Chapter 1

Introduction

1.1 Historical notes

In the nineteenth century the profession of a specialized scientist was created and the main scientific activity moved to university-like institutions. As a result scientific research flourished. One of the major and at the same time one of the oldest branches of physics was *mechanics*. Its foundation dates back to 1687, when *Isaac Newton* (1642–1727) formulated the principles of mechanics and the gravitational law. The theory was further developed, among others, by *Joseph Louis Lagrange* (1736–1813), who formulated the dynamical equations, *Carl Friedrich Gauss* (1777–1855), who introduced the ‘principle of least constraints’, as well as *William Rowan Hamilton* (1805–1865) and *Carl Gustav Jacob Jacobi* (1804–1851), who worked out a new scheme of mechanics. They stated that motions of objects in nature always occur with least action, which was defined as the time integral over the so-called Lagrange function.

On the basis of these discoveries *thermodynamics* was developed as a new branch of physics. *Julius Robert Mayer* (1814–1878) and *James Prescott Joule* (1818–1889) found out that heat fully corresponds to energy. The first and the second law of thermodynamics were first explicitly stated in a book by *Rudolf Emanuel Clausius* (1822–1888) in 1850. Clausius also shaped the concept of entropy in 1865. Maxwell’s velocity distribution for the kinetic theory of gases was then explained by Boltzmann (1844–1906) with statistical mechanics. At the end of the 19^{th} century this lead to the important problem of blackbody radiation, i.e. the quest for a theoretical understanding of the spectrum emitted by a perfect absorber (see chapter 1.2.1).

Electrodynamics and optics were two separate disciplines until *Heinrich Hertz* (1857–1894) proved in 1888 that light possesses all characteristics of an electromagnetic wave. The first quantitative description of an electrical force (attractive or repulsive) was made by *Charles Auguste de Coulomb* (1736–1806) in 1785. *André Marie Ampère* (1775–1836) was the first to speak of electrodynamics in 1822. In 1826 *Georg Simon Ohm* (1787–1854) formulated what is nowadays known as Ohm's law. In 1833 Gauss and *Wilhelm Weber* (1804–1891) invented the telegraph. One of the most important contributions was made by *Michael Faraday* (1792–1867) who discovered electromagnetic induction and electrolysis. Based on this work *James Clerk Maxwell* (1831–1879) found a complete system of equations that describes all electromagnetic phenomena.

We conclude our excursion into the evolution of physics till the beginning of the 20^{th} century with a short glance at *atomism*. In ancient Greece, *Demokritus* introduced the idea of atoms as indivisible building block of matter. This idea was reintroduced in the 17^{th} century after it had been mostly forgotten throughout the middle ages. Chemists focused on matter that could not be separated by chemical methods. Physicists, on the other hand, tried to explain phenomena such as pressure, temperature, specific heat and viscosity in terms of the particles (molecules) that gases consist of. This approach is called the kinetic theory of gases. Out of this statistical mechanics evolved. At the beginning of the 20^{th} century the atomic hypothesis was at last widely accepted among the scientific community. It was not until 1905, however, that a theoretical proof for the existence of atoms was made simultaneously by *Albert Einstein* (1879–1955) and *Marian Smoluchowski* (1872–1917) in their work on Brownian motion. Still the structure of an atom and the ways in which the atoms of different elements differ were not yet understood at all. All in all, one can say that atomic physics was in its infancy at the turn of the century.

In the late 19^{th} century some very important discoveries were made: In 1885 *Wilhelm Conrad Röntgen* (1845–1923) discovered what he called X-rays. This phenomenon reminded *Antoine-Henri Becquerel* (1852-1908) of his work on phosphorescent stone and he began to search for a stone with similar properties. He finally found one – a uranium salt – and realized that he had observed a new kind of radiation emitted by radioactive material. This radiation later on turned out to be a very powerful tool for investigating atomic structure. In 1897 *Joseph John Thomson* (1856–1940) was able to identify the first elementary particle, the electron, and to determine its charge to mass ratio. The reaction of the scientific world was rather unenthusiastic. Some physicists didn't even believe in the concept of atoms. Others thought that atom and electrons were too small to be made objects of speculation. Later, Lord Kelvin and J.J. Thompson together developed a theory of atomic structure.

The 20th century

There were some physicists at the end of the 19^{th} century who believed that physics had come to some kind of an "end of evolution" and that there was hardly anything interesting left to be found out. Classical mechanics was able to describe almost all phenomena that had been detected and thus seemed to be satisfactory. It was a simple and unified theory.

Physicists distinguished two completely different categories of objects – matter and radiation: According to Newtonian mechanics *matter* is built out of localizable corpuscles with a well-defined position and velocity. One can thus compute the time evolution of a system as soon as one knows this data at a given moment. The corpuscular theory could even be extended to the microscopic scale of solid bodies (i.e. to molecules or atoms). According to thermodynamics and statistical mechanics macroscopic parameters thus derive from the motion of the (microscopic) particles. *Radiation*, on the other hand, could well be explained with Maxwell's laws that are able to link electromagnetism, optics and acoustics. As light was capable of interference and diffraction, which are clearly associated with waves, light was eventually considered to be a form of radiation.

At the beginning of the 20^{th} century some experiments and theoretical problems implied, however, that this distinction between radiation and matter was not entirely valid. Physicists were confronted with a bunch of data that seemed hard to explain within the framework of what we now call classical physics and were even forced to look for different and at first strange new concepts. This lead to the idea of *quantization* of physical entities and to *wave-particle dualism*. The important achievements of quantum physics in the first three decades of the new century include the following:

- **1900** Max Planck derives his formula for blackbody radiation by introducing a constant h that determines the sizes of energy packages, called *quanta*, of electromagnetic radiation.
- **1905** Albert Einstein explaines the photoelectric effect in terms of the same constant.
- **1906** J.J. Thompson discovers the proton.
- **1910** Robert Millikan measures the elementary electric charge.
- **1911** After observations on the scattering of alpha particles caused by atoms, Ernest Rutherford introduces the first modern picture of the atom.
- **1913** Niels Bohr explains spectral lines and the stability of atoms by postulating quantization of angular momentum.
- **1923** Arthur Compton gives an explanation for the scattering of photons on electrons by assigning the momentum $\vec{p} = \hbar\vec{k}$ to photons.
- **1924** Wolfgang Pauli formulates his exclusion principle.
- **1925** Louis de Broglie's doctoral thesis states that matter particles like photons are associated to waves of wavelength $\lambda = h/p$.
- **1925** Werner Heisenberg invents matrix mechanics, which assigns noncommuting matrix operators to dynamical variables.

- **1926** Erwin Schrödinger finds his equation, which describes wave mechanics.
- **1927** Werner Heisenberg derives the uncertainty relation.
- **1927** Max Born suggests the probabilistic interpretation of the wavefunction.
- **1928** Paul Adrien Maurice Dirac discovers the Dirac equation, which combines quantum mechanics with special relativity. This lead him to predict the existence of antimatter.
- **1932** Anderson's discovery of positrons in cosmic ray showers confirms Dirac's prediction.
- **1932** Chadwick observes a neutron (predicted by Rutherford in 1920).

We next discuss some of the problems mentioned above in more detail.

1.2 Limitations of classical physics

1.2.1 Blackbody radiation

A blackbody is by definition a surface that absorbs radiation entirely. One can imagine a blackbody to be a closed container with a well-absorbing surface and with a small window brought to a uniform temperature, i.e in thermal equilibrium. Radiation entering the container through the small window is reflected several times within the blackbody (see figure 1.1) and has a negligible chance for reemerging through the window. Hence this container is a perfect absorber. According to Kirchhoff's law the ratio of the emission power, or emittance, to the absorption coefficient is the same for all bodies at the same temperature. Since a blackbody has a maximum absorption coefficient it must therefore also be the most efficient emitter.

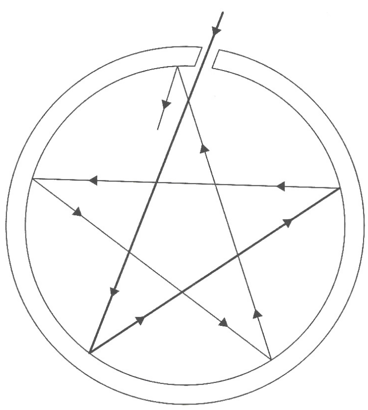

Figure 1.1: Schematic illustration of a blackbody

Rayleigh and Jeans used electrodynamics and thermodynamics to deduce a formula for the energy $u(\nu)$ per frequency interval that is emitted by such a blackbody:

$$u_{RJ} = \frac{8\pi\nu^2}{c^3} k_B T, \tag{1.1}$$

where $k_B = 1.381 \cdot 10^{-23} J/K$ is Boltzmann's constant and c is the speed of light. This formula fits the experimentally observed curve for low frequencies quite well but it deviates from the experimental value and diverges at larger ones (cf. figure 1.2)! The formula predicts an infinite total energy emission and hence cannot possibly be correct. This indicates an inconsistency between statistical mechanics and electrodynamics.

Wien also tried to describe the radiation of a blackbody. Upon general considerations he came to the conclusion that the proper term for $u(\nu)$ must be of the form

$$u(\nu, T) = \nu^3 g\left(\frac{\nu}{T}\right), \tag{1.2}$$

where g is a function that cannot be determined from thermodynamics. In order to specify this function one has to go beyond thermodynamical reasoning and use a more detailed theoretical approach. Finally Wien, Lord Rayleigh and J. Jeans managed to derive an expression for g that could explain the experimental data for higher frequencies quite well.

Planck tried to interpolate the two approximations of Wien and Rayleigh & Jeans. By guesswork he found a perfect fit to the experimental data, but he was confronted with the problem that he was lacking a theoretical derivation for this formula. Thirty-one years after this discovery Planck described this situation as follows:

> I can characterize the whole procedure as an act of desperation, since, by nature, I am peaceable and opposed to doubtful adventures. I had fought for six years with the problem [...] without arriving at any successful result. [...] I knew the formula describing the energy distribution [...] hence a theoretical interpretation had to be found at any price, however high it might be.

He made an assumption that might at first seem strange (and therefore at first was not accepted by the physicists of his time): He postulated that the energy for radiation with the frequency ν exists only in multiples of $h\nu$, where h is a constant of nature, the so called *Planck's constant*

$$h = 6.6260755 \cdot 10^{-34} Js. \tag{1.3}$$

According to this hypothesis energy is no longer a continuous quantity, but it consists of small quanta of energy $h\nu$, called *photons*. Planck thus arrived at the following expression for the energy per frequency interval $u(\nu)$:

$$\boxed{u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}}. \tag{1.4}$$

This formula fits strikingly well to the experimentally obtained curves. It looks similar to the Rayleigh-Jeans approximation, but the factor $[e^{\frac{h\nu}{k_B T}} - 1]^{-1}$ prevents the expression from diverging at higher frequencies (see figure 1.2).

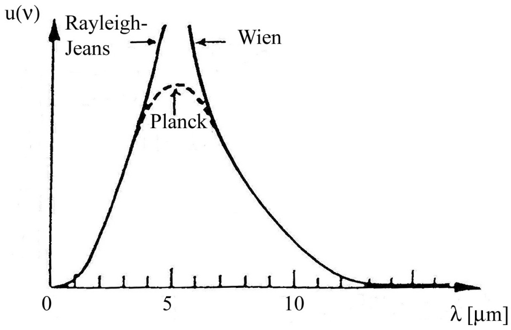

Figure 1.2: Comparison of the results for the spectrum of a blackbody according to Wien, Rayleigh-Jeans and Planck

Although Planck received a Nobel prize in 1918 for his ideas, his explanation of the spectrum of blackbody radiation did not take the world by storm at first. It seemed as if he had constructed a theory derived from experiment, but based on a hypothesis with no experimental basis.

1.2.2 The photoelectric effect

Five years later *Einstein* built on the ad hoc hypothesis of the quantization of energy to explain the phenomenon of the photoelectric effect. This effect was first observed by Hertz in 1887: If an alkali metal is irradiated by light with a frequency larger than a certain minimum frequency (which depends on the metal) electrons are emitted by this metal. It is interesting that the velocity of the electrons (and thus their energy) is only dependent on the frequency of the light beam hitting the metal, but not on its intensity. Classical physics is not able to explain the ν–proportionality of this effect. Assuming light to be an electromagnetic wave, the electrons of the metal should absorb an energy that is increasing with the intensity of the light beam until their velocity is high enough to overcome the potential well. According to this, we should be able to observe a delay between the start of the irradiation and the onset of the emission of electrons. This delay has not been measured until today, even though by now we would be able to do so (if it existed). Classical physics thus fails to explain this effect correctly.

Einstein took up the idea of Planck and even went a bit further. He assumed that light consisted of particles, called photons, with the energy $h\nu$. When one of these corpuscles encounters an electron of the metal, it is absorbed and the electron receives its energy $h\nu$ (at one instant). If this energy is large enough for the electron to overcome the potential of the atom, it escapes. The energy of such an electron would be

$$\frac{1}{2}mv^2 = h\nu - W, \tag{1.5}$$

where W is the work needed to free an electron from the potential well. This theory is in complete accord with the experiment.

At this time the whole extent of the idea of energy or light quanta could not yet be perceived. Planck thought that his hypothesis was a mere complement to the theories known so far. Years later it became evident that they were in fact revolutionary. Nernst wrote in 1911:

> It appears that we find ourselves at present in the midst of an all-encompassing re-formulation of the principles on which the erstwhile kinetic theory of matter has been based.

Although Einstein himself contributed to the development of this new theory, he turned out to be a strict opponent to some of its consequences. In 1944 he wrote in a letter to Max Born:

> You believe in the God who plays dice, and I in complete law and order in a world which objectively exists, and which I, in a wildly speculative way, am trying to capture. I hope that someone will discover a more realistic way [...] than it has been my lot to find. Even the great initial success of Quantum Theory does not make me believe in the fundamental dice-game, although I am well aware that our younger colleagues interpret this as a consequence of senility. No doubt the day will come when we will see whose instinctive attitude was the correct one.

Einstein was appreciated for his work with a nobel prize in 1921.

1.2.3 Bohr's theory of the structure of atoms

At the end of the 19^{th} century Gustav Kirchhoff and Robert Bunsen examined the spectrum of gas atoms. If you energize a tube filled with gas of atoms of a certain kind, the gas begins to glow at a sufficient voltage. It emits a line spectrum, i.e. the emerging light has a discrete set of wavelengths. It turned out that every atom has a characteristic spectrum. The atomic number Z and the wavelengths of the spectrum are related by the *Rydberg-Ritz-formula*:

$$\boxed{\frac{1}{\lambda} = RZ^2\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \tag{1.6}$$

λ	...	wavelength of spectral line
R	...	Rydberg's constant, for big Z; $R_\infty = 10,97373\frac{\mu}{m}$
Z	...	atomic number
n,m	...	whole numbers with $n > m$

At first there was no theoretical explanation for this formula. In 1911 *Rutherford* and his coworkers Hans Geiger and Ernest Marsden deduced from scattering experiments of α-particles off a golden foil that the positive charge of the atom is cumulated in a small center, the nucleus. They imagined that the electrons move along circular or elliptical orbits around the nucleus, just like the planets move around the sun. Within the framework of classical physics, the moving electron would radiate (because its circular trajectory is equivalent to an accelerated movement) and thus loose energy until it would eventually fall into the nucleus within 10^{-8} seconds.

Many attempts were made to overcome these and similar difficulties without any significant success. Physicists tried to find a solution to this problem within the framework of the newly arisen quantum theory. It appeared natural to do so since the discrete lines in the spectra of atoms seemed to be related to the fact that the energy of an oscillator assumed values that were integral multiples of the energy packets $h\nu$. In 1913 a so far unknown physicist, *Niels Bohr*, who worked with Rutherford in Manchester and had therefore come to know his model of the atom, had an idea to avoid this 'disaster'. He set up two *postulates*:

- The electron moves around the nucleus in discrete circles according to classical mechanics. In these (stationary) states with energy E_n the atom does not radiate and the momentum is given by:

  $$\oint p\, dr = nh \tag{1.7}$$

  The line integral extends over the electron's orbit around the nucleus.

- When an atom undergoes a change from energy E_n to E_m it emits a photon with the energy

  $$E = E_n - E_m \tag{1.8}$$

  and correspondingly with the frequency

  $$\nu = \frac{E_n - E_m}{h}. \tag{1.9}$$

Let us consider the first postulate in more detail. If the electron moves along a circular trajectory, the line integral is

$$2\pi r p = nh \tag{1.10}$$

or, with $p = \hbar k = \frac{h}{\lambda}$,

$$2\pi r = n\lambda. \tag{1.11}$$

The circumference of the electron's orbit thus is a multiple of the wavelength λ of the electron and the orbits are quantized. We will now calculate the radius and the energy for such an orbit.

The electron moves in a circular orbit around the nucleus. The centripetal force thus balances the Coulomb force between the electrons and the protons,

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}. \tag{1.12}$$

So the radius of the atom is

$$r = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{mv^2}. \tag{1.13}$$

With $\vec{p} = m\vec{v}$ we find

$$r = \frac{1}{4\pi\epsilon_0} m \frac{Ze^2}{p^2} \tag{1.14}$$

Using the above quantization rule,

$$p = \frac{nh}{2r\pi}, \tag{1.15}$$

the radius becomes

$$r_n = \frac{n^2}{Z}\frac{\epsilon_0 h^2}{me^2\pi} = \frac{n^2}{Z}a_0 \tag{1.16}$$

$$a_0 = \frac{\epsilon_0 h^2}{me^2\pi} \tag{1.17}$$

r_n ... radius of the electron's orbit, for $n = 1, 2, 3, ...$ different radii
a_0 ... *Bohr radius*

Each radius belongs to a certain energy E_n. The energy for an electron in an orbit with the radius r_n is

$$E_n = \underbrace{\frac{mv^2}{2}}_{E_{kin}} - \underbrace{\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r_n}}_{E_{pot}} \tag{1.18}$$

Using equation (1.12) we find

$$mv^2 = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r_n}. \tag{1.19}$$

Inserting this and formula (1.16) into the expression for E_n we find

$$E_n = \frac{1}{8\pi\epsilon_0}\frac{Ze^2}{r_n} - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r_n} = -\frac{1}{8\pi\epsilon_0}\frac{Ze^2}{r_n}, \tag{1.20}$$

$$E_n = -\frac{me^4}{8\epsilon_0^2 h^2}\frac{Z^2}{n^2}. \tag{1.21}$$

Let us now return to the initial problem: the spectrum emitted by atoms and the Rydberg-Ritz formula (1.6). If an electron falls from the energy level E_n to a lower level E_m it emits a photon with a wavelength λ corresponding to $E_n - E_m$. According to (1.21):

$$\frac{hc}{\lambda} = \Delta E = E_n - E_m = \frac{me^4}{8\epsilon_0^2 h^2} Z^2 \left(\frac{1}{m^2} - \frac{1}{n^2}\right) \tag{1.22}$$

So we end up with formula (1.6):

$$\frac{1}{\lambda} = \frac{me^4}{8\epsilon_0^2 h^3 c} Z^2 \left(\frac{1}{m^2} - \frac{1}{n^2} \right) = RZ^2 \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \tag{1.23}$$

$R = \frac{me^4}{8\epsilon_0^2 h^3 c}$ ... *Rydberg's constant*

We thus find the following picture of the structure of an atom:

- The *bound electrons* of an atom move along circular orbits with different radii. The radii are quantized and correspond to *discrete energy values*. These values are all *negative*.
- There is a *minimum energy* $E_0 = -\frac{me^4}{8\epsilon_0^2 h^2} Z^2$ (formula (1.21) with $n = 1$), the *ground state* of the atom. If an electron is excited to a higher energy level ($n = 2, 3, 4 \ldots$), it always returns to an energy as low as possible, whereby it emits light of a certain frequency.
- For $r_n \to \infty$ the energy of an electron becomes $\lim_{n\to\infty} E_n = 0$. For *E > 0* the atom is ionized and *all (continuous) values of the energy* are allowed.

Many years later, Werner Heisenberg recalled the work on the development of the atomic model:

> I remember discussions with Bohr which went through many hours till very late at night and ended almost in despair; and when at the end of the discussion I went alone for a walk in the neighbouring park I repeated to myself again and again the question: Can nature possibly be so absurd as it seemed to us in these atomic experiments?

Niels Bohr was awarded the nobel prize in 1922.

1.2.4 The Compton effect

The Compton effect also confirms the photon theory. Consider free electrons irradiated by x-rays (see figure 1.3). One observes that the wavelength of the incoming x-rays is different from the wavelength of the outgoing ones.

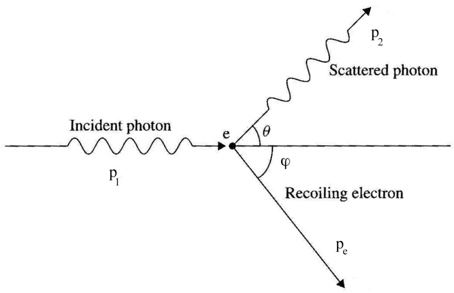

Figure 1.3: The experimental setup for the Compton effect

$$\lambda_{in} \neq \lambda_{out} \tag{1.24}$$

The difference $\Delta\lambda$ is related to the angle θ between the direction of propagation of the x-rays and of the scattered beam according to

$$\boxed{\Delta\lambda = 2\frac{h}{mc} sin^2\frac{\theta}{2}} \tag{1.25}$$

It is not possible to understand the shift of the wavelength of the radiation from a classical point of view. If we regard the x-rays as waves, the electrons should absorb energy and then re-emit radiation of the same wavelength λ. So, what is the origin of this $\Delta\lambda$?

Compton managed to explain this effect using the idea of photons. The irradiation of the electrons can thus be understood as an elastic collision between a photon and an electron. The photon loses energy to the electron and, since its wavelength is inversely proportional to the energy, it has to increase.

Since photons travel at the speed of light their energy and momentum are related by the relativistic formula $E^2 = m_0^2c^4 + p^2c^2$ with rest mass $m_0 = 0$, i.e. $|p| = E/c$. The *Planck-Einstein relation* $E = h\nu$ and the relation between frequency ν and wave vector $\vec{k}$ in vacuum thus imply

$$E = h\nu = \hbar\omega, \tag{1.26}$$

$$\vec{p} = \hbar\vec{k}. \tag{1.27}$$

Considering the elastic collision of a photon with an electron we can use the conservation of *momentum*

$$\vec{p}_1 = \vec{p}_2 + \vec{p}_e, \tag{1.28}$$

or

$$\hbar\vec{k}_1 = \hbar\vec{k}_2 + \vec{p}_e \tag{1.29}$$

$\vec{p}_1,\vec{k}_1$	...	momentum, wave vector before the impact
$\vec{p}_2,\vec{k}_2$	...	momentum, wave vector after the impact
$\vec{p}_e$	...	momentum of the electron

and the conservation of *energy*

$$\underbrace{p_1c}_{\text{moving photon}} + \overbrace{m_ec^2}^{\text{resting electron}} = \underbrace{p_2c}_{\text{moving photon}} + \overbrace{\sqrt{p_e^2c^2+m_e^2c^4}}^{\text{moving electron}} \tag{1.30}$$

or, with $pc = E = \hbar\omega$ and $\omega = kc$

$$\hbar k_1 + m_e c = \hbar k_2 + \sqrt{p_e^2 + m_e^2c^2} \tag{1.31}$$

Combining (1.29) and (1.31) and eliminating $\vec{p}_e$, where the scalar product of $\vec{k}_1$ and $\vec{k}_2$ is

$$\vec{k}_1\vec{k}_2 = k_1k_2cos\theta \tag{1.32}$$

with θ being the angle between $\vec{k}_1$ and $\vec{k}_2$, we finally end up with formula (1.25).

1.2.5 Interference phenomena

So far, we have considered situations of electromagnetic waves behaving in a corpuscular manner. We have come to the conclusion that it is problematic to describe some phenomena in a classical way. In the following we will see that the new corpuscular theory is insufficient too and that a combination of wave and particle aspects of matter is needed.

Problems with the newly introduced photon theory arise when we observe phenomena such as diffraction or interference. Is there a way to find an explanation for these things based upon the photon theory? Consider *Young's double-slit experiment* (see figure 1.4), in which light falls on a wall with two slits. Behind that wall there is a detector like a photographic plate in order to observe the interference pattern that is produced by the wall. The blackening of the photographic plate is proportional to the distribution of the light intensity.

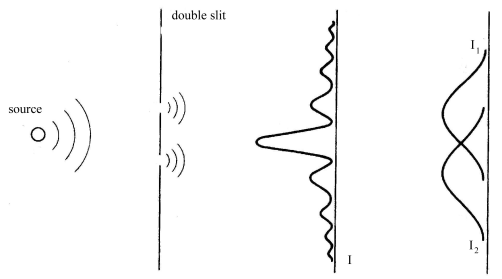

Figure 1.4: Young'd double slit experiment

The two beams produced by slit one and slit two interfere and thus the total intensity on the screen depends on the phase between the two beams. If these beams are represented by the two wave functions

$$\psi_1 = |\psi_1| e^{i\varphi_1} \tag{1.33}$$

$$\psi_2 = |\psi_2| e^{i\varphi_2} \tag{1.34}$$

where φ_1 and φ_2 are the phases of the two waves, and thus functions of $(\vec{r}, t)$, the overall intensity on the photographic plate is

$$I = |\psi|^2 = |\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + \underbrace{|\psi_1 \psi_2|[e^{i(\varphi_1 - \varphi_2)} + e^{i(\varphi_2 - \varphi_1)}]}_{\text{interference term}}, \tag{1.35}$$

which is not only the sum of the two intensities I_1 and I_2,

$$I \neq I_1 + I_2 = |\psi_1|^2 + |\psi_2|^2. \tag{1.36}$$

One could try to explain this result with the interaction of the photons that passed through slit one and those that passed through slit two. If we diminish the intensity of the light beam that falls on the wall and increase the exposure time so that the overall amount of photons that are detected on the plate behind the wall remains the same, the photons eventually pass the two slits one after another and thus cannot interact. But the interference pattern on the photographic plate is found to stay the same!

It seems as if in this case the wave-aspects of light would dominate. But if we diminish the intensity of the light beam and keep the exposure time short, we are still able to detect localized impacts on the photographic plate, i.e. single photons. Here the wave theory is insufficient. On the other hand, even if these photons pass the double slit one by one (without possible interaction) they still generate the interference pattern. The result of this experiment leads to a paradox: As mentioned before the intensity distribution of a double slit is *not* simply the sum of two single slits. Although a photon is far too small to "know" whether there is a second slit or not, it nevertheless seems to be aware of it and moves accordingly. While all photons are emitted under essentially the same conditions, their trajectories are different. The initial state of a system thus no longer determines its evolution in time. There is only a *statistical probability* for different locations (for example, photons are more likely to hit the photographic plate at a maximum of the intensity of the interference pattern than at a minimum).