# Higher Spin Lifshitz Holography

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*Upcoming Work*, MG, D Grumiller, S Prohazka, S J Rey
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Der Wissenschaftsfonds.

## Outline

# Motivation

- Higher Spin Holography with Lifshitz Scaling
- Role of Geometry in Higher Spin Theories
- Understanding the Mechanisms of Holography

# 3D Gravity

- Einstein-Hilbert action with negative cosmological constant

$$S = \frac{1}{16\pi G_N} \int_{\mathcal{M}} \sqrt{-g} \left( R + \frac{2}{\ell^2} \right)$$

- Vacuum solution is $AdS_3$

$$ds^2_{AdS} = \ell^2 \left( - \cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\phi^2 \right)$$

- All solutions are locally $AdS_3$
- BTZ Black Hole

$$\begin{aligned} ds^2_{\mathrm{BTZ}} = \ell^2 \Big[ & d\rho^2 - \left( e^{2\rho} + 16(M^2 - J^2) \right) dx^+ dx^- \\ & + 4(M - J)(dx^+)^2 + 4(M + J)(dx^-)^2 \Big] \end{aligned}$$

# Chern-Simons Formulation

- Gravity in Asymptotically $AdS_3$ can be formulated as $\mathfrak{sl}_2(\mathbb{R}) \oplus \mathfrak{sl}_2(\mathbb{R})$ Chern-Simons theory with level $k = \frac{1}{4G_N}$

$$S = \frac{k}{4\pi}\left(S_{\mathrm{CS}}\left[A\right] - S_{\mathrm{CS}}\left[\overline{A}\right]\right)$$

$$S_{\mathrm{CS}}\left[A\right] = \int_{\mathcal{M}} \mathrm{tr}\left(\mathrm{A} \wedge \mathrm{dA} - \frac{2}{3}\mathrm{A}^3\right)$$

where

$$e = \frac{\ell}{2}\left(A - \overline{A}\right) \qquad \omega = \frac{1}{2}\left(A + \overline{A}\right)$$

- Gauge transformations $\delta_\epsilon A = d\epsilon + [\epsilon, A]$, $\delta_{\overline{\epsilon}}\overline{A} = d\overline{\epsilon} + \left[\overline{\epsilon}, \overline{A}\right]$
- Diffeomorphisms generated by $\xi^\mu$ are given by

$$\epsilon = \xi^\mu A_\mu \qquad \overline{\epsilon} = \xi^\mu \overline{A}_\mu$$

# Brown-Henneaux Boundary Conditions

- Denote $\mathfrak{sl}_2$ generators by $L_0, L_{\pm 1}$
- Convenient to partially gauge fix

$$A = g^{-1} dg + g^{-1} a g \quad \overline{A} = g dg^{-1} + g \bar{a} g^{-1} \quad g = e^{\rho L_0}$$

- Impose Asymptotic AdS boundary conditions

$$\begin{aligned} a &= \left(L_1 + \mathcal{L}(x^+) L_{-1}\right) dx^+ + o(1) \\ \bar{a} &= \left(L_{-1} + \overline{\mathcal{L}}(x^-) L_1\right) dx^- + o(1) \end{aligned}$$

- Solutions include AdS, BTZ black holes, more

$$ds^2 = \ell^2 \left[ d\rho^2 - \left(e^{2\rho} + e^{-2\rho} \mathcal{L}\overline{\mathcal{L}}\right) dx^+ dx^- + \mathcal{L}(dx^+)^2 + \overline{\mathcal{L}}(dx^-)^2 + \cdots \right]$$

# Canonical Analysis

- Locally, all solutions are flat, so gauge equivalent to the vacuum
- At the asymptotic boundary, some first class constraints become second class, and thus generate new states, rather than gauge transformations
- Asymptotic Symmetry Algebra is two copies of Viraosoro with $c_L = c_R = 6k$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n^3\delta_{n,-m}$$

- CFT vacuum defined by $L_n |0\rangle = 0$ for all $n \geq -1$ (similar for barred sector)
- States generated by $L_{n_1} \cdots L_{n_m} |0\rangle$ for $n_i < -1$, called boundary gravitons

## Symmetries & Vacuum

- AdS solution preserves 3+3 symmetries corresponding to the wedge algebra of the ASA, $\mathfrak{sl}_2 \oplus \mathfrak{sl}_2$, and is thus identified with the CFT vacuum
- All solutions locally preserve 3+3 symmetries, since all solutions locally flat, but global realization is such that they excite infinite numbers of charges, not the wedge algebra

# Higher Spin Generalization

- Enlarge $\mathfrak{sl}_2$ to $\mathfrak{sl}_N$
- Choice of embedding $\mathfrak{sl}_2 \hookrightarrow \mathfrak{sl}_N$ determines other field content
- Spins of other fields given by weight under gravitational $\mathfrak{sl}_2$ action
- Typical choice: Principal embedding, integer spins $2, \ldots, N$.

$$
\begin{aligned}
g_{\mu\nu} &= \frac{1}{2}\mathrm{tr}\left[\mathrm{e}_\mu \mathrm{e}_\nu\right] \\
\phi_{\mu\nu\rho} &= \mathrm{tr}\left[\mathrm{e}_{(\mu}\mathrm{e}_\nu \mathrm{e}_{\rho)}\right] \\
&\vdots
\end{aligned}
$$

# $\mathfrak{sl}_3$ Conventions

- $\mathfrak{sl}_2$ generators $L_0, L_{\pm 1}$
- Spin 3 generators $W_0, W_{\pm 1}, W_{\pm 2}$
- Commutators

$$[L_n, L_m] = (n-m)L_{n+m} \qquad [L_n, W_m] = (2n-m)W_{n+m}$$

$$[W_n, W_m] \propto L_{n+m}$$

- Traces

$$\mathrm{tr}\left(\mathrm{L_n L_m}\right) \propto \delta_{n,-m} \qquad \mathrm{tr}\left(\mathrm{W_n W_m}\right) \propto \delta_{n,-m}$$

$$\mathrm{tr}\left(\mathrm{L_n W_m}\right) = 0$$

# AdS Boundary Conditions

$$A = g^{-1}dg + g^{-1}ag \qquad \overline{A} = gdg^{-1} + g\bar{a}g^{-1} \qquad g = e^{\rho L_0}$$

$$a = \left(L_1 + \mathcal{L}(x^+)L_{-1} + \mathcal{W}(x^+)W_{-2}\right) dx^+ + o(1)$$
$$\bar{a} = \left(L_{-1} + \overline{\mathcal{L}}(x^-)L_1 + \overline{\mathcal{W}}(x^-)W_2\right) dx^- + o(1)$$

- Asymptotic Symmetry Algebra: two copies of $\mathcal{W}_3$ with central charges $c_L = c_R = 6k$
- Vacuum: metric is AdS$_3$, spin-3 field is 0, invariant under $\mathfrak{sl}_3 \times \mathfrak{sl}_3$ symmetry

# General Procedure

- Add boundary term to cancel variation of the action

$$S_{\mathrm{CT}} = -\frac{k}{4\pi}\int_{\partial\mathcal{M}} \mathrm{tr}\left(\mathrm{A}^2 - \overline{\mathrm{A}}^2\right)$$

- Split connection into background and fluctuations
- Impose consistent boundary conditions on fluctuations. In particular
  - Find closed set of boundary condition preserving gauge transformations
  - Require finite, conserved, integrable asymptotic charges
- Determine Asymptotic Symmetry Algebra by computing Poisson Brackets and quantizing

# Examples

- Lobachevsky boundary conditions

$$\widehat{ds}^2 = d\rho^2 \pm dt^2 + \sinh^2 \rho d\phi^2$$

$\mathcal{W}_N^{(2)} \times \hat{\mathfrak{u}}(1)$ Asymptotic Symmetry Algebra

[1209.2860], [1211.4454]

- Null-Warped $AdS_3$ boundary conditions

$$\widehat{ds}^2 = d\rho^2 + e^{2\rho} dt d\phi + \frac{9}{4} e^{4\rho} d\phi^2$$

$\mathcal{W}_3^{(2)}$ Asymptotic Symmetry Algebra

*(Unpublished, with E Perlmutter and D Grumiller)*

# Lifshitz Geometry

- Lifshitz geometries are dual to Lifshitz field theories, which feature anisotropic scaling between space and time with a relative factor $z$

$$t \to \lambda^z t \qquad\qquad x \to \lambda x$$

- Metric

$$\begin{aligned} ds_z^2 &= \ell^2 \left( -r^{2z} dt^2 + \frac{dr^2}{r^2} + r^2 dx^2 \right) \\ &= \ell^2 \left( -e^{2z\rho} dt^2 + d\rho^2 + e^{2\rho} dx^2 \right) \end{aligned}$$

- Isometries and Lifshitz Algebra

$$\xi_{\mathbb{H}} = \partial_t \qquad \xi_{\mathbb{P}} = \partial_x \qquad \xi_{\mathbb{D}} = -zt\partial_t + \partial_\rho - x\partial_x$$

$$[\xi_{\mathbb{H}}, \xi_{\mathbb{P}}] = 0 \qquad [\xi_{\mathbb{D}}, \xi_{\mathbb{H}}] = z\xi_{\mathbb{H}} \qquad [\xi_{\mathbb{D}}, \xi_{\mathbb{P}}] = \xi_{\mathbb{P}}$$

# $z = 2$ Lifshitz Background

- Background connection

$$\hat{a} = L_1 dx + \frac{4}{9} W_2 dt$$
$$\hat{\bar{a}} = L_{-1} dx + W_{-2} dt$$

- Background metric

$$ds^2 = \ell^2 \left( -e^{4\rho} dt^2 + d\rho^2 + e^{2\rho} dx^2 \right)$$

- Non-trivial background spin-3 field

$$\phi_{\mu\nu\lambda} dx^\mu dx^\nu dx^\lambda = -\frac{5\ell^3}{4} e^{4\rho} dt (dx)^2$$

# Higher Spin Fluctuations on Lifshitz Background

- Boundary conditions

$$a^{(0)} = \left(4t\mathcal{W}L_0 - \mathcal{L}L_{-1} - \frac{16t^2}{9}\mathcal{W}W_2 + \frac{16t}{9}\mathcal{L}W_1 + \mathcal{W}W_{-2}\right) dx$$
$$\bar{a}^{(0)} = \left(-\overline{\mathcal{L}}L_1 - 9t\overline{\mathcal{W}}L_0 + \overline{\mathcal{W}}W_2 + 4t\overline{\mathcal{L}}W_{-1} - 9t^2\overline{\mathcal{W}}W_{-2}\right) dx$$

- Theory includes states with metrics that would not typically be called asymptotically Lifshitz
- On-shell solutions have explicit $t$-dependence

# Asymptotic Symmetry Algebra

- Asymptotic charges are $\mathcal{L}(x), \mathcal{W}(x), \overline{\mathcal{L}}(x), \overline{\mathcal{W}}(x)$, $t$-independent
- Asymptotic Symmetry Algebra: two copies of $\mathcal{W}_3$ with central charges $c_L = c_R = 12k\mathrm{tr}(\mathrm{L}_0)^2 = \frac{3\ell}{2\mathrm{G_N}}$

$$\delta_{\epsilon_L}\mathcal{L} = \mathcal{L}'\epsilon_L + 2\mathcal{L}\epsilon_L' - \frac{k}{\pi}\epsilon_L^{(3)}$$
$$\delta_{\epsilon_L}\mathcal{W} = \mathcal{W}'\epsilon_L + 3\mathcal{W}\epsilon_L'$$
$$\delta_{\epsilon_W}\mathcal{L} = 2\mathcal{W}'\epsilon_W + 3\mathcal{W}\epsilon_W'$$
$$\delta_{\epsilon_W}\mathcal{W} = \left(\frac{3\pi}{k}\mathcal{L}\mathcal{L}' - \frac{3}{8}\mathcal{L}^{(3)}\right)\epsilon_W + \left(\frac{3\pi}{k}\mathcal{L}\mathcal{L} - \frac{3}{8}\mathcal{L}^{(2)}\right)\epsilon_W' - \frac{45}{16}\mathcal{L}'\epsilon_W'' - \frac{15}{8}\mathcal{L}\epsilon_W^{(3)} + \frac{3k}{16\pi}\epsilon_W^{(5)}$$

## Symmetries of the Background

- Background is invariant under $8+8$ linearly independent gauge transformations of the form

$$\epsilon_L = l_{+1} - x l_0 + x^2 l_{-1}$$
$$\epsilon_W = w_{+2} - x w_{+1} + x^2 w_0 - x^3 w_{-1} + x^4 w_{-2}$$
$$\epsilon_{\overline{L}} = \overline{l}_{-1} - x \overline{l}_0 + x^2 \overline{l}_{+1}$$
$$\epsilon_{\overline{W}} = \overline{w}_{-2} - x \overline{w}_{-1} + x^2 \overline{w}_0 - x^3 \overline{w}_{+1} + x^4 \overline{w}_{+2}$$

- Special case: Lifshitz isometries

$$\xi_{\mathbb{H}}: \qquad w_{+2} = \frac{4}{9} \qquad \overline{w}_{-2} = 1$$
$$\xi_{\mathbb{P}}: \qquad l_{+1} = 1 \qquad \overline{l}_{-1} = 1$$
$$\xi_{\mathbb{D}}: \qquad l_0 = 1 \qquad \overline{l}_0 = 1$$

# Symmetries & Global Structure

- Symmetries of the background enhanced to the full wedge algebra $\mathfrak{sl}_3 \times \mathfrak{sl}_3$, thus background is dual to the CFT vacuum on the plane
- All states locally have $8 + 8$ symmetries, but globally realized non-polynomially, leading to infinite towers of non-trivial charges
- No other states are invariant under precisely the complete wedge algebra

# Interpretations and Duality

- Perturbative spectrum is equivalent to that of spin-3 gravity in $AdS_3$: duality?
- Unclear if duality holds also at non-perturbative level
- Polynomial $x$ and $t$ dependence of boundary conditions and boundary condition preserving gauge transformations imply boundary should be non-compact

## Conclusions

- Also looser boundary conditions for Lifshitz background, possibly related to $\mathcal{W}_3^{(2)}$
- Are there other backgrounds which are also related by duality?
- Do such dualities exist for more general higher spin theories?
- Metric and higher spin fields need to be placed on equal footing—all massless degrees of freedom
- To really talk about geometry, we should use a local probe (e.g. scalar field in $HS(\lambda)$ theory)

## Thank You