

# Theory of higher spin particles

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String Ensemble





# Outline

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- Future talks by the Wiener String Ensemble
- Which questions to answer to understand theory of higher spin particles?
- Standard Model particles
- Spin of a particle
- Gauge theory
- Higher spin theory

## Mini-Lectures at Pint of Science

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- 12.5. Various lectures at various restaurants [Pint of Science]

## Presentation with Münze Austria

- 2.6. Daniel Grumiller & Timon Zipfelmaier: *Anlässlich der Prägung einer Schwarzen Loch Münze* [TUtheSky]

## Philosophy lecture at University of Vienna

- 30.6. Richard Dawid: Stringtheorie: Spekulation oder abgesicherte Theorie? [SkyLounge]

## Opened lectures at Strings 2022

- 22.7. Netta Engelhardt (MIT) [Festsaal der ÖAW]
- 23.7. TBA [Festsaal der ÖAW]

<https://indico.cern.ch/event/1085701/>



**Strings**  
Vienna 2022  
July 18-22

organized by University of Vienna,  
in cooperation with TU Vienna and ÖAW

universität wien | TU WIEN TECHNISCHE UNIVERSITÄT WIEN | ÖAW | FWF Der Wissenschaftsfonds. | CITY OF VIENNA

What is particle?

What is spin?

What is higher spin particle?

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How do we define a theory?

How do we define the theory of higher spin particles?



# Particles

- Everything we see around us is made of particles



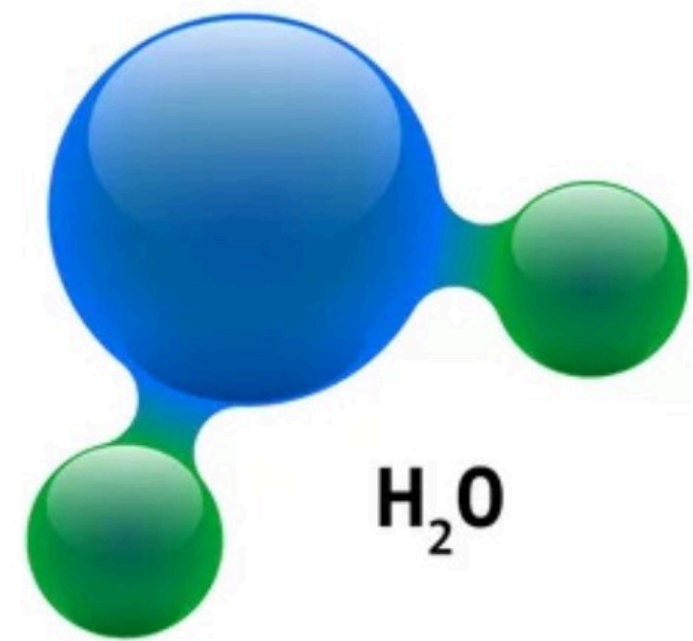
Matter



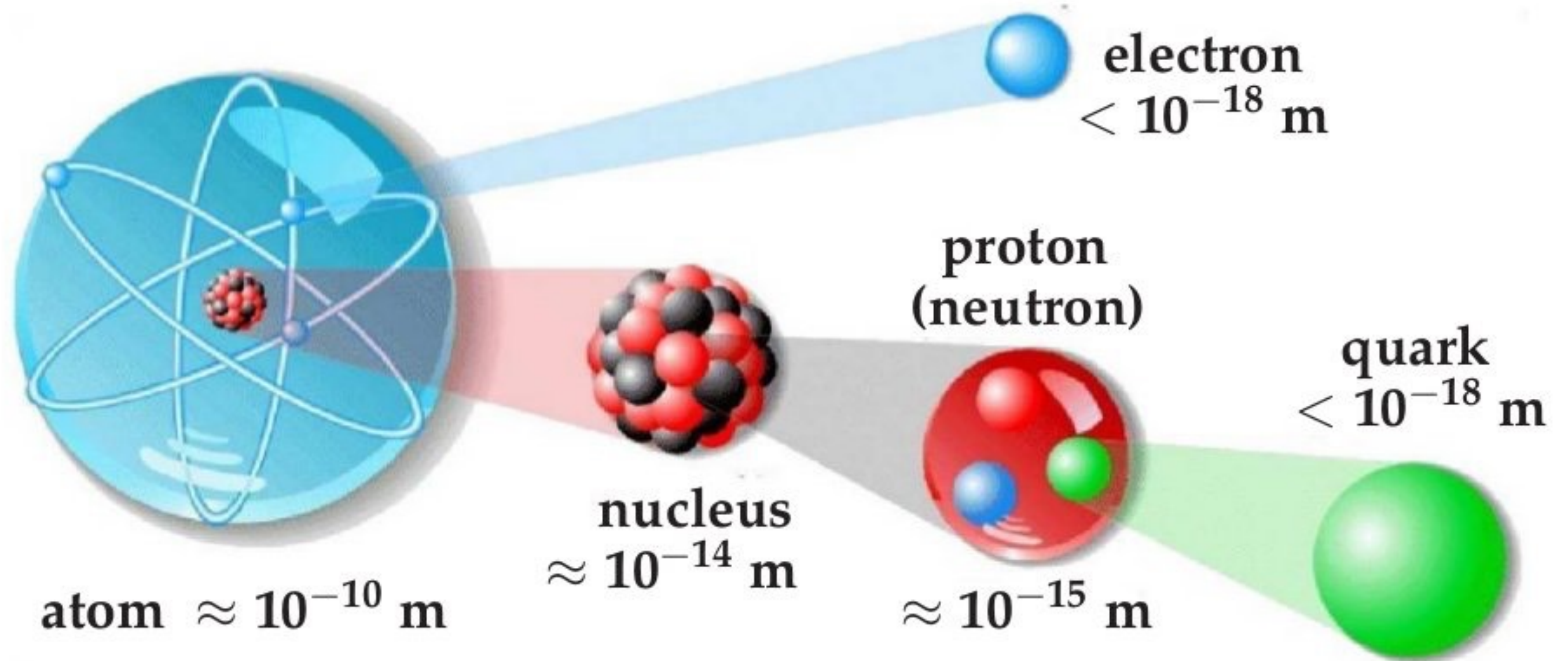
Molecules



Atoms



H<sub>2</sub>O



- Quarks, gluons and electrons are some of the elementary particles

# Particles

- Particles that we know exist today are organised in the table of Standard Model of Elementary Particles
- They are divided in quarks, leptons, gauge bosons and scalar boson
- **Quarks** are basic ingredients that build nuclei of an atom
- They can interact via all force carriers
- **Leptons** interact with all force carriers beside gluon
- **Electrons** build atoms

## Standard Model of Elementary Particles

	three generations of matter (fermions)			interactions / force carriers (bosons)	
	I	II	III		
mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 124.97 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
	<b>u</b> up	<b>C</b> charm	<b>t</b> top	<b>g</b> gluon	<b>H</b> higgs
	$\approx 4.7 \text{ MeV}/c^2$	$\approx 96 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
	<b>d</b> down	<b>S</b> strange	<b>b</b> bottom	<b><math>\gamma</math></b> photon	
	$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	$\approx 91.19 \text{ GeV}/c^2$	
	-1	-1	-1	0	
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
	<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau	<b>Z</b> Z boson	
	$< 1.0 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$	$\approx 80.39 \text{ GeV}/c^2$	
	0	0	0	$\pm 1$	
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
	<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b>W</b> W boson	

QUARKS

LEPTONS

GAUGE BOSONS  
VECTOR BOSONS

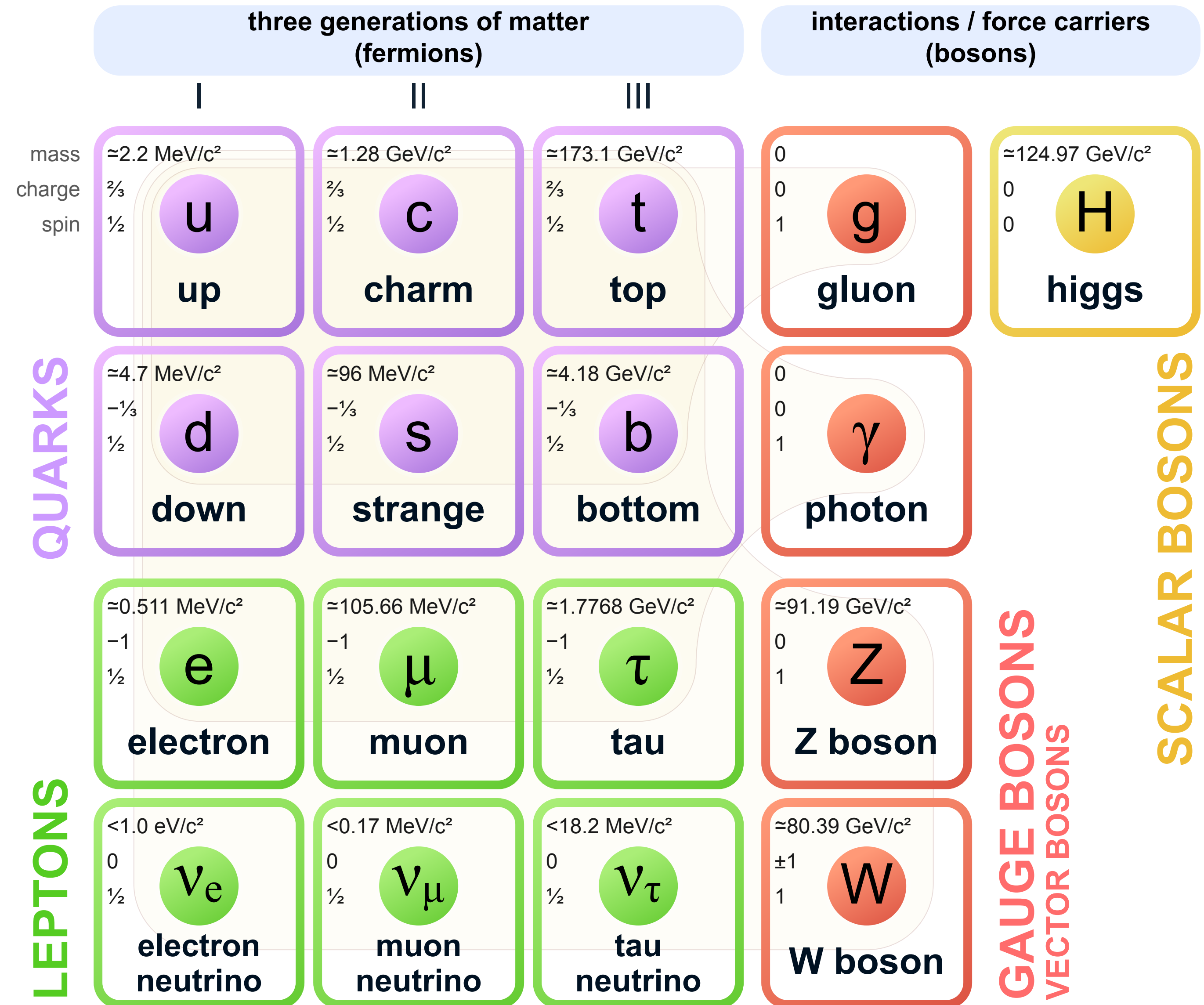
SCALAR BOSONS



# Standard Model of Elementary Particles

## Particles

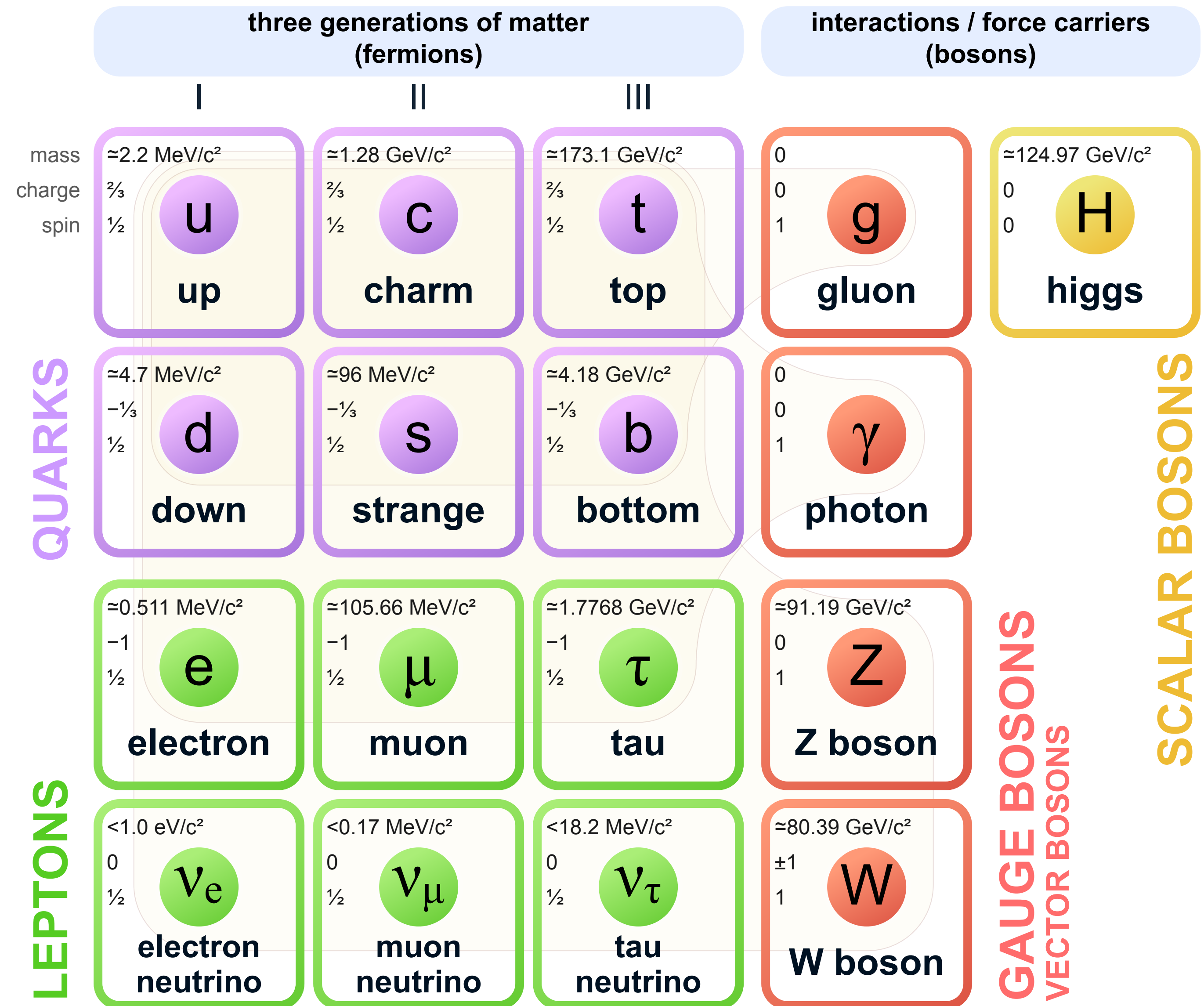
- **Muons** - byproduct of cosmic ray interaction with atmosphere
- **Tau** and **heavier quarks** - appear in exotic nuclei and are found in accelerators
- **The force carriers**, are responsible for the interaction between the particles
- The **Higgs particle** is responsible for giving a mass to other particles



# Particles

- **Historically**, people new about existence of electric force for long time:
- The Ancient Greeks knew that amber attracted small objects when rubbed with fur
- Sir Joseph John Thompson then in 1897 discovered electron particle
- There was a zoo of hadrons discovered beginning of the 20th century, but it wasn't until mid 20th century Murray Gell-Mann and Georg Zweig came up with classification of hadrons
- Boom of particle discoveries at Stanford linear collider in 60s and 70s: u,d,s,c

# Standard Model of Elementary Particles



# Particles

- They are grouped by their properties:
- mass,
- charge,
- spin
- and their gauge group: weak, strong, electromagnetic interaction, and gravity

charge of an electron

$$1 \text{ eV}/c^2 = \frac{(1.602\,176\,634 \times 10^{-19} \text{ C}) \cdot 1 \text{ V}}{(2.99\,792\,458 \times 10^8 \text{ m/s})^2} = 1.782\,661\,92 \times 10^{-36} \text{ kg}$$

speed of light

mass  
charge  
spin

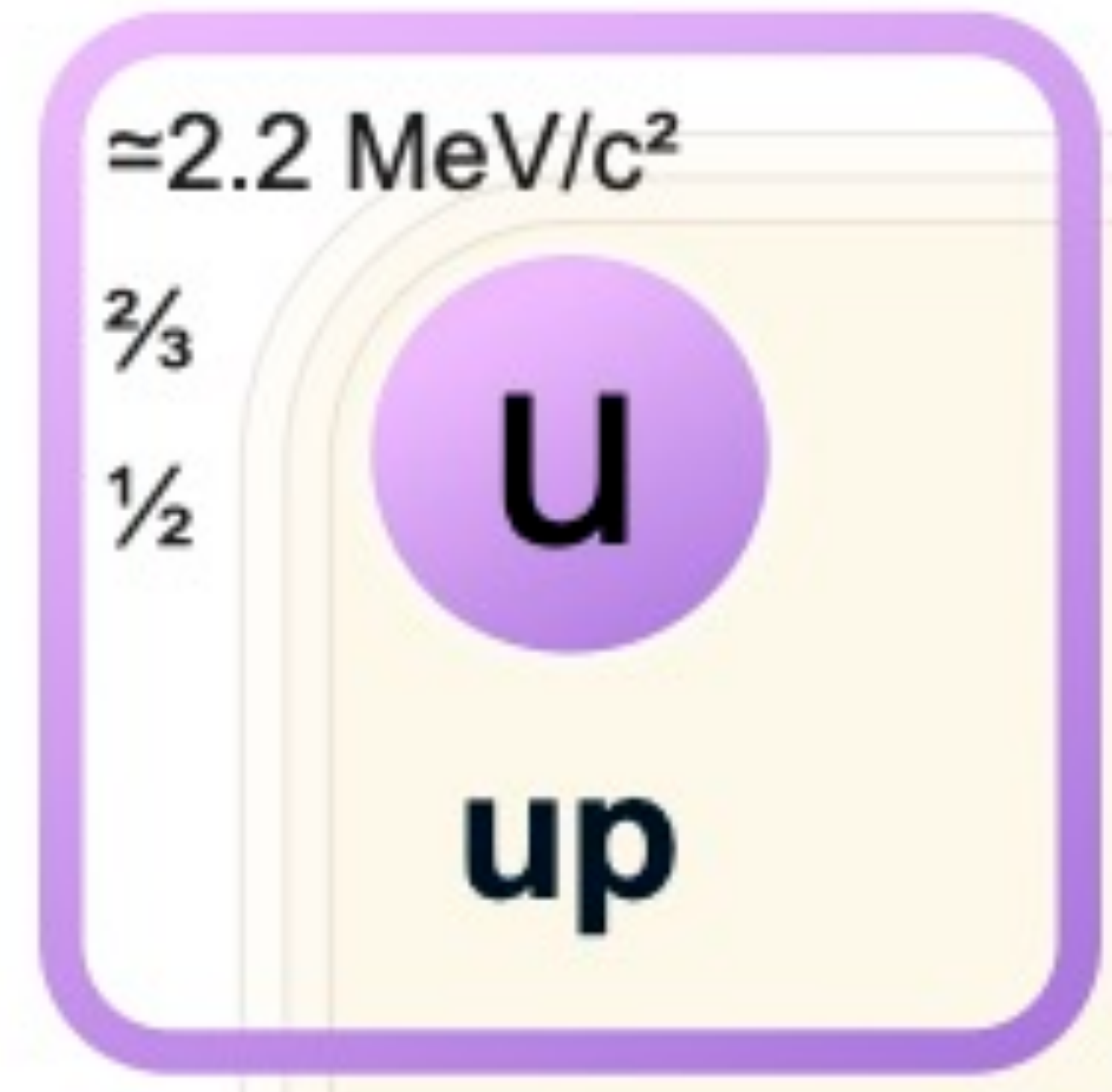
$\approx 2.2 \text{ MeV}/c^2$

$\frac{2}{3}$

$\frac{1}{2}$

u

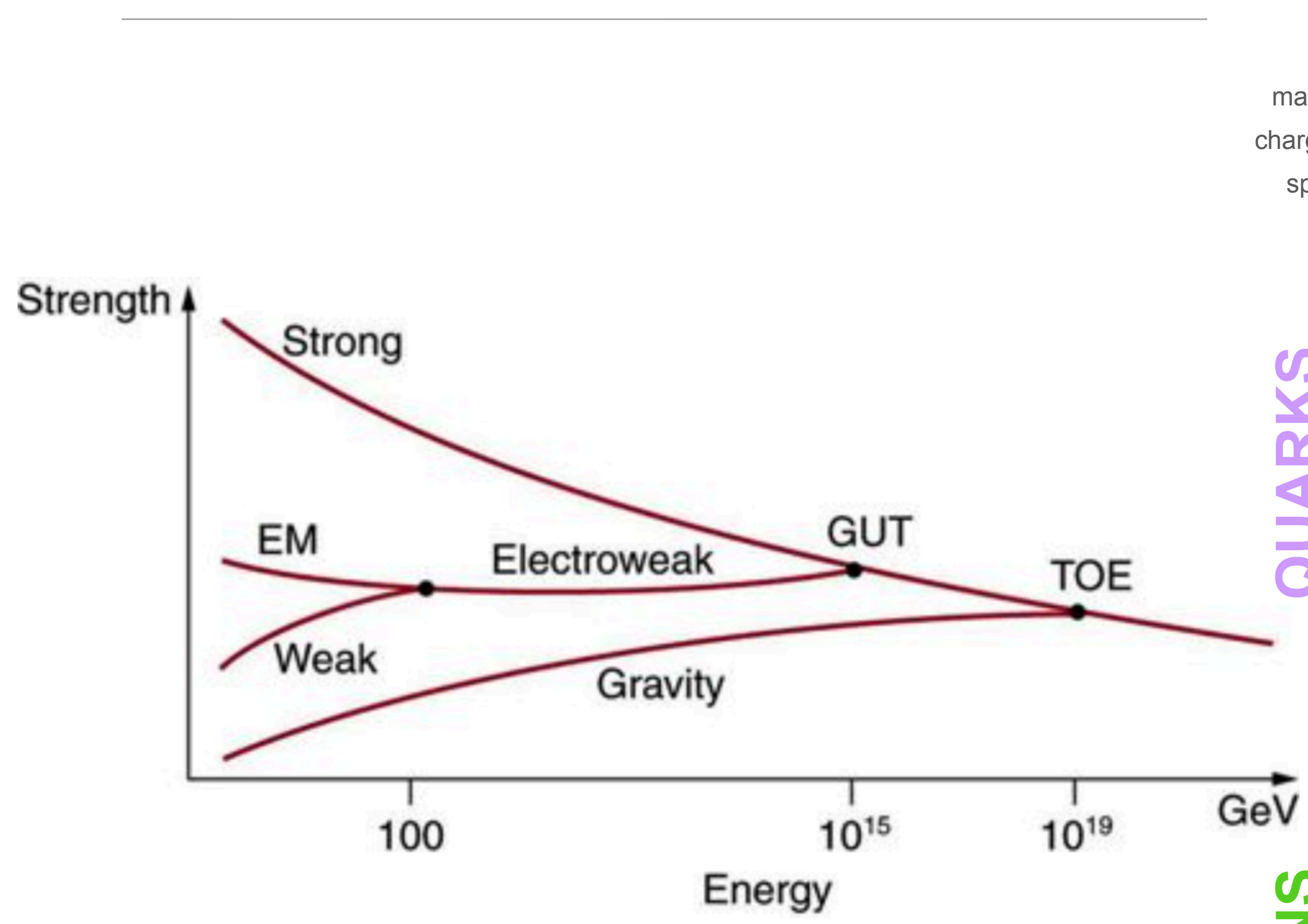
up



# Standard Model of Elementary Particles

three generations of matter (fermions)      interactions / force carriers (bosons)

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					<b>GAUGE BOSONS</b> <b>VECTOR BOSONS</b>
					<b>SCALAR BOSONS</b>



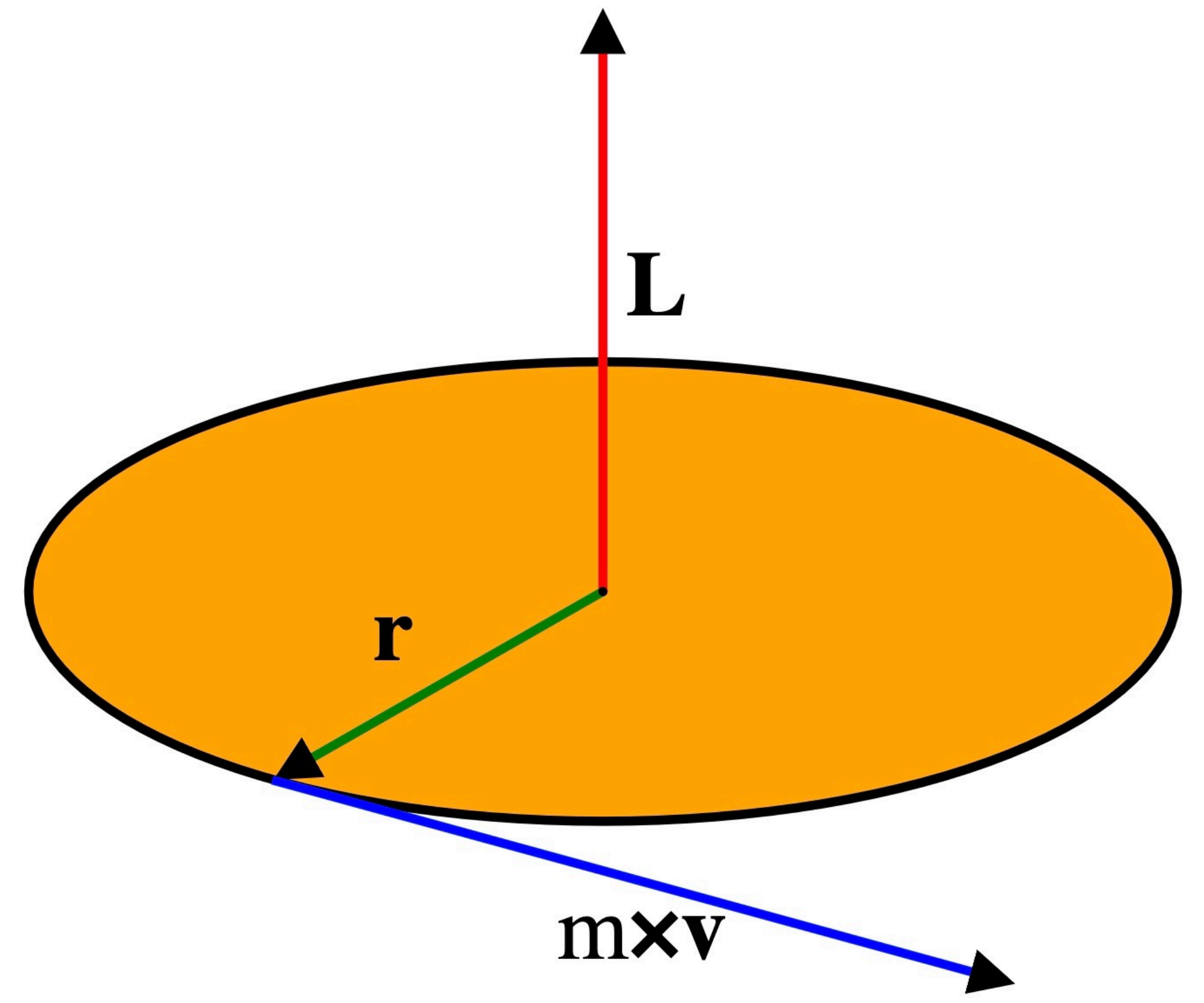
- These are the particles that we know exist today, and that we have measured
- What about other particles?
- One of the candidates for that underlying and unifying theories of everything is theory of higher spin particles, or higher spin theory
- We have seen that that the know particles have spins 0,  $1/2$  and 1
- There is also yet undiscovered particle of spin 2 that we know about which is graviton
- First, let's see what is spin actually?

# Spin

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- What is angular momentum?
- Orbital analog of linear momentum  
body of mass  $m$  moving with speed  $v$   
has linear momentum  $\vec{p} = m\vec{v}$

$$\vec{L} = m\vec{v}\vec{r}$$



# Spin

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- It is conserved 
$$\vec{L} = m\vec{v}\vec{r}$$
- Example 1. Changing the radius of the mass will cause faster rotation  
[https://www.youtube.com/watch?v=49zi\\_73pm1Q](https://www.youtube.com/watch?v=49zi_73pm1Q)
- Example 2. In a closed system the angular momentum can transfer from one part of the system to another  
<https://www.youtube.com/watch?v=ISImuPcmiC4>

# Example 1.

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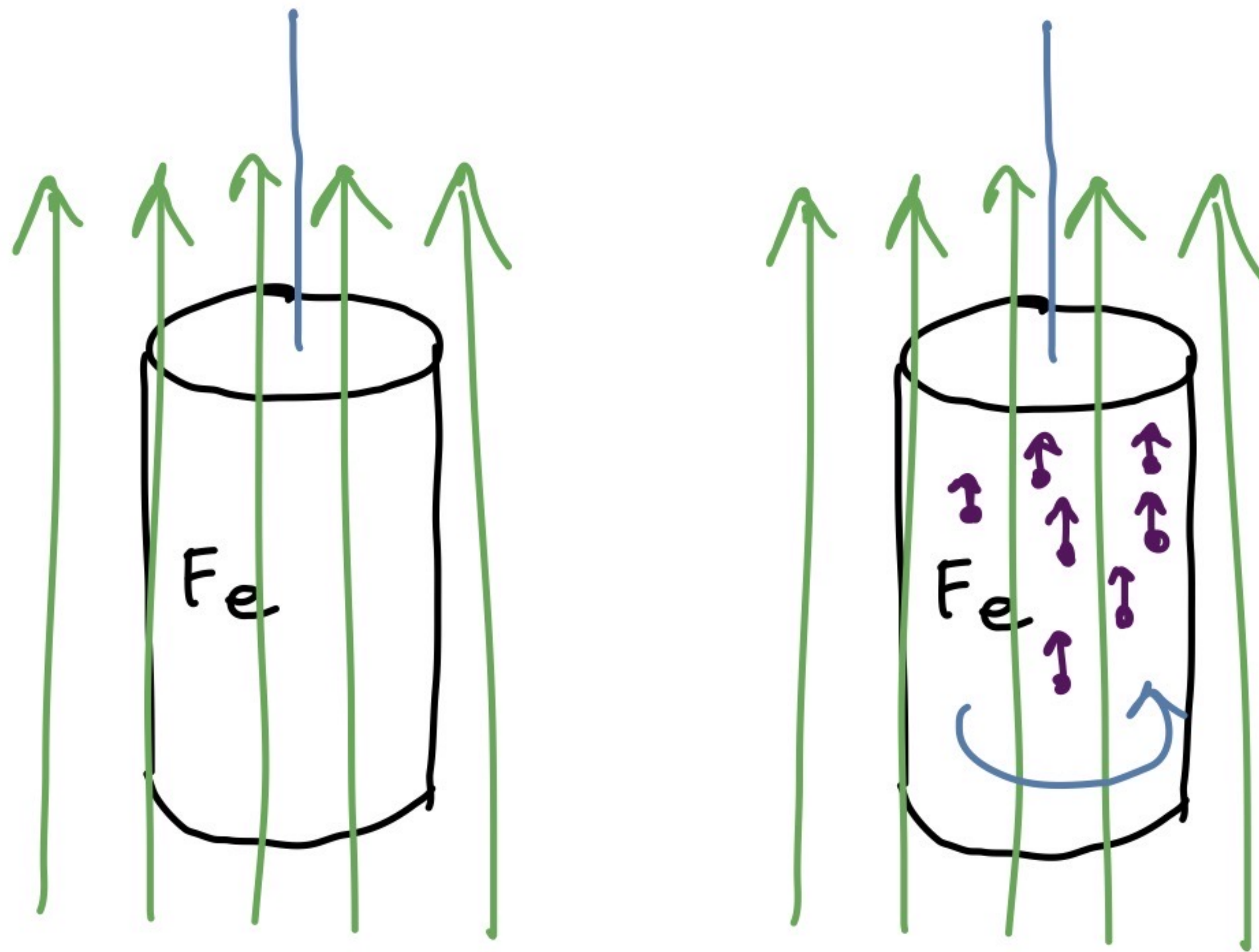




## Example 2.



# Spin



- external magnetic field magnetised the iron
- electron in the outer shell of iron aligned their spins
- they act as tiny bicycle wheels
- their shifted angular momenta needs to be compensated by rotation of the cylinder
- we imagine spin of an electron as bicycle wheels, BUT
- electrons definitely DO NOT spin as bicycle wheels
- they possess specific angular momentum

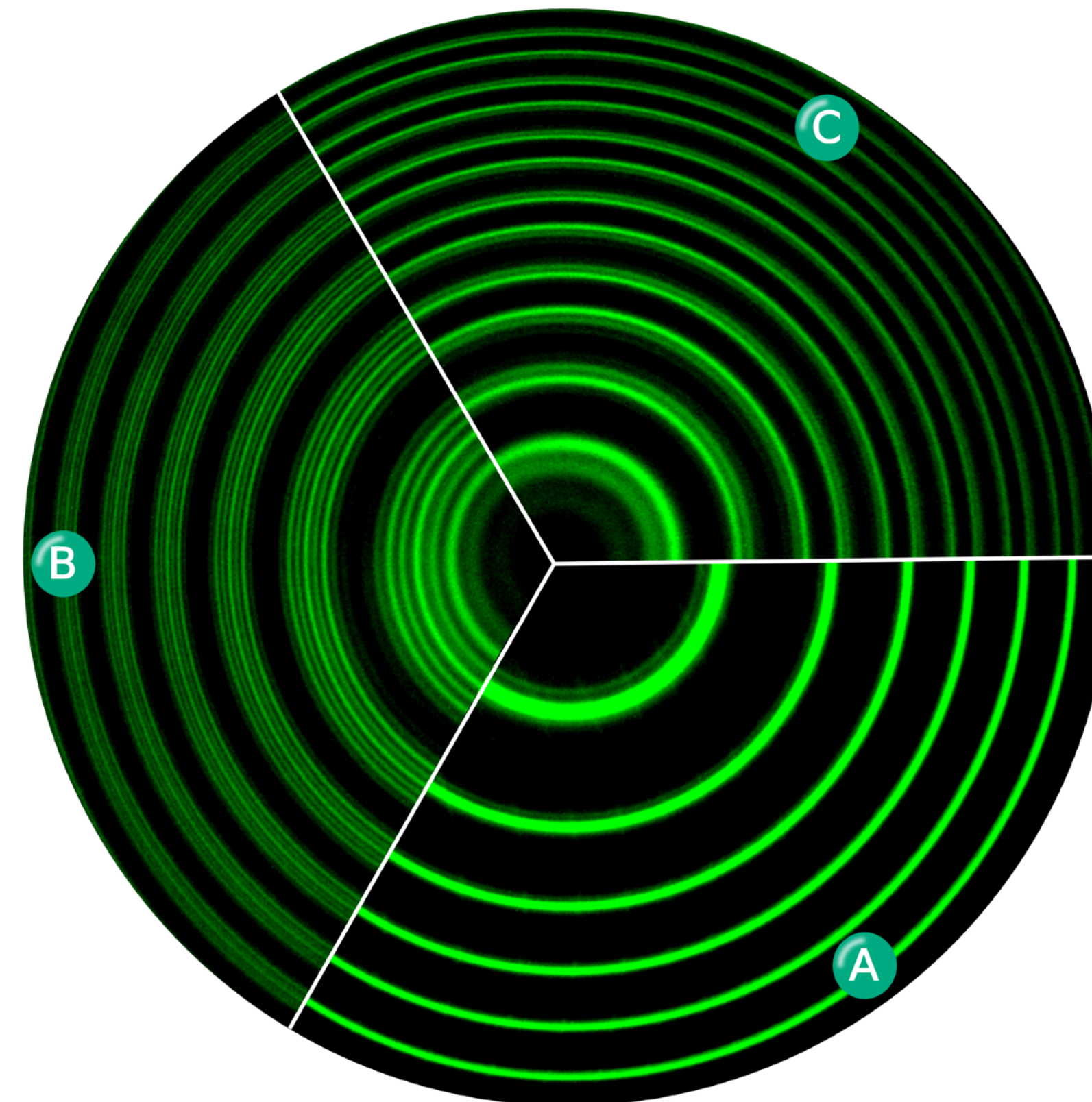
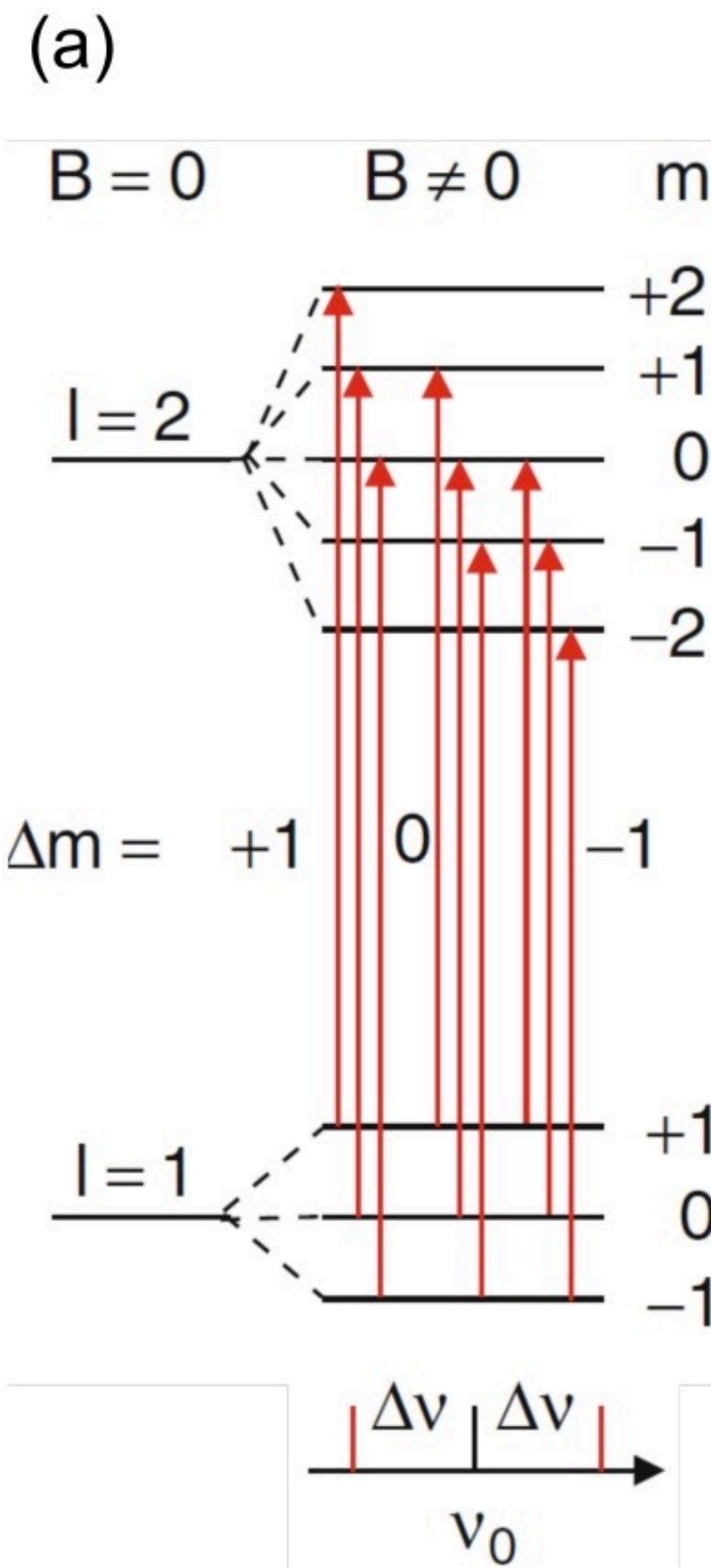
Einstein - de Haas effect, 1915

# Spin

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- Spin of an electron somehow exists without classical rotation
- It is a quantum property of a particle, like mass or various charges
- It is related to fundamental structure of matter and we want to see what it really is

- First indication of spin-like properties was Zeeman effect
- he was looking at specific wavelengths of photons when electrons jump between energy levels
- these levels split when atom is put in external magnetic field



- Zeeman effect was explained by Lorenz:
- Electrons that rotate around atom have magnetic moment, and create magnetic field, like tiny magnet
- Different alignment of that orbital magnetic field along external magnetic field turn one energy level into three
- Anomalous Zeeman effect: Sometimes this energy levels split further, as if electron has its own magnetic moment
- So we get alignment of both, orbital magnetic moment, and electron's internal magnetic moment that contribute to new energy levels

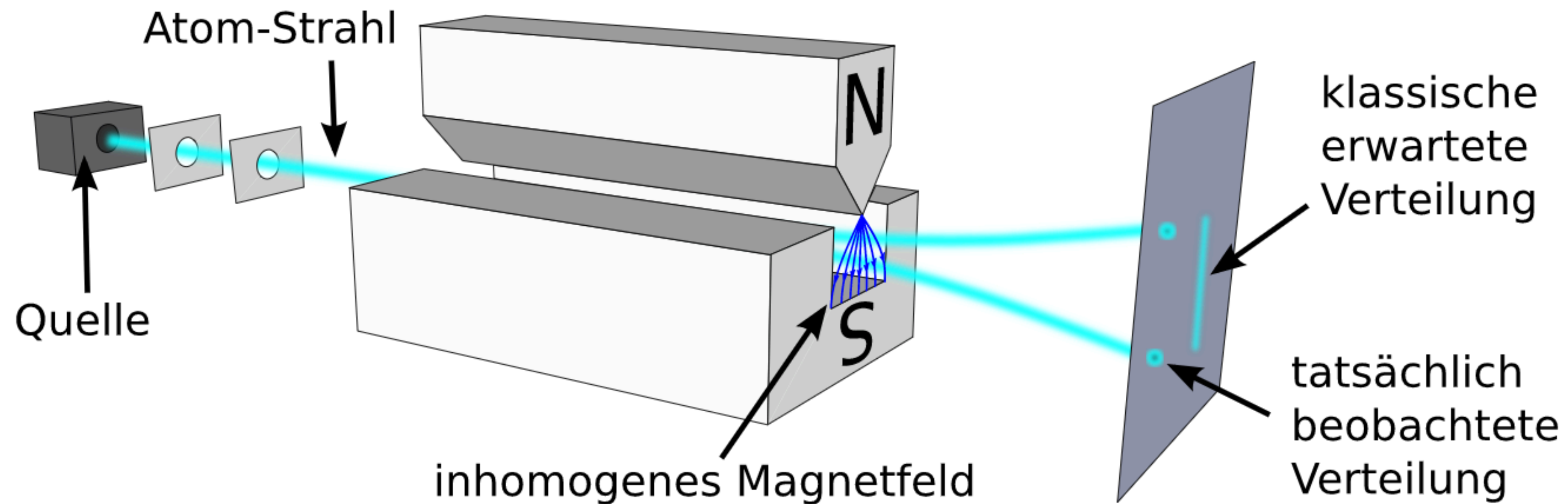
- For that to explain the spin of an electron we need to think of them as spinning balls of charge, but that brings problems
- To achieve the experimentally measured values, electrons should be rotating faster than a speed of light
- That was discovered by Austrian physicist, Wolfgang Pauli
- He had to assume that electrons have maximal possible size
- Pauli didn't like that idea and described the property as "classically non-describable two-valuedness"

 electrons are not spinning, but they act as if they have angular momentum

- Electrons have quantum spin
- it is completely quantum mechanical
- it gives them intrinsic angular momentum
- due to it, charged particles get magnetic field
- experiment that finally convinced community into the quantum properties of spin of a particle was Stern-Gerlach experiment in 1921:
- Atoms of silver are sent through magnetic field that is stronger above (near north pole) and weaker going down (at south pole)

# Stern-Gerlach experiment

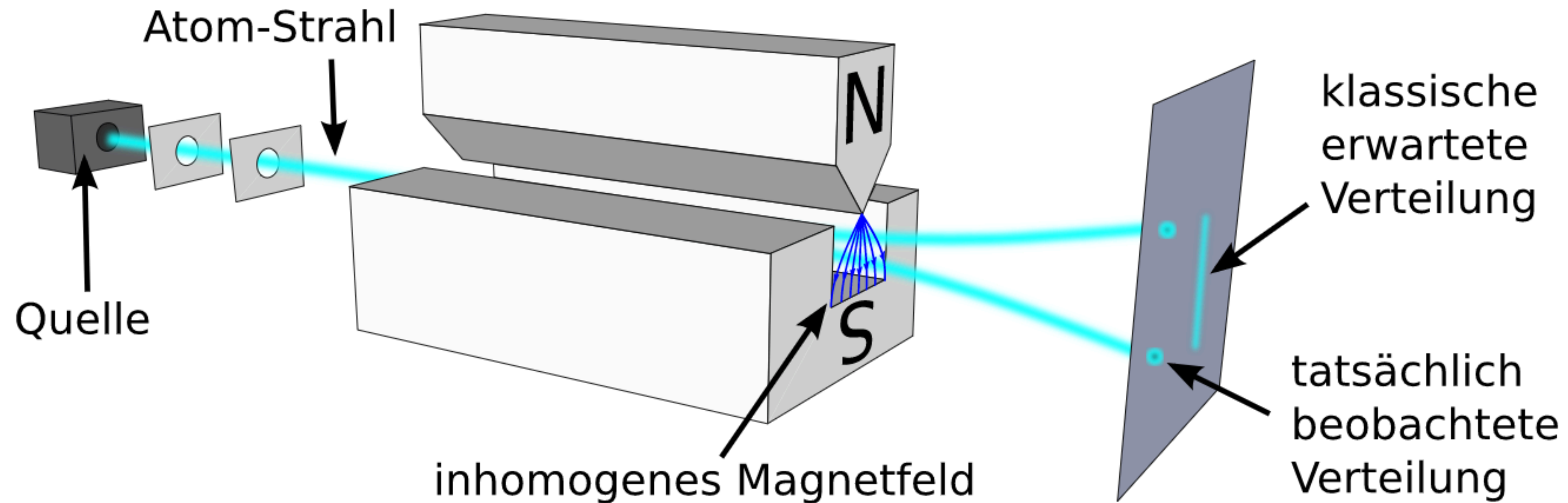
- silver atoms have one electron in their outer shell which grants the atom magnetic moment
- because of that moment external magnetic field will induce a force on the atoms
- the direction of that force depends on where the little magnetic moments are pointing, relative to magnetic field





# Stern-Gerlach experiment

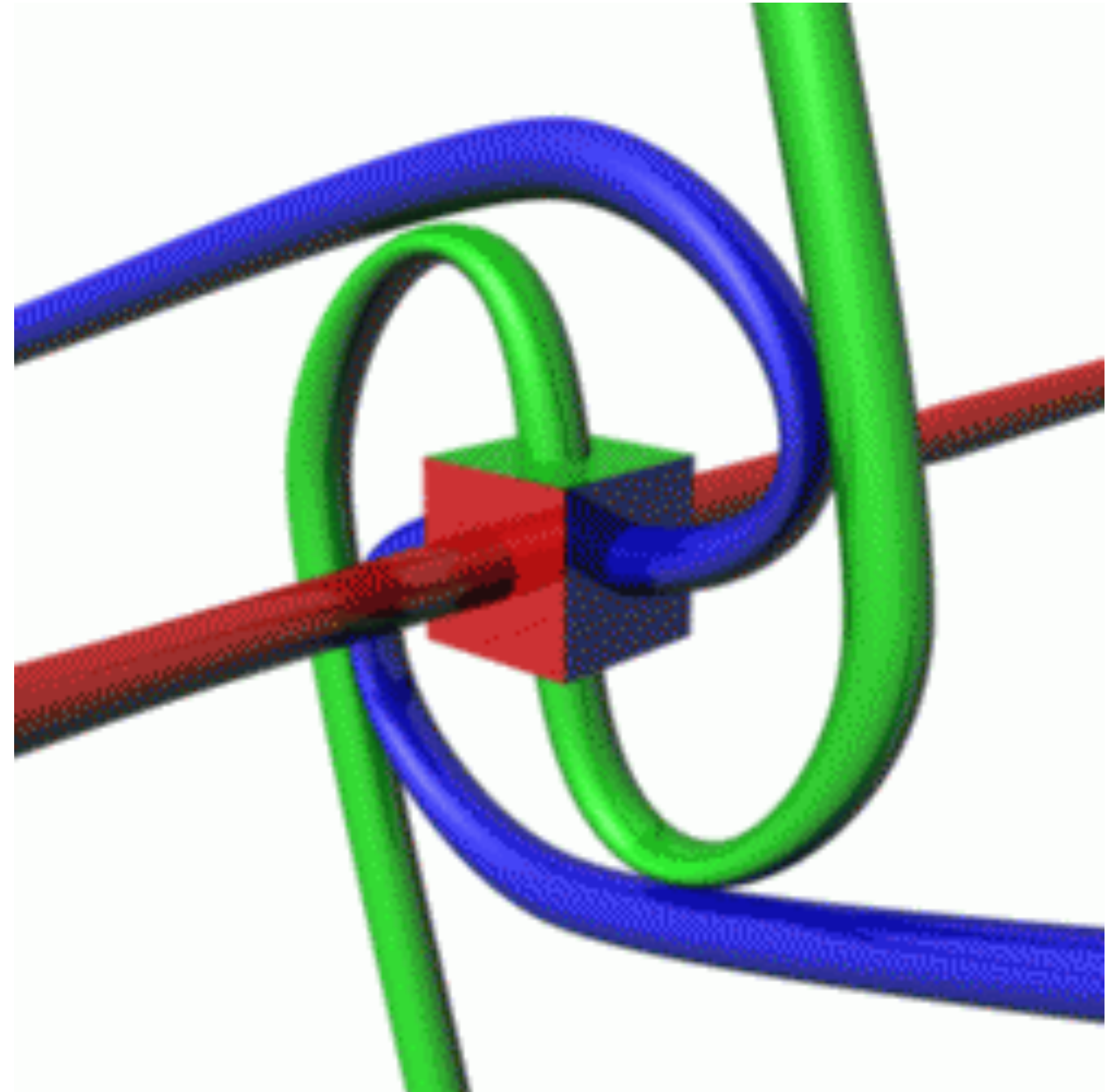
- in classical picture the atoms with perfectly aligned moments are deflected the most
- most of them would be somewhere in the middle because of their random orientation
- However, this is not what is observed, the observed spots where the atoms hit the screen are the two most extreme deflections



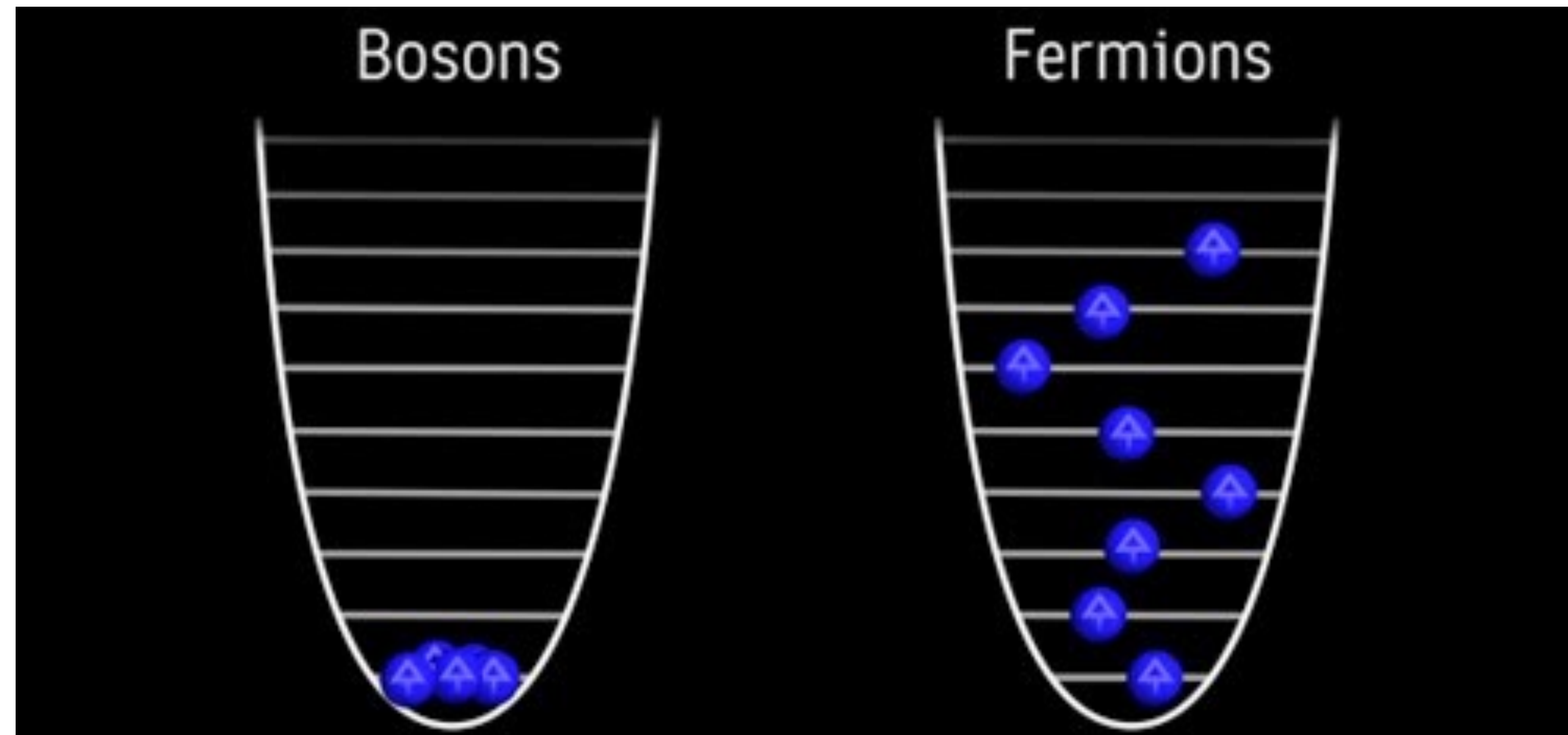
# Stern-Gerlach experiment

- if we add another set of magnets but rotated to make magnetic field horizontal, and let the beam of atoms through it, classical dipoles oriented with 90 degrees to the field, would not feel a force on them
- However the silver atoms again land on the screen in two maximally distanced positions, but rotated horizontally
- Direction of magnetic moment of the electrons is fundamentally quantum
- Direction of the property of the spin is quantised - it takes only specific values
- That direction depends on the direction in which we choose to measure it

- Spin of particles like an electron, that is  $1/2$  is in quantum mechanics described by spinors
- We can think of it as an object that needs to be rotated two times for 360 degrees in order for it to come to original state
- One can imagine the cube attached to walls by ribbons, we need to rotate it two times for it to come to original position



- Another property of the spin  $1/2$  particles is Pauli's exclusion principle
- If we have particles with spin  $1/2$  we cannot put two same particles in one state
- The particles of integer spin, for example spin  $0,1,2$  are called bosons, and we can put arbitrarily many such particles in the same state



# Gauge theory

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The name comes from the rod used to measure railways.

It is not important the size of the gauge as long as the same size is used along the railways.

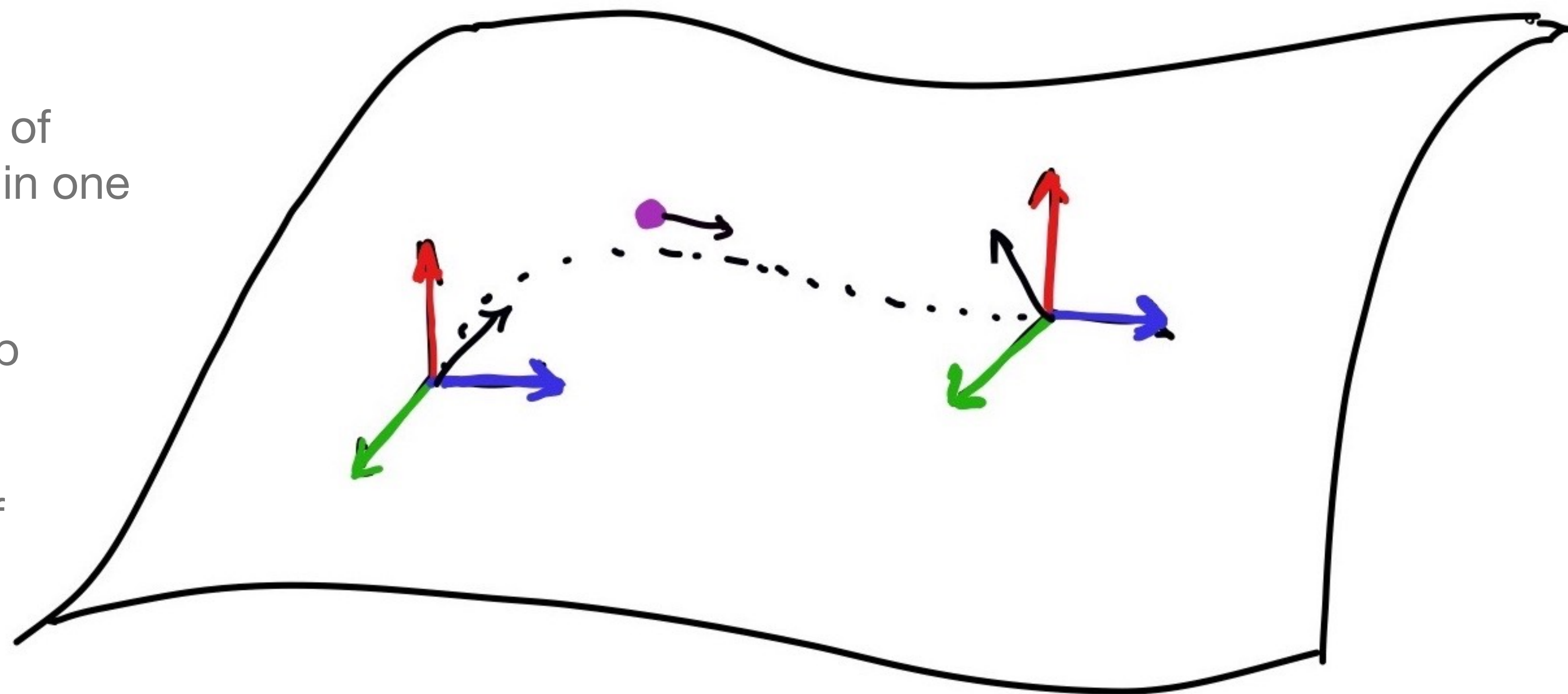
It was later used by Hermann Weyl to describe importance of scale in physics



# Gauge theory

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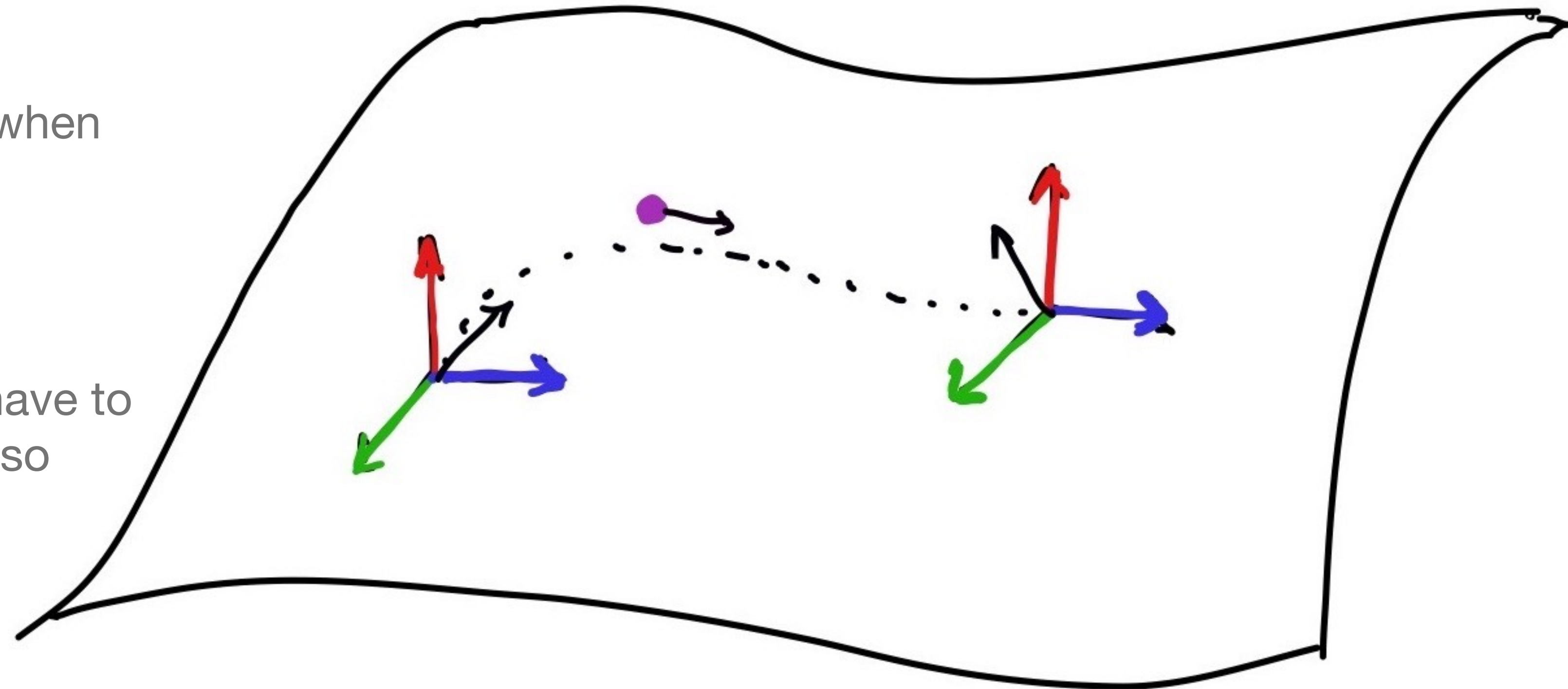
- Lets say that we have a particle, one of those quarks, for example up quark in one position on the manifold
- On another place we have another up quark
- We want to examine their property of color
- We draw the color space as axis in direction of red, green and blue
- The quark is given as a vector in this space
- We can set vectors pointing in the same direction and say two quarks are equal, which is a global property



# Gauge theory

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- We can also say that they are equal when each of them rotates locally
- This is where gauge comes into play
- To compare it to another quark, we have to transport one vector along the path, so that the vector keeps its direction
- To do that we need a connection
- The role of the connection field is played by a gauge field, which is in this case gluon
- The symmetry that allows us to do that is SU(3) symmetry
- All quarks have SU(3) symmetry



- They are invariant in the SU(3) color space
- **Quantum chromodynamics (QCD)**
- **Strong force**

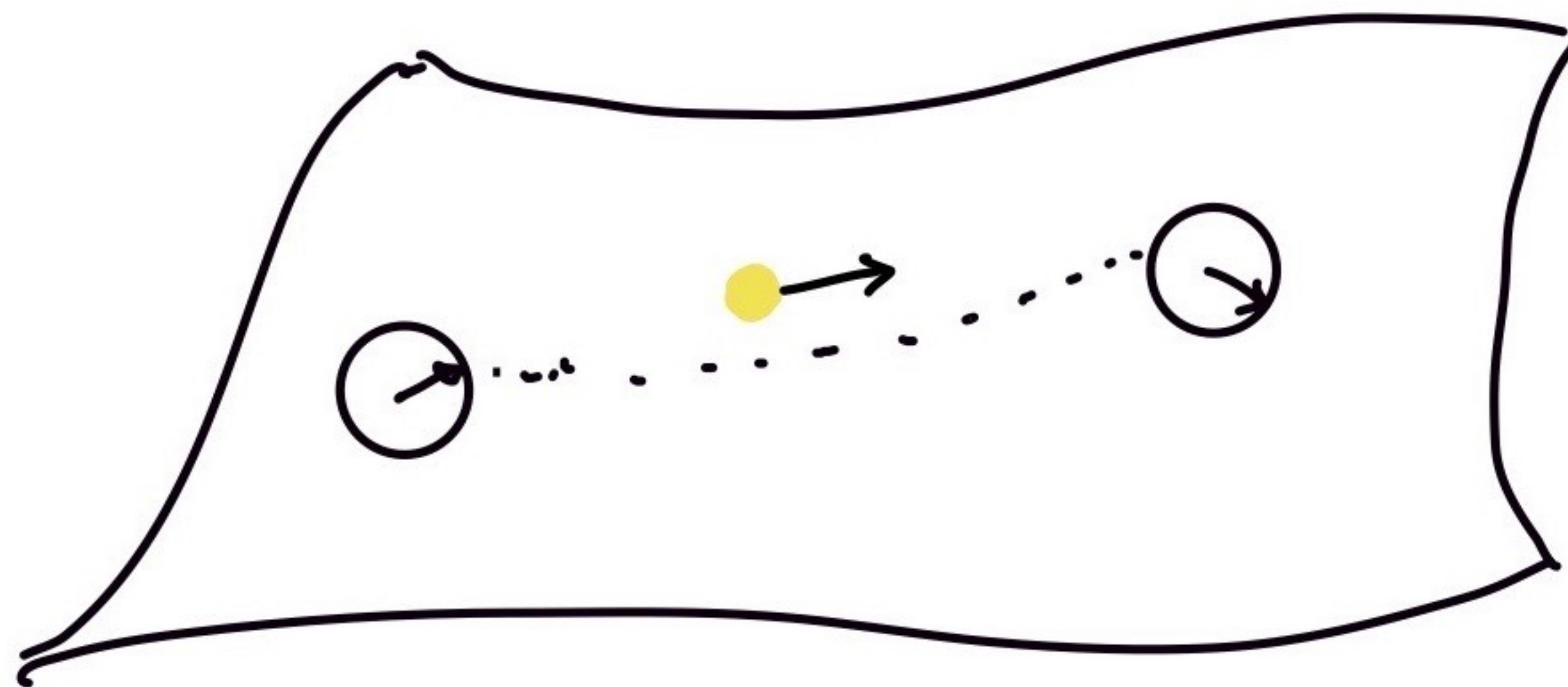
# Gauge theory

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- **Electromagnetism** (19th century)
- Second example are electrons  $\psi_{e^-}$
- If we have two electrons at two points in space we want to compare to see if they are the same
- They are described by two complex numbers
- There is U(1) symmetry

$$\psi_{e^-} \rightarrow e^{i\theta} \psi_{e^-}$$

- U(1) rotation in the complex plane

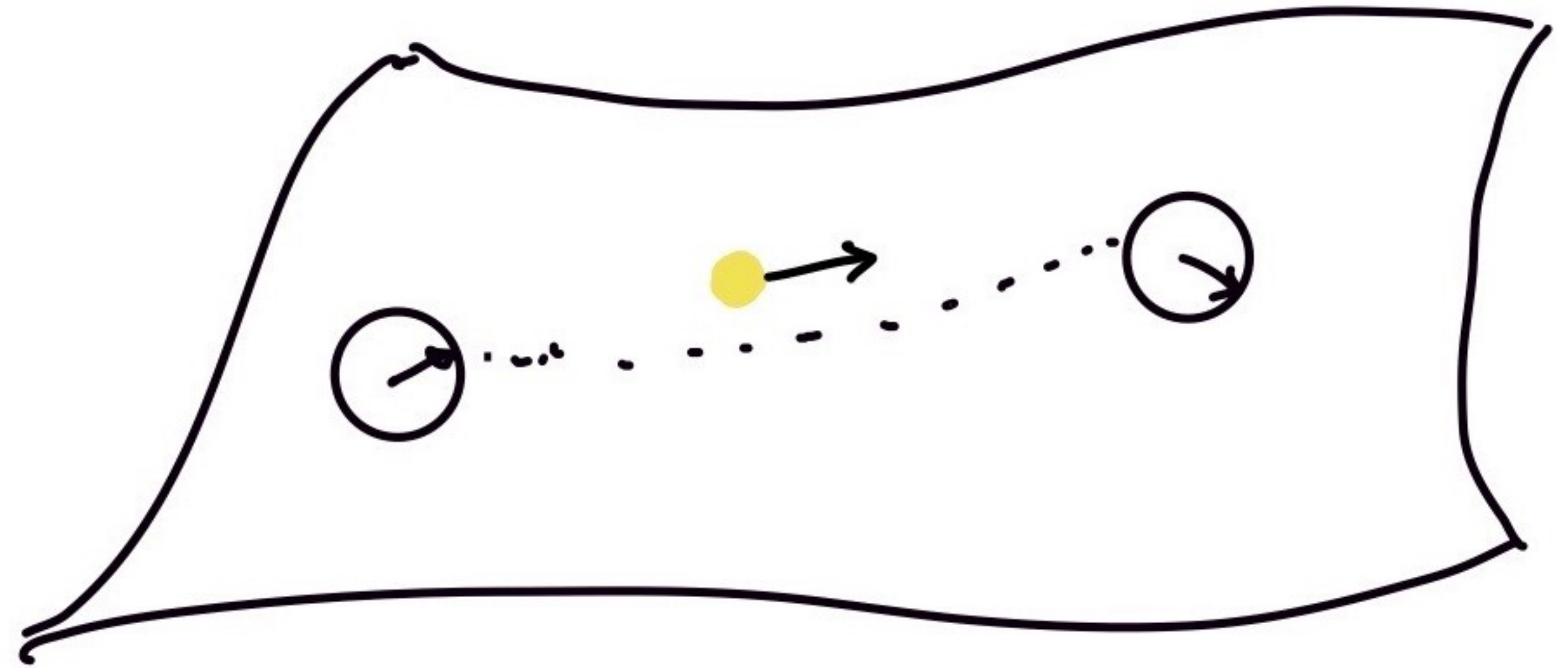




# Gauge theory

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- electron is a complex field which I rotate by an angle  $\theta$
- I can rotate it separately at every point in space  $\theta$
- Whole theory is symmetric invariant under
- $\theta$  is not physical but implies existence of photon field  $\gamma(x)$
- **$\gamma(x)$  photon field is a connection in the  $U(1)$  symmetry of the complex rotation of the electron field**
- that is the reason we have electromagnetism

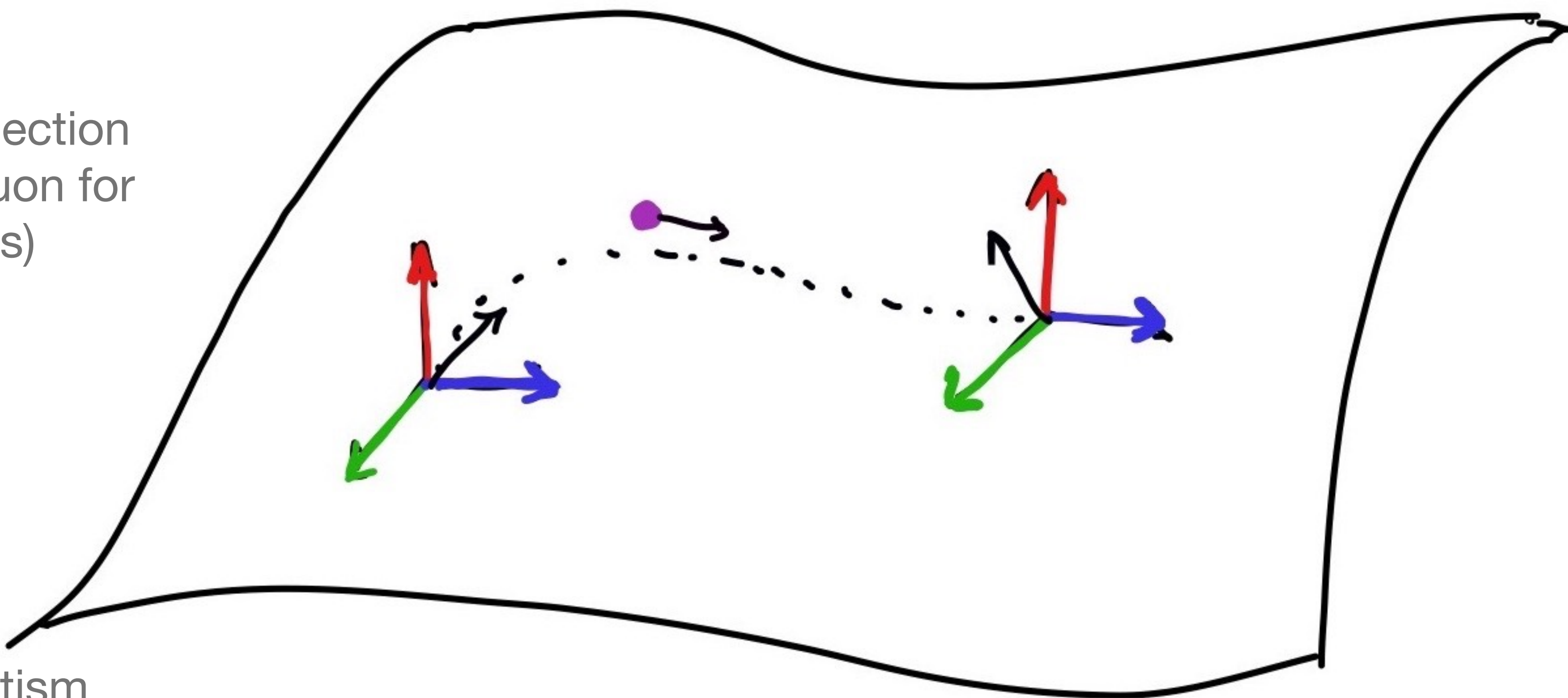


- there is a gauge symmetry, to parallel transport the electron from one point to another, we need a connection which is a separate field (photon) with its own dynamics

# Gauge theory

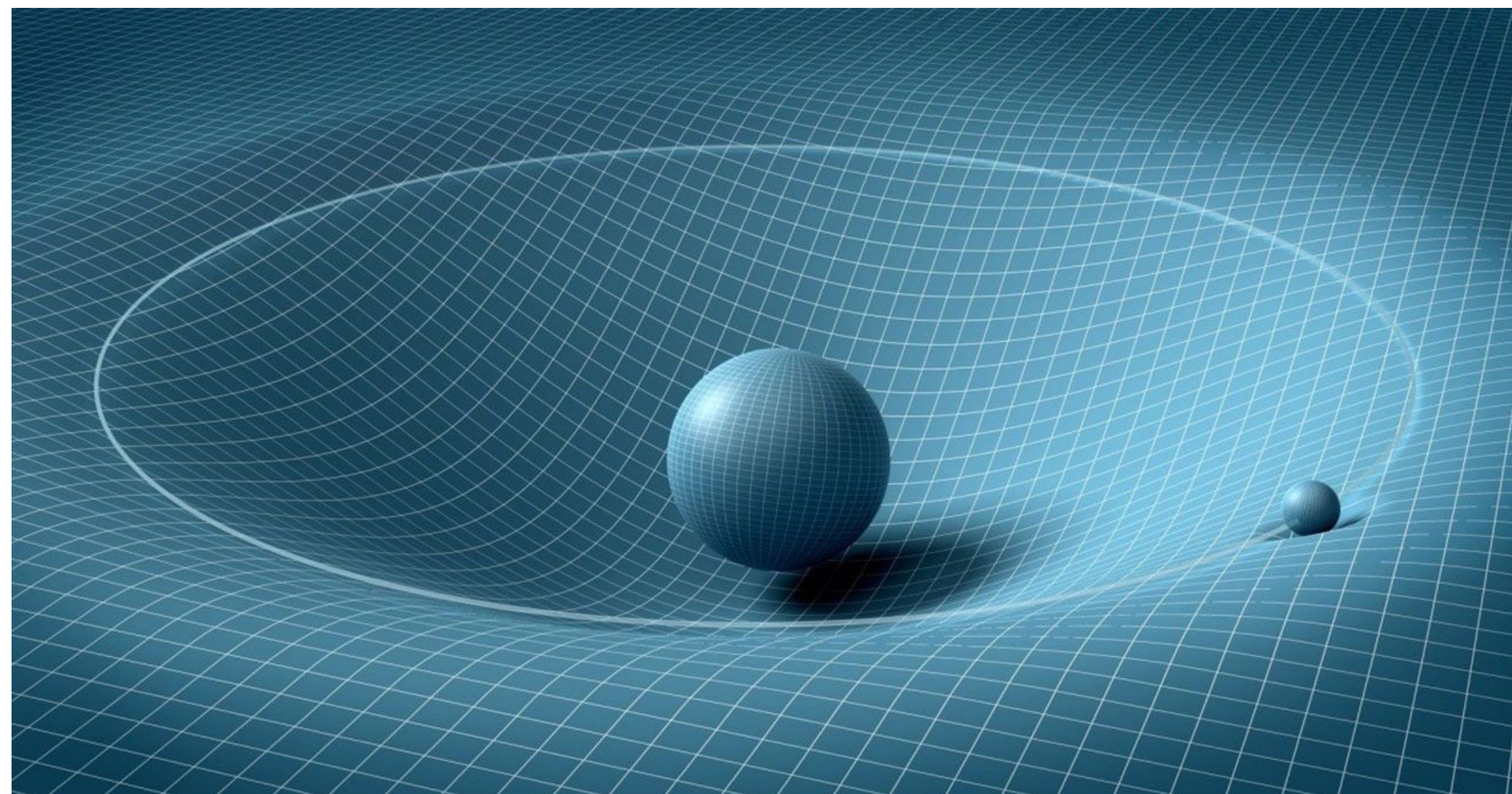
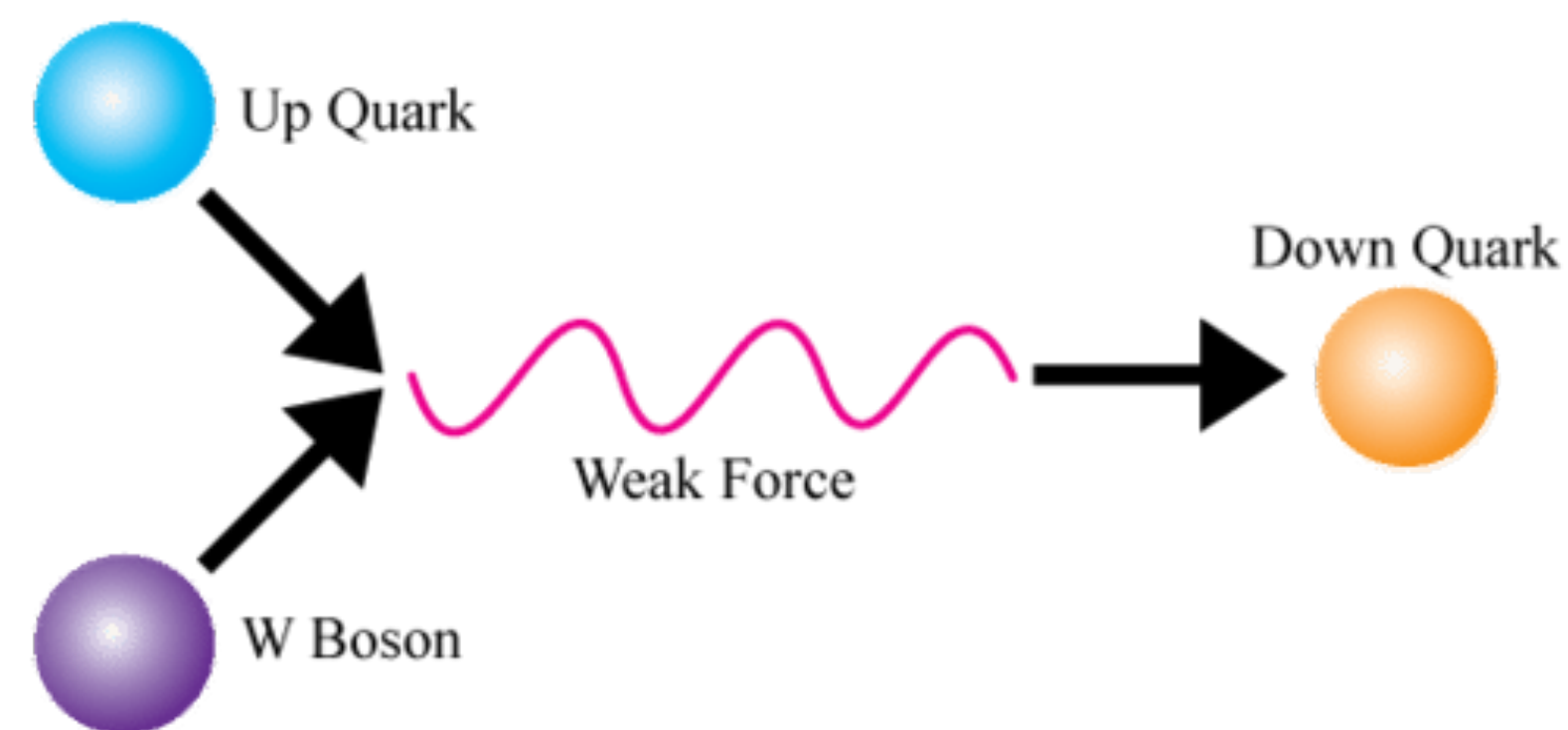
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- **Strong force** in this picture has connection field gluon (there is 8 of them, one gluon for every different way I can rotate quarks)
- It is a consequence of SU(3) color symmetry
- Specific version of Yang-Mills theory
- In 1950-is Chen Ning Yang and Robert Mills compared electromagnetism and said in higher dimension there should be these non-Abelian theory (the order how we do the rotations is important)
- internal symmetry



# Gauge theory

- **Weak force**
- rotations of  $\begin{pmatrix} u \\ d \end{pmatrix}$ ,  $\begin{pmatrix} e \\ \nu_e \end{pmatrix}$
- SU(2) symmetry: gauge fields are gauge bosons W and Z, the symmetry is broken
- **Gravity**
- idea came from Ryoyu Utiyama in 60-s, soon after YM proposed gauge theory
- gauge group could be SO(3,1): rotations+boosts, called Lorentz group
- SO(3,1) - because in 4 dimensional spacetime we have 3 spacetime directions and 1 time direction



# Gauge theory

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- We have also translations
- Poincaré group=translations+SO(3,1)
- its best to think of gravity as gauge theory of Poincaré group
- gravity is special, transformations of spacetime itself
- gauge boson of gravity is **graviton**
- it has spin 2, and it is massless
- its waves are gravitational waves
- when we want to quantise it there are infinities
- Existence of gauge symmetries implies connection fields which give rise to physical particles
- Gauge bosons are physical excitations from vibrations in the group's gauge field
- Existence of these fields giving rise to interactions, giving rise to forces of nature, comes from a gauge symmetry
- Existence of these symmetries places a constraint on what the theory is

# Action

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- Action describes change of physical system in time
- In the simplest example we have a mass  $m$  moving with velocity  $v$  in direction it moves  $S = m\vec{v}\vec{r}$
- In general action is an integral over Lagrange density

$$S = \int (L_{kinetic} + L_{potential} + L_{interaction}) d^4x$$

$$L_{total}(\phi(x), \partial\phi(x))$$

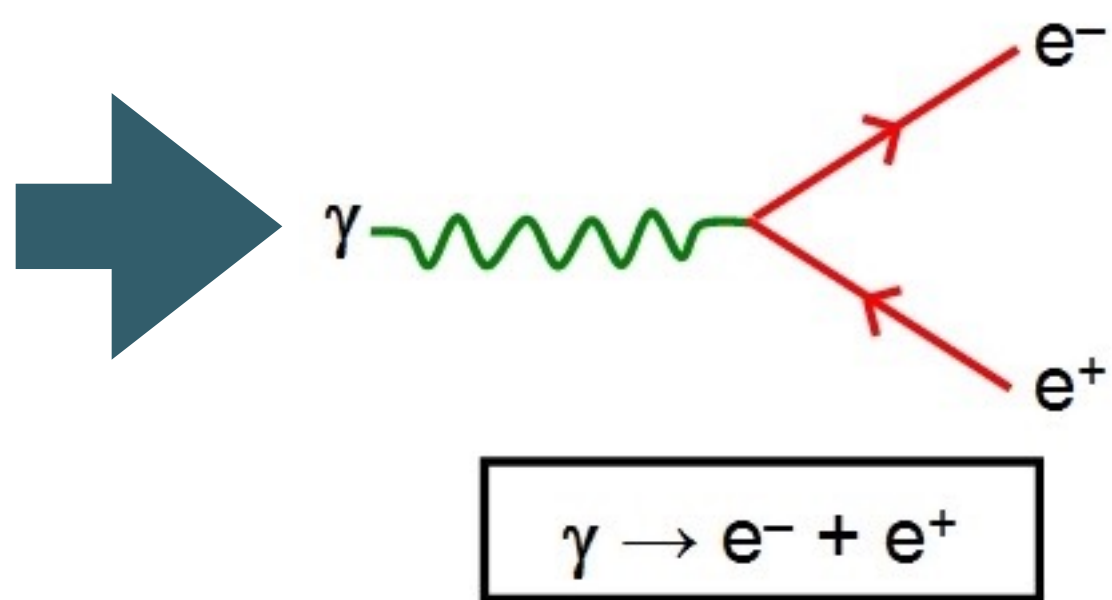
- Action needs to be invariant under these symmetries
- From the action one obtains equations of motion of particles

- Let's take a look at the simplest example of **electromagnetism**
- Usually the terms together need to be invariant under  $\psi_{e^-} \rightarrow e^{i\theta} \psi_{e^-}$
- Kinetic energy of a gauge (connection) field comes from the EM curvature: E (electric) and B (magnetic) fields
- Potential term - we can try to guess to be of the form:  $(\psi_{e^-})^2$
- This is not gauge invariant:  $(\psi_{e^-})^2 \rightarrow e^{2i\theta} (\psi_{e^-})^2$

• We need

$$|\psi_{e^-}|^2 = \boxed{\psi_{e^-}^* \psi_{e^-}} \rightarrow e^{-i\theta} \psi_{e^-}^* e^{i\theta} \psi_{e^-} = \boxed{|\psi_{e^-}|^2}$$

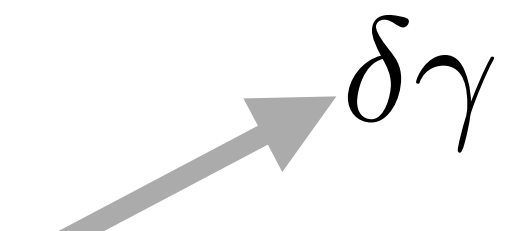
positron ←
electron →
mass term →

- interaction term  $\psi_{e^-}^* \psi_{e^-} \gamma$   conservation of charge  

$\gamma \rightarrow e^- + e^+$

- potential term for photon  $\gamma$ ?

- U(1) gauge transformations:  $\gamma(x) \rightarrow \gamma(x) + \partial\theta$



- $\gamma^2$  is not invariant under U(1) gauge transformation, we can't write it
- there is no mass term, photon is massless

# Higher spin theory

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- It has a long history, around 80 years, in which it had to overcome important conceptual issues
- Do we observe higher spin particles in nature?
- Yes, as a hadron resonances with spin smaller than  $1\frac{1}{2}$
- The idea first appeared when Ettore Majorana in 1932 proposed a field equation which describes infinite tower of HS fields with inverse mass - spin dependence
- In 1936 Paul Dirac considered equations for a single free field of arbitrary half integer spin



# Higher spin theory

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- Markus Fierz and Wolfgang Pauli in 1939 considered spin  $3/2$  particle, but they had difficulties coupling to electromagnetism, they almost discovered supergravity
- To solve infinities that appear when quantising gravity people thought of supergravity, introducing gravitino of spin  $3/2$ , needs  $2/3$  of turn to get back to itself
- supplementing graviton with gravitino also does not work
- one possibility to solve problem of quantising gravity is to add particles beyond spin 2
- Once you go beyond spin 2 you need all spins,  $3,4,5,6,\dots$
- All these particles are also predicted by string theory, but in the string theory they are massive

# Higher spin theory

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- In 60-s there were problems and no-go theorems when looking at higher spin fields on flat space (Weinberg, Coleman, Mandula)
- To better understand the theory of fundamental particles, we have seen that we need to probe it at higher and higher energies
- And at very high energies particles look like massless
- This suggested in the 80-s that we should look extremely high, ultra-Planckian energies, when all these higher spin particles are massless

# Higher spin theory

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- Higher spin gravity theory is study of these highly energetic higher spin particles which are massless
- It is exotic extension of general relativity
- Christian Fronsdal was first to describe bosonic free massless symmetric higher spin fields in 1978
- Fang and Fronsdal did the same for fermionic fields

# Higher spin theory

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- example 1. Electromagnetism

- field equations:  $f(\partial\partial\gamma) = 0$

- gauge symmetry:  $\delta\gamma = \partial\theta$

- example 2. linearised gravity

- field equations:  $f(\partial\partial h_{\mu\nu}) = 0$

- gauge symmetry:  $\delta h_{\mu\nu} = \partial_\mu\xi + \partial_\nu\xi$

# Higher spin theory

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- Particle of arbitrary spin

- field equations:  $f(\partial\partial\phi_{\mu_2\dots\mu_s}) = 0$

- gauge symmetry:  $\delta\phi_{\mu_1\dots\mu_s} = \partial_{(\mu_2}\xi_{\mu_2\dots\mu_s)}$

- gauge invariance also requires:  $\xi_{\mu_1\dots\mu_{s-3}}\lambda^\lambda = 0$

# Higher spin theory

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- First people looking of higher spin particle were Ginsburg (spin  $3/2$  particle)
- Student of Ginsburg, Efim S. Fradkin - proposed a theory of interacting particles of spin higher than 2 which are massless
- His students Mikhail Vasiliev, Arkady Tseytlin, Rusland Metsaev continued the study
- In the 90-s Vasiliev wrote equations of motion for higher spin fields in AdS space

# Higher spin theory

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- Some of the underlying problems that we are still trying to solve is what is the underlying geometric structure
- great success of GR was that it could provide geometric view on gravity
- It was unification of mathematical view invented by Riemann and other geometers that all objects regardless of which mass “fall” in the same way with same acceleration
- Something similar should exist for higher spin theories
- Second point is how to go from such massless to massive particles
- These massive particles should be at very high energy, so something that is still missing is some sort of Higgs mechanism

# Summary

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- We have seen how which are the elementary particles today known in nature and their properties
- These properties are mass, charge and spin
- Spin can be thought of as quantum angular momentum intrinsic to the particle
- The particles are described by the gauge theory
- To get consistent description of the gauge theory that involves gravity we have to look at the particles of all spins
- It appears in string theory, and it can give string theory after it's symmetry breaking



Thank you for the attention!

There will be future activities until July:

<https://indico.cern.ch/event/1085701/page/23439-outreach>