

Geometry: Double C* fibrations over complex plane, 3-cycles are double circle fibrations over segments between degenerations [Ooguri,Vafa]

Orientifold symmetry as Z2 reflection



a) Generically there are two O(1) instantons, A and B

$$W = f_1 e^{-T_B} + f_2 e^{-T_A}$$

b) Line of marginal stability, in which instanton A disappears. One O(1) instanton B and one U(1) instanton C/C'

Exp(-T_A) is generated by 2-instanton process involving B,C,(C')

Lifting of fermion zero modes and 4d susy breaking

Relation between superpotential and higher F-term by lifting fermion zero modes?

- Consider an instanton which can misalign and become non-BPS
- Introduce a mechanism to lift extra fermion zero modes to make it contribute to the superpotential
- Contradiction with counting of goldstinos is possible only if ...

⇒ 4d supersymmetry breaking upon misalignment due to mechanism lifting fermion zero modes!

Ex: Flavor mass to flow to Nf=Nc-1 SQCD

⇒ D-term on instanton implies a non-zero D-term on 4d branes

Ex: Closed string fluxes

⇒ Mass of extra z.m. ≈ susy variations of gravitino and dilatino

Ex: Lifting by other instantons ⇒ Previous marginal stability turns to threshold stability of new multi-instanton system

[see O(1)xU(1) →O(1) example; also Cvetic, Richter, Weigand]

Fluxes and D-brane instantons

- Interplay of fluxes and D-brane instantons, at different levels

  - Mutual consistency conditions: Freed Witten anomalies

    Bianchi identity for worldvolume gauge field   dF=H3

    D-brane instantons do not break isometries gauged by the flux

    [Kashani-Poor, Tomasiello]

  - Lifting of fermion zero modes of the D-brane instanton

    - Index for a modified Dirac operator [Bergshoeff, Kallosh, Kashani-Poor, Sorkin, Tomasiello]

    - Lifting computable as G3 λλ disk diagram in fluxless CFT

      [Billo, Ferro, Frau, Fucito, Lerda, Morales]

  - Instantons that do not contribute to the superpotential of fluxless compactification can contribute in the presence of fluxes

    - E.g: 3-form flux does not lift N=2 goldstinos of D3-brane instantons but can lift deformation zero modes

    Is there a macroscopic effective field theory description?

The 4d effective field theory picture [AU]

- Drawbacks of microscopic picture:

- For fixed CY, need to evaluate superpotential for each flux choice
- Local in moduli space
- Requires a microscopic picture of the flux

- There must exist a consistent description in 4d effective theory

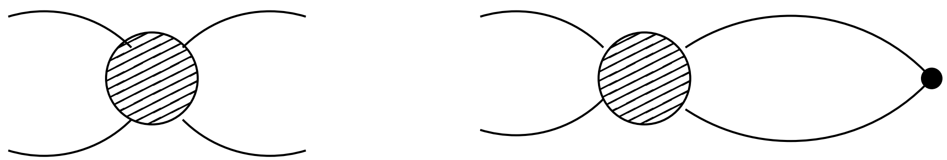
At large radius, flux scale α'/R^3 much smaller than KK scale $1/R$

Should describe all effects of fluxes as a deformation of the fluxless 4d effective theory (potential in moduli space of exact theory)

- Works indeed if fluxless effective theory includes higher F-terms

Effects of instantons with additional zero modes

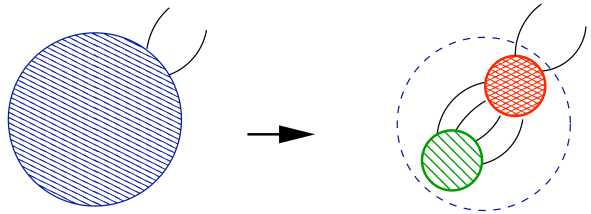
Upon inclusion of the flux superpotential, they turn into non-perturbative superpotentials, via integration of the massive moduli



- Recovers standard results, and many more

Comments

- Universality of contributions to non-perturbative F-terms



(insensitive to D-terms inside instanton world-volumes)
Presumably related to universality of category of holomorf. branes & topological strings

- Relation to non-perturbative effects in topological strings?
[Mariño, Schiappa, Weiss]

- Lessons from matrix models? [Garcia-Etxebarria, to appear]

- Lifting of zero modes in multi-instanton processes
Define index for multi-instanton systems, robust under splitting?

- Any relation to other brane splittings? multicenter black holes, gauge theories, ...

Expect many other surprises from D-brane instantons

Conclusions

- Several basic ingredients, grounded on interesting maths

D-brane model building ⇔ holomorphic A/B branes
theory of calibrations

Flux compactifications ⇔ generalized complex geometry,
group theory of gauged supergravities

Non-perturbative effects ⇔ holomorphic A/B branes
presumably modular functions... (N=2)

& their interplay... (e.g. generalized calibrations, twisted K-theory)

- Many other open interesting avenues

Eventually, what are the maths of non-supersymmetric vacua?